COMPARISON OF ANALYTICAL AND NUMERICAL RESULTS FOR BROADBAND COUPLING IMPEDANCE*

L. Hänichen **, Wolfgang F. O. Müller, Thomas Weiland,
Technische Universität Darmstadt, Institut für Theorie Elektromagnetischer Felder,
Schlossgartenstraße 8, 64289 Darmstadt, Germany,

Ahmed Al-Khateeb
Yarmouk University, Department of Physics,
Irbid, Jordan

Oliver Boine-Frankenheim,
GSI-Darmstadt, FAIR-Accelerator Theory Group,
Planckstraße 1, 64291 Darmstadt, Germany

Abstract

Beam coupling impedances have been first introduced at CERN [1] and have been found as an appropriate quantity to describe collective instabilities caused through beam-induced fields in heavy ion synchrotron accelerators such as the SIS-18 and the planned SIS-100 at the GSI facility. The impedance contributions caused by the multiple types of beamline components need to be determined to serve as input condition for later stability studies. This paper will present an approach exploiting the abilities of commercial FDTD wake codes such as CST PARTICLE STUDIO® for a benchmark problem with cylindrical geometry. Since exact analytical formulae are available, the obtained numerical results will be compared. Special attention is paid towards the representation of the particle beam as the source of the EM fields and conductive losses.

INTRODUCTION

For the prediction of beam instabilities tracking codes can be used where coupling impedances are taken into account as accumulated kick per turn. Especially for geometry where the coupling impedance can not be determined analytically, numerical 3D codes have to be used. The objective of this approach is to calculate the coupling impedance for the case of resistive pipe and cross-check with the analytical results. The paper is organized as follows: First the underlying definitions for coupling impedance and the source terms for beam excitation will be given and analytical formulations for particular cases will be summarized according to references. Then simplifications will be taken into account that relate to numerical modeling and time domain wakefield methods. Finally simulation results will be compared with the analytical formulations. The last section will draw conclusions and name issues that require further attention.

* Work supported by GSI under contract DAWEI2.

** haenichen@temf.tu-darmstadt.de

ANALYTICAL APPROACH

Allowing for arbitrary transverse beam current distributions, the coupling impedances are generally defined as follows[2]:

$$Z_{||}(\omega) = \frac{1}{q^2}\int d^3x E \cdot J_{beam}, \qquad (1)$$

$$Z_{x,y}(\omega) = \frac{i}{q^2\Delta}\int d^3x \rho_{x,y} \cdot (E_{x,y} \mp vB_{y,x}), \qquad (2)$$

where E is the longitudinal electric field and $E_{x,y}$ and $B_{y,x}$ are the transverse electric and magnetic fields excited by the beam. Lets assume a particle beam in form of a circular lamina of radius a that moves along the z-axis with a constant longitudinal velocity $v = \beta c$, where β is the relativistic factor and c is the vacuum speed of light. After solving the time harmonic wave equation for cylindrical geometry and applying exact field matching, the relevant field component is substituted in the given definition for the coupling impedance and evaluated at $\varrho = a$ which is the surface of the beam.
The total coupling impedance can be split into two parts, namely the space charge impedance $Z_{\|}^{\mathrm{sc}}$ and the resistive wall impedance $Z_{\|}^{\mathrm{rw}}$. In a smooth cylindrical beam-pipe of radius b, the longitudinal space-charge impedance for a beam of uniform transverse charge distribution is [3],[5],

$$Z_{\|}^{\mathrm{sc}}(\omega) = -j\,\frac{nZ_0}{2\beta\gamma_0^2}\,\frac{4\gamma_0^2}{k_z^2a^2}\left[1 - 2I_1^2(\sigma_0 a)F_0\right] \qquad (3)$$

$$F_0 = \frac{K_1(\sigma_0 a)}{I_1(\sigma_0 a)} + \frac{K_0(\sigma_0 b)}{I_0(\sigma_0 b)}\;, \quad \sigma_0 = \frac{k_z}{\gamma_0}\;, \qquad (4)$$

where $Z_0 = \mu_0 c$ is the vacuum impedance, L is the ring circumference, $\gamma_0^{-2} = 1 - \beta^2$ is the relativistic Lorentz factor, k_z is the longitudinal wave vector, and $n = k_z R$ is the harmonic number with R being the ring radius. Here I_α and K_α stand for the α^{th} order modified Bessel functions of first and second kinds, respectively.

For a thick resistive wall of electric conductivity σ_w, the longitudinal resistive-wall impedance is [3],

$$Z_{\parallel}^{\mathrm{rw}}(\omega) = j\frac{nZ_0}{\beta\gamma_0^2\sigma_0 b}\frac{4\left[\frac{I_1^2(\sigma_0 a)}{(\sigma_0 a)^2}\right]}{I_0^2(\sigma_0 b)}\frac{\eta}{\left[1+\eta\frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\right]} \quad (5)$$

$$\eta = \frac{\omega\varepsilon_0\gamma_0}{j\underline{\gamma}\left(\sigma_w - j\omega\varepsilon_0\right)}, \quad \underline{\gamma}^{-2} = \frac{1}{\gamma_0^2} - j\frac{\mu_0\sigma_w\omega}{k_z^2} \quad (6)$$

COMPUTATIONAL MODEL

We want to use the available commercial wakefield codes allowing for reproduceable and interchangeable results. CST PARTICLE STUDIO® has been chosen which is a FDTD wake code based on the Finite Integration Technique [4]. The excitation is processed by a leading gaussian bunch that is passed through the model. The fields seen by a trailing test particle are integrated along the distance yielding the wake potential $W(s)$. The wake potential is the convolution of the charge distribution with the wake function $G(s)$ which is connected to the coupling impedance through Fourier transform. The impedance is given by:

$$Z(\omega) = FT\{G(s)\} = \frac{FT\{W(s)\}}{FT\{\lambda(s)\}}. \quad (7)$$

In the following a circular pipe of conductivity σ_w, length l and radius b is assumed. There are two major differences between the analytical and the numerical approach, being firstly the finite length of the simulated model and secondly the profile which is point-like in the simulation. Depending on the spacing of the transverse mesh, the current is blurred across the corresponding meshcell making the current and charge densities dependent on the meshwidth. The first issue is taken care of by normalizing all length dependent expressions by substituting L by 1 meter and $R = L/(2\pi)$ respectively and dividing the calculated impedances by l. For the second issue we will approximate as a point-like beam and apply the following limit:

$$\lim_{a\to 0} Z_{||}^{rw} = \frac{4inZ_0}{\beta\gamma_0^2\sigma_0 b}\frac{\eta[I_0(\sigma_0 b)]^{-2}}{\left[1+\eta\frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\right]}\underbrace{\lim_{a\to 0}\left[\frac{I_1(\sigma_0 a)}{(\sigma_0 a)}\right]^2}_{\frac{1}{4}}. \quad (8)$$

NUMERICAL RESULTS

We want to consider the longitudinal coupling impedance for an ultra-relativistic beam ($\beta = 1.0$) due to finite wall resistance. The electric field of a free ultra-relativistic particle has only transverse components so Z^{sc} must vanish. The finite conducting walls will introduce longitudinal components that contribute to the coupling impedance. The model consists of a cylindrical vacuum region extending in z-direction with a diameter of 0.02 m and 0.1 m length. The vacuum region is embedded in a block of lossy metal. The mesh step is set to 1 mm in all directions. Due to symmetry of the monopole field, magnetic symmetry planes can be inserted for x-z- and y-z- planes. The bunch charge is set to 10 nC and length is set to $\sigma_z = 0.25$ m. When observing resistive effects, it is mandatory to determine the zero-effect first by using a model with perfect conducting walls. As expected both real and imaginary part are close to zero (order of 10^{-4}). The remaining signal consists of noise and an oscillating component with an increase in amplitude towards higher frequencies. The oscillation is related to the truncation of the time domain calculation by a rectangular window function. The presence of oscillation in the frequency range of interest depends on the choice of the spectra of the exciting bunch and the time frame after which the wakefield is calculated. We now consider wall material with finite conductivity. According $\delta_s = \sqrt{2/\omega\mu_0\sigma_w}$ the skin depth at 1 GHz sec^{-1} is 40 μm. For capturing field penetration into the wall, a rule of thumb suggests to discretize the skin depth with a couple of meshcells.

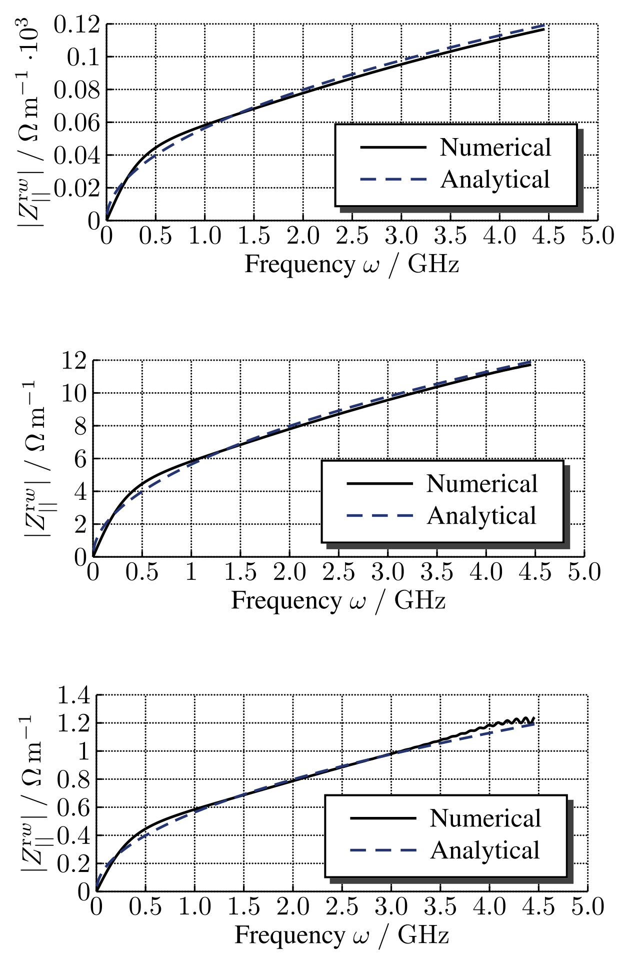

Figure 1: Longitudinal resistive wall coupling impedance for a pipe with circular cross-section, calculated for three different conductivities: 100, 10000 and 1000000 S/m (from top to bottom)

An intrinsic property of the underlying hexagonal mesh is that a refinement by locally enforcing smaller mesh-steps will always lead to a global mesh refinement along the corresponding meshline through its intersections with the orthogonal meshplanes. Having mentioned the mesh structure, the given aspect ratio between transverse dimensions and wall the thickness, it can be seen that this leads to a very large number of meshcells. For this reason an impedance boundary condition has been selected to approximate the resistance of a thick wall. Figure 1 shows the scaling with wall conductivity σ_w. For very good conductors the simulation gets sensitive towards noise that can be observed at the wake function already. The resistive wall impedance matches the with the analytical values in a wide range, especially in the higher frequency range. Figure 2 demonstrates the linear scaling behaviour of the resistive wall impedance. Figure 3 shows a closer view on the lower frequency range. The deviation from the analytical solution could be related to the impedance boundary condition which does not perform well at low frequencies.

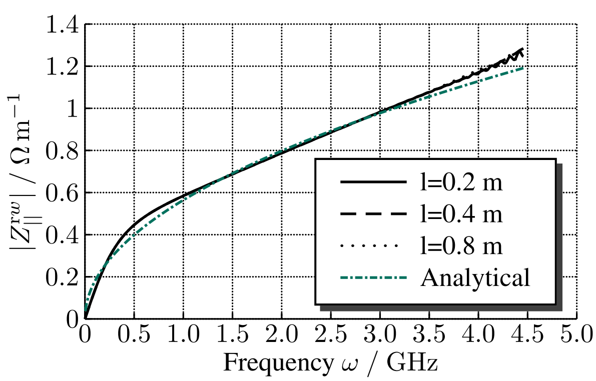

Figure 2: Longitudinal resistive wall coupling impedance for three model lengths: The result values have been normalized to 1 meter.

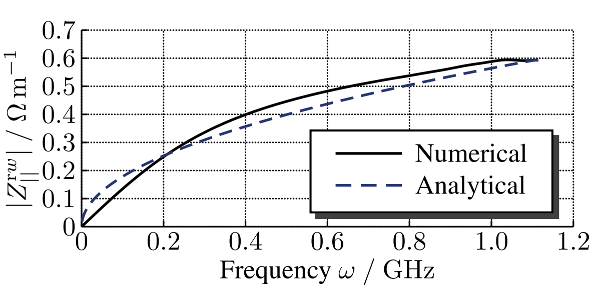

Figure 3: Low frequency range for longitudinal resistive wall coupling impedance ($\sigma_w = 10^6$ S/m).

For improving mechanical stability and ease of manufacturing, a corrugated vacuum chamber has been proposed for SIS-100. Simulations show that if the spectrum of the exciting bunch is increased towards higher frequencies, the first propagable modes will be excited, causing resonant peaks in the impedance. The corrugation pattern increases both the resistive and inductive behaviour which results in an overall increase of the continuous part of the coupling impedance. Figure 4 shows an example for a section of finite conducting corrugated wall.

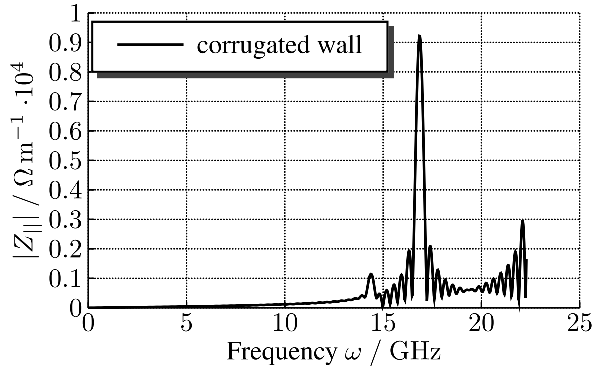

Figure 4: Longitudinal coupling impedance for corrugated wall (diameter 10 cm, corrugation period 6 mm, corrugation amplitude 4 mm).

SUMMARY AND OUTLOOK

It is possible to use FDTD wakefield codes such as CST PARTICLE STUDIO® for the calculation of longitudinal coupling impedances within certain limitations. For capturing effects through penetrating fields in metallic conductors, a very fine mesh resolution is required locally.
This usually leads to high computational cost and requires a different model for finite conducting layers such as surface impedance. These surface impedance models do not perform well in the low frequency range. The need to calculate the coupling impedances for a wide frequency range from kHz to GHz while retaining resolution and accuracy in the low frequency region, requires long simulation periods to shift the bunch frequency spectrum towards lower frequencies. For non-relativistic beam velocities, space charge contributes predominantly to the coupling impedance. The representation of the exciting beam is limited to a single point beam which does not allow for space charge studies connected with the transverse beam profile.

REFERENCES

[1] V.C. Vaccaro, CERN report ISR - RF / 66 - 35 (1966).

[2] R.L. Gluckstern, Analytical Methods for Calculating Coupling Impedances, CERN report 2000-011, p.5 (2000).

[3] A. M. Al-Khateeb, O. Boine-Frankenheim, R. W. Hasse, and I. Hofmann, Phys. Rev. E **71**, 026501-1 (2005).

[4] T. Weiland, R. Wanzenberg, Wake Fields and Impedance, DESY report M-91-06, (1991).

[5] A. M. Al-Khateeb, R.W. Hasse, O. Boine-Frankenheim, W. M. Daqa, and I. Hofmann, Phys. Rev. ST Accel. Beams **10**, 064401 (2007).