FIVE CELL METHOD OF TUNING BIPERIODIC LINEAR STANDING WAVE π/2 ACCELERATING STRUCTURES*

S. Kulinski, E. Plawski, M. Wojciechowski, A. Soltan Institute for Nuclear Studies, Swierk, Poland

Abstract

The five parameter method of tuning of biperiodic π/2 linear accelerating structure is presented. The method consists in analytical calculation of the five parameters determining the dispersion relation of such structure: two eigen frequencies f_a and f_c of accelerating and coupling cavities, the first coupling coefficient k_{ac} and two second coupling coefficients k_{aa} and k_{cc}, using five measured dispersion frequencies. Usually the process of tuning is based on sets of 3 cavities however, to include directly also the second coupling coefficients k_{aa} and k_{cc}, one should consider sets composed of five cells. Using the dispersion relation, a set of five equations for five unknowns is obtained. This set is solved by successive elimination of unknowns by expressing them in terms of $F_a = f_a/f_{\pi/2}$. For F_a one obtains biquadratic equation. Coefficients of this equation are expressed as functions of measured quantities: dispersion phases and frequencies. Knowing F_a all other parameters are easily calculated together with the Stop Band $SB = |f_a - f_c|$. In this way, on each step of building up the structure one can control precision of measurements and the Stop Band.

INTRODUCTION

The well known advantages of standing wave biperiodic accelerating structure operating in the π/2 mode are good stability and small sensitivity to mechanical, temperature and assembly imperfections. The structure consists of a chain of accelerating and coupling cavities, the latter being much shorter than the accelerating ones, serve only for coupling and in the π/2 mode are not excited to the first order. The dispersion curve given by the relation (1) of such structure consists of two branches which for the well tuned structure should coincide at the shift angle $\varphi = \pi/2$

$$k_{ac}^2\cos^2\varphi = \left(1-\frac{\omega_a^2}{\omega^2}+k_{aa}\cos2\varphi\right)\left(1-\frac{\omega_c^2}{\omega^2}+k_{cc}\cos2\varphi\right) \quad (1)$$

Using this dispersion relation (1) we will solve the following problem: given five measured dispersion frequencies, find five parameters ω_a, ω_c, k_{ac}, $k_a a$ and k_{cc} defining the acceleration structure and realizing acceptable, sufficiently small, stop band $SB = |f_a - f_c|$.

*Work supported by EU Structural Funds Project no POIG.01.01.02-14-012/08-00

SYSTEM OF EQUATIONS FOR FIVE ELEMENTS METHOD OF TUNING BIPERIODIC ACCELERATING STRUCTURE

To formulate the equations for five elements method it will be useful to introduce some conventions and notations.

We will assume that for phases φ between 0, π the dispersion curve $f(\varphi)$ is monotonically increasing, and has the shape presented in Fig. 1.

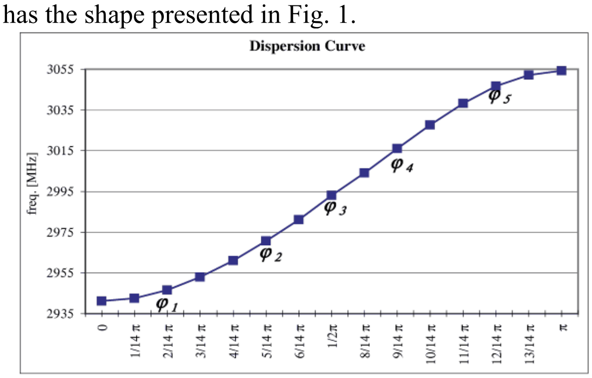

Figure 1: Arbitrary dispersion curve.

Each point $P_i(\varphi_i, f_i)$ of the dispersion curve is defined by two parameters (φ_i, f_i), with $i = 1\ldots5$, which we assume to be given by measurements. Taking into account the symmetry of the dispersion curve in respect to central phase $\varphi_3 = \pi/2$, in each set of five points taken into consideration, except central phase π/2, following two pairs of correlated phases will be chosen:

(φ_1, φ_5), where $\varphi_5 = \pi - \varphi_1$ (2)

(φ_2, φ_4), where $\varphi_4 = \pi - \varphi_2$ (3)

According to this choice we will have

$\cos\varphi_5 = -\cos\varphi_1$, $\cos^2\varphi_5 = \cos^2\varphi_1 = 1/c_1$ (4.1)

$\cos\varphi_4 = -\cos\varphi_2$, $\cos^2\varphi_4 = \cos^2\varphi_2 = 1/c_2$ (4.2)

$\cos2\varphi_5 = \cos2\varphi_1 = b_1$ (4.3)

$\cos2\varphi_4 = \cos2\varphi_2 = b_2$ (4.4)

For any set of five points P_i lying on the dispersion curve chosen according to the above procedure using expressions (2) to (4.4), together with corresponding frequencies and dispersion relation (1), one obtains following set of five equations:

$$k_{ac}^2 = c_2(1 - a_2F_a^2 + b_2k_{aa})(1 - a_2F_c^2 + b_2k_{cc}) \quad (5.1)$$

$$k_{ac}^2 = c_2(1 - a_2F_a^2 + b_2k_{aa})(1 - a_2F_c^2 + b_2k_{cc}) \quad (5.2)$$

$$0 = (1 - F_a^2 - k_{aa})(1 - F_c^2 - k_{cc}) \quad (5.3)$$

$$k_{ac}^2 = c_2(1 - a_4F_a^2 + b_2k_{aa})(1 - a_4F_c^2 + b_2k_{cc}) \quad (5.4)$$

$$k_{ac}^2 = c_1(1 - a_5F_a^2 + b_1k_{aa})(1 - a_5F_c^2 + b_1k_{cc}) \quad (5.5)$$

where:

$$F_a = \frac{f_a}{f_3},\ F_c = \frac{f_c}{f_3},\ a_i = F_i^{-2},\ i = 1,2,..5 \quad (6)$$

We will solve the system of equations (5.1) to (5.5) by successive elimination of unknowns by expressing them in terms of unknown F_a. As a result we obtain biquadratic equation (7) for F_a^2 .

$$PF_a^4 - QF_a^4 + R = 0 \quad (7)$$

Where

$$P = c_1(a_1 + b_1)(a_5 + b_1) - c_2(a_2 + b_2)(a_4 + b_2) \quad (8)$$

$$Q = c_1(1 + b_1)(a_1 + a_5 + 2b_1) - c_2(1 + b_2)(a_2 + a_4 + 2b_2) \quad (9)$$

$$R = c_1(1 + b_1)^2 - c_2(1 + b_2)^2 \quad (10)$$

Using definitions of b_k and c_k, (k=1,2), given by expressions (4.1) to (4.4) one finds

$$c_k(1 + b_k) = 2$$

$$c_1(1 + b_1)^2 - c_2(1 + b_2)^2 = 2(b_1 - b_2)$$

Taking above into account one obtains

$$R = 2(b_1 - b_2) \quad (11)$$

$$Q = 2[(a_1 + a_5) - (a_2 + a_4) + R] \quad (12)$$

Solution to the equation (7) is

$$F_{a_{12}}^2 = \frac{Q \pm \sqrt{\Delta}}{2P} \quad (13)$$

where the discriminant Δ is equal to

$$\Delta = Q^2 - 4PR \quad (14)$$

Knowing F_a^2 the remaining unknowns k_{aa}, k_{cc}, F_c^2 and k_{ac} can be calculated). In this way all parameters defining the dispersion relation of the π/2 biperiodic accelerating structure and the stop band $SB = f_a - f_c$ are found.

Usually calculated parameters can be different for different five element sets of dispersion frequencies, although, they should define the same structure. This effect is due to the errors in dispersion frequencies measurements and these errors influence also the calculated parameters of dispersion relation. Since results of measurements are random, one can assume, that errors distribution is normal and the most probable values of calculated parameters will be an arithmetic mean of all individual results. For structure composed of N (N is odd) cells, the number N_s of different combinations of sets composed of five cells, chosen according to the prescription given by equations (2) and (3) will be

$$N_s = (N-1)(N-3)/8$$

The knowledge of the calculated most probable values of the biperiodic accelerating structure parameters will be very useful in the process of tuning the structure.

THE TUNING

Principally process of tuning in the five cells method is similar to that of three cells method. Tuning starts with 3 elements sets: ½ accelerating + coupling+ ½ accelerating cavities. The aim of tuning is to obtain minimum field level in the coupling cavities. Joining successively the three elements pretuned sets one obtains sets composed of 5, 7… 2N+1 cavities. Beginning from the set of 5 cavities, according to the above described method, one can calculate, all five parameters defining the successively growing up accelerating structure. On each step of building up the structure different checks and corrections are available.

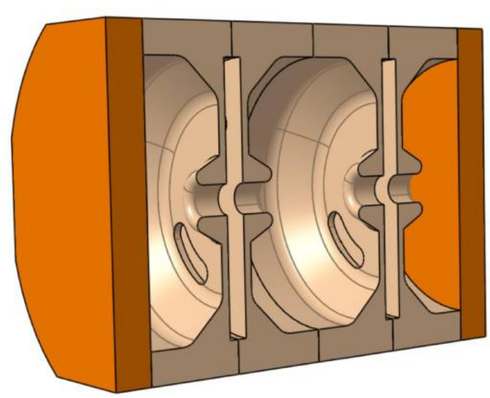

Figure 2: RF quintuplet.

The first check depends on the sign of the discriminant Δ (14) of eq.(7). If $\Delta < 0$, then the frequencies f_a, f_c and $f_{\pi/2}$ are complex and there is attenuation in time of the electromagnetic wave propagating in the structure.

Analysing the dependence of the quantities P, Q, R and Δ on the parameters $a_k(f_k)$ one can find, which measured frequencies f_k of the dispersion curve belonging to the considered five elements set, should be changed. Similar situation can happen with the sign of the square of coupling coefficient k_{ac}. If $k^2_{ac} < 0$ for some set of frequencies f_k, then the analysis of the expressions for k^2_{ac} (eq. (5.1) or (5.2)) will give indications in which direction one should go with changes of f_k to obtain $k^2_{ac} > 0$.

However, the most important corrections are possible due to the knowledge of average calculated values of dispersion curve parameters. Since these values are the most probable parameters defining dispersion relation of the measured structure, we can solve the following problem: given the parameters f_a, f_c, k_{ac}, k_{aa}, k_{cc} of the dispersion curve, find dispersion frequencies $f_i(\varphi_i)$ of the structure.

A simple program TUNING was written to solve the equation (7). As the input data for the program only the measured dispersion curve of the tuned structure is needed.

Below an example of tuning of a short accelerating structure is given. The first dispersion curve presents measured data without corrections, and the second presents data after tuning.

Figure 3: Biperiodic 3GHz accelerating stack.

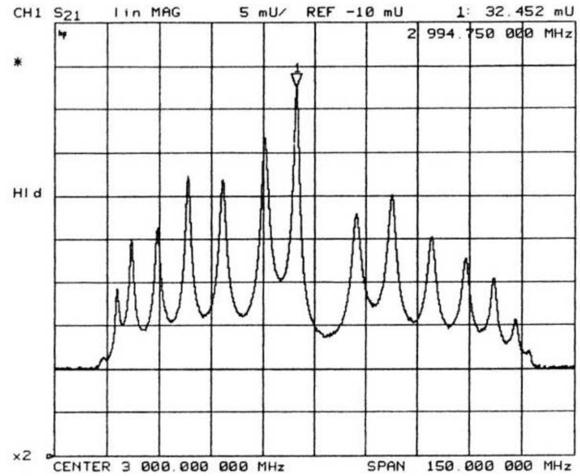

Figure 4: Frequency spectrum during tuning (8 cell stack).

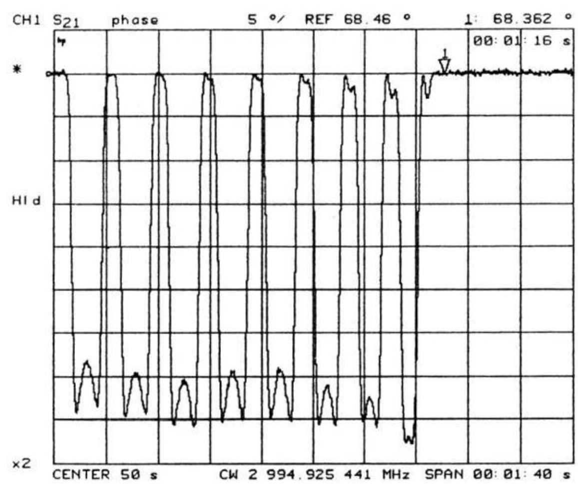

Figure 5: Accelerating field distribution in 8 cell stack.

REFERENCES:

[1] T.P.Wangler, RF Linear Accelerators, Wiley-VCH, 2004
[2] S.O. Schriber , Proc.1976 Lin. Acc. Conf., Chalk River