RINGING IN THE PULSE RESPONSE OF LONG AND WIDEBAND COAXIAL TRANSMISSION LINES DUE TO GROUP DELAY DISPERSION

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Abstract

In particle accelerators coaxial cables are commonly used to transmit wideband beam signals covering many decades of frequencies over long distances. Those transmission lines often have a corrugated outer and/or inner conductor. This particular construction exhibits a significant amount of frequency dependent group delay variation. A comparison of simulations based on theoretical models, numerical simulations and S_{21} network analyzer measurements up to 2.5 GHz is presented. It is shown how the non-linear phase response and varying group delay leads to ringing in the pulse response and subsequent distortion of signals transmitted through such coaxial transmission lines.

INTRODUCTION

Corrugated coaxial lines (like heliax or flexwell) are widely used in particle accelerator installations for the wideband transmission of RF diagnostic signals over comparatively large distances in the order of several hundred meters. The need for using such long and fairly large diameter cables is related to the need of low attenuation up to several GHz and very good electromagnetic shielding. The application of preamplifiers in the accelerator tunnel is often limited due to ionizing radiation issues. For reasons of easy installation corrugated cables are often preferred compared to smooth conductor versions. Braided cables often do not reach the required immunity level. However, such corrugated coaxial transmission lines exhibit a not very widely known dispersion effect due to the presence of the corrugation. This dispersion term which shows up in addition to the well know dispersion that comes along with the frequency dependent attenuation is typically manifested as a phase term proportional to the 3rd power of the frequency (linear phase subtracted) and gives a quadratic term for the group delay dependence on frequency. This dispersion leads to a ringing in wideband impulse transmission applications even when the spectrum of the transmitted signal is always below the waveguide mode cut-off frequency of the coaxial line considered. Time domain responses of corrugated cables have been measured and compared with smooth cables which do not exhibit the ringing [1]. The group delay dispersion related pulse distortion (ringing) is less relevant for telecommunication applications since there the combination of large cable length and very wideband signal transmission rarely occurs.

Dispersion compensation FIR filters have been used on beam signals transmitted over long cables to reconstruct the original pulse shape [2, 3].

THEORY

A homogeneous transmission line coaxial cable can be characterized by four parameters [4], its inductance per length *L'*, capacity per length *C'*, effective resistivity per length *R'* and insulation (losses) of the dielectric material expressed here as a conductance per unit length *G'*. The complex propagation constant is readily computed from [4]

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} \tag{1}$$

where ω is the angular frequency and $j=\sqrt{-1}$. In general the parameters *R'*, *L'*, *G'* and *C'* can depend on frequency. In a smooth coaxial cable at high frequencies the dependence of *R'* and *L'* on frequency is given by the skin-effect with the frequency dependent part proportional to $\sqrt{\omega}$ while the frequency dependence of *C'* and *G'* is weak for low loss dielectrics and determined by the material properties.

A corrugation of the outer conductor changes the boundary condition and essentially alters the frequency dependence of the inductance per unit length. Before the appearance of numerical computer simulations the problem of corrugated surfaces has been extensively treated with analytical and approximate methods, see for example [5]. Common to these methods is that they use the periodicity of the corrugation to derive an effective surface impedance that will allow to compute the propagation constant and wave impedance. It is instructive to resort to a simple planar model used in [5] originally to compute the wave retarding effect of corrugations used in electron tube devices. Approximating eqn. (54) of ref. [5] for a rectangular corrugation with a width of the corrugation equal to half the corrugation period *L* yields a phase advance ψ_0 per length *L* of the corrugation period, for a small corrugation depth (small when compared to the wavelength) of

$$\frac{\psi_0}{L} = \frac{\omega}{c}\left[1 + \frac{s}{4b}\left(1 + \frac{1}{3}\frac{\omega^2 s^2}{c^2}\right)\right] \tag{2}$$

s denotes the corrugation depth and *b* the distance between inner and outer conductor of the coaxial cable without corrugation slots. *c* is the speed of light in the homogenous insulation material of the cable. The additional retardation consists of a constant term proportional to the corrugation depth *s* and a cubic phase term corresponding to an additional group delay term with quadratic dependence on frequency. In practice the corrugation does not have the rectangular shape as assumed for the derivation of (2), however comparison with measurements and numerical simulations will show that the essential characteristics are still captured fairly well by this simple model.

MEASUREMENTS

Measurements were carried out on a 100 m long sample of 7/8” corrugated LDF50 cable from Andrew corporation, manufactured for the LHC RF system, in transmission mode (S_{21}) using a vector network analyzer. The cable is of the foam filled type and has a specified velocity factor of 0.89. For the analysis of the measured data the attenuation was fitted using the usual skin-effect formula [4] for high frequencies together with a small amount of losses from the dielectric, assumed to depend linearly on the frequency. In a 50 Ohm coaxial cable with smooth surfaces and low loss dielectric the overall attenuation is dominated by the ohmic losses on the surface of the inner conductor under the influence of the skin effect. The losses from the outer conductor amount to 28 % of the ohmic losses for a velocity factor of 0.89. The corrugation of the outer conductor will increase these losses, but since the outer conductor accounts only for a smaller share of the overall losses, the resulting attenuation of a coaxial cable with corrugated outer conductor will not be increased by a large amount. Along with the skin effect related attenuation term proportional to $f^{1/2}$ the inner inductance will also increase accordingly leading to a phase term also proportional to $f^{1/2}$. Dielectric losses can be modelled from measurements

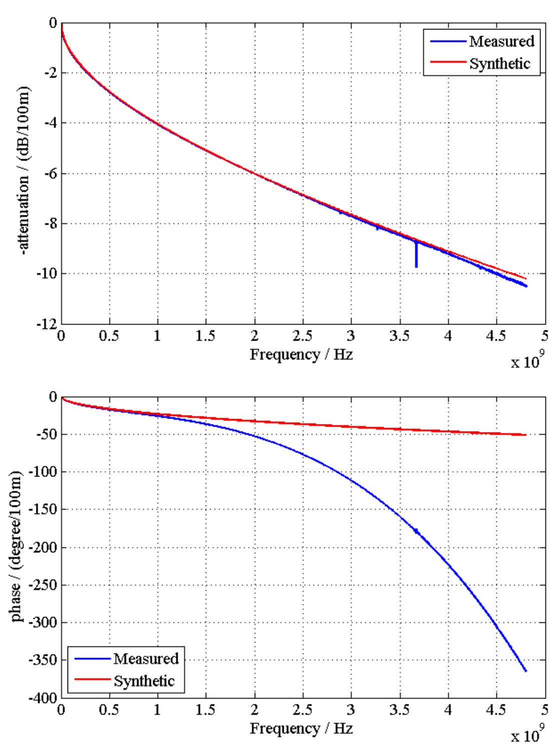

Figure 1: Measured (blue) and modelled (red) attenuation (top) and phase (bottom) of 7/8” coaxial cable.

with a linear frequency dependence using a tan $\delta = 5\text{x}10^{-5}$ which fits well with published data for bulk polyethylene (PE) of ~$2\text{x}10^{-4}$ and scaling with an air/dielectric filling factor of 0.8 for the PE foam of the cable, consistent with a propagation velocity of 0.89 of the cable and a permittivity of 2.3 of the bulk PE. Figure 1 shows the frequency dependent attenuation and phase measured (blue) and modelled (red) of the 7/8” coaxial cable. The red curve represents the phase term originating from the skin-effect. When this term is subtracted from the measured data the remaining phase follows a cubic dependence. This cubic term will be compared in the following section with numerical simulations and the simple model equation (2).

NUMERICAL SIMULATIONS

All simulations of the corrugated coaxial cable were performed using CST Microwave Studio. Within Microwave Studio the frequency domain solver (FDS) was used over a frequency range from 100 MHz to 5 GHz which is the cut-off frequency specified by the manufacturer. This cut-off could be verified via measurements. The FDS solves Maxwell’s equations in the frequency domain assuming a time harmonic dependence between the fields and the excitation. The equations are solved for one frequency at a time in the course of a frequency sweep, with in this case 10000 frequency samples. For each frequency sample, the linear equation system will be solved by an iterative, sparse direct solver. The solution comprises the field distribution as well as the S-parameters at the given frequency.

To minimize the calculation time only a very small part of the structure was used. Since the corrugations are periodic the length was chosen to be one period and periodic boundary conditions were applied to simulate an infinite periodic structure.

The dimensions of the simulated cable correspond to the real parameters of the cable structure. These parameters are listed in Table 1.

Table 1: Parameters of the Simulated Corrugated Cable

Outer conductor:	
Rmax	12.5 mm
Rmin	10.5 mm
Inner conductor:	
R	4.5 mm
Length of period of corrugation	6.2 mm
Line Impedance:	approx. 50 Ω

Three different types of cables were simulated:

1. The inner and outer conductor were assumed to be perfectly conducting (PEC). The dielectric used was vacuum ($\varepsilon = 1$).

2. The inner and outer conductor were assumed to be perfectly conducting. The dielectric used was polyethylene foam (ε = 1.3)
3. The inner conductor was assumed to be perfectly conducting. For the outer conductor copper with a thickness of 1 mm was used. The dielectric used was polyethylene foam (ε = 1.3)

All the simulations showed that the phase of a corrugated cable deviates cubically from a similar coaxial cable without corrugation and of equal length. Figure 2 compares this cubic term as simulated by CST Microwave studio (green) with the cubic term from eqn (2) (red) and the measured data (blue). Numerical simulations fit very well with measurements. The model (red) eqn. (2) fits well if a corrugation depth of s=1.4 mm is chosen. This value is smaller than the actual depth of 2 mm and can be viewed as an effective depth considering the fact that the cable corrugations are rather sinusoidal in shape and not rectangular slots as in the underlying theory to eqn (2).

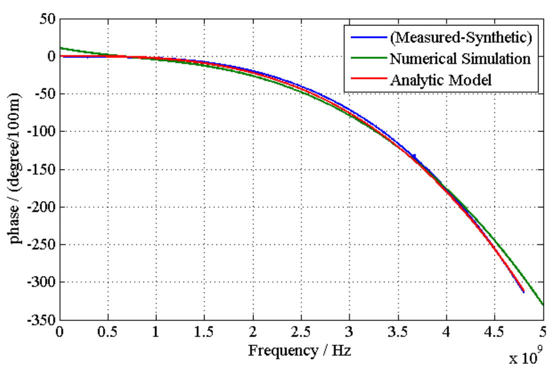

Figure 2: Cubic phase term as calculated numerically (green), measured (blue) and estimated from an analytic model (red). The corrugation depth in the model was 1.4 mm.

RINGING AND COMPARISON WITH OTHER PERIODICAL STRUCTURES

The non-linear phase response leads to a ringing in the pulse and step response as already has been measured in time domain [1].

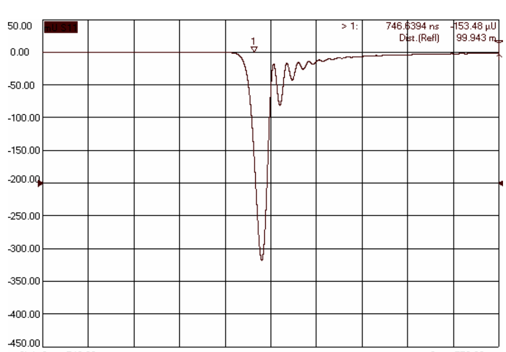

Figure 3: Ringing in the pulse response; computed by the network analyzer from measured S11 with short circuit at the far end (inverted signal).

In Fig. 3 we present a synthetic response calculated by the network analyzer from measured S_{11} data.

A frequency dependent group delay as presented here can also be found in more general terms in periodic structures. An example is the meander structure used for the chopper of the LINAC4 and SPL accelerator projects at CERN [6, 7]. A similar quadratic dependence also leads there to an inherent ringing in the time domain.

CONCLUSIONS AND OUTLOOK

We have shown in a comparison of analytical, numerical and measured data the impact of the group delay related dispersion on pulse dispersions in corrugated coaxial cables. This effect which is fundamentally known since more than 50 years is relevant for applications where wideband pulse signals have to be transmitted over large cable length. It would be desirable if cable manufacturers could provide in their cable specification sheets the deviation from linear phase over 100 meter length. The frequency dependent attenuation alone just leads to a rounding off effect in the step response whereas the frequency dependent group delay causes a ringing. In practice we have to deal with a combination of both effects which can be quantified by measurements or numerically. Hardware compensation of the attenuation only is fairly straight forward by means of suitable equalizer networks. However, the simultaneous compensation of both the frequency dependent attenuation and group delay dispersion becomes hardware wise much more demanding if not impossible over larger frequency ranges. Obviously a numerical compensation of such effects is rather simple in the frequency domain. Corrugated coaxial cables may not be very suitable for specific applications such as periodic notch filters used in microwave stochastic beam cooling since the above mentioned dispersion would disturb considerably the required strict periodicity of the notch position.

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