EMITTANCE INFLUENCE FOR ZUMBRO LENS IN PROTON RADIOGRAPHY

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Abstract

The capability of the chromatic aberrations correction of Zumbro Lens lies on the angle-position correlation, which is obtained by passing the beam through an expander and the beam can considered as from a point source. However, even after a long distance drift downstream the expander, the angle-position correlation isn't perfect because of the existence of finite emittance. This paper discusses the influences of the emittance to the chromatic aberrations correction and the optimization of beam status in phase space when illuminating the object. The emittance's influences to spatial resolution and object material indentification capability are also disscussed.

INTRODUCTION

Short pulsed, intense X-rays have been used to radiograph high-speed moving objects of high densities for many years. The transmitted X-rays are detected to obtain the areal density of the object. The X-rays' source size, incident flux, scattered backgrounds and detector efficiency affect the quality of the projected image. Presently, the advance of X-ray radiography has almost come to its limit.

Protons can be used to radiograph objects with higher spatial resolution, higher detector eficiency, multi-frame, multi-axis capability and material identification capability [1, 2, 3, 4, 5, 6]. When protons pass through the object, Multiple Coulomb Scattering (MCS) process make the exiting beam has an angular distribution. A following magnetic lens can be used to cure the MCS resulted image blurring. Because of the momentum spread in the proton beam, the chromatic aberrations of the lens must be corrected. Zumbro Lens can be used for a beam highly correlated in phase space to make the position dependent chromatic aberrations vanish.

But the illuminating particles are not perfectly correlated in phase space. The finite emittance of the incident proton beam make particles distribute around the correlation line. The proton beam's status in phase space can be optimized to minimize the emittance's influence to the chromatic aberrations correction. The finite emittance also degrades the spatial resolution and material identification capability, which are also disscussed.

CHROMATIC ABERRATIONS CORRECTION AND EMITTANCE

In the process of protons passing through the object, three types of interactions occur:

- the nuclear interaction through strong force, protons are lost from the beam;
- proton's Multiple Coulomb Scattering with nuclei, protons are deflected, which causes the image blurring;
- proton's Multiple Coulomb Scattering with electrons, protons loss energy, which causes energy spread and chromatic aberrations in the downstream imaging optics.

The MCS deflected protons have a near Gaussian angular distribution. The rms value of this angular distribution, in milliradians, is [5]:

$$\phi_{MCS} = \frac{14.1}{p\beta}\sqrt{\frac{L}{L_R}}\left[1+\frac{1}{9}\log_{10}\left(\frac{L}{L_R}\right)\right] \quad (1)$$

where p is the beam momentum in GeV/c, L is the path length through the object, and $L_R \propto Z^2/A$ is the radiation length of the material.

A section of specially designed optics, Zumbro Lens, can be used to focus the beam at the image plane. Under certain condition, Zumbro Lens can make all position dependent chromatic aberrations vanish.

At the image plane, the final position of an off momentum proton with initial coordinates (x, θ) is:

$$x_f = R_{11}(\delta)x + R_{12}(\delta)\theta, \quad (2)$$

where $\delta = \Delta p/p$ is the relative momentum spread, and R_{11}, R_{12} are elements of transport matrix from the object plane to the image plane.

If the initial beam is strongly correlated with correlation $\theta = wx$ in phase space, particles in such a beam would lie about the correlation line. Real particle has an angle deviation ϕ from the correlation line, $\theta = wx + \phi$.

To first order in δ, the final position of this particle is:

$$x_f = R_{11}x + R_{12}\theta + (R'_{11} + wR'_{12})x\delta + R'_{12}\phi\delta + \cdots, \quad (3)$$

where, primes indicate momentum derivatives.

When $w = -R'_{11}/R'_{12}$, all position dependent chromatic aberrations vanish.

In order to reduce the value of angular dependent chromatic aberration in Eq. 3, $\langle\phi^2\rangle$ need to be minimized. Another import measure of the performance of the lens is the field-of-view (FOV), which is determined by the maximum beam size when the beam travels through the lens. Obviously, smaller $\langle\phi^2\rangle$ would result in larger FOV.

The proton beam arrives at the object plane has a certain finite emittance ϵ. ϕ in Eq. 3 represents all deviations from

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the correlation line, which includes both MCS produced in the object and finite emittance in the incident beam. The incident emittance angles are added in quadrature to the MCS angles, $\langle \phi^2 \rangle = \langle \phi^2_{MCS} \rangle + \langle \phi^2_\epsilon \rangle$.

Note that, the beam that arrives at the object is expanded upstream to be large enough to illuminate the object by passing through the expander (typically, stainless steel plates). This expansion process increases the emittance of the beam by increasing the angular distribution. Hence, the illumating beam's emittance angular distribution is comparable with the MCS angular distribution obtained during passing the object.

In order to obtain a beam of size large enough to illuminate the object, the value of β at the object position is determined with the a specific emittance, $\langle x^2 \rangle = \epsilon\beta$. With known ϵ and β, the proton beam's phase space distribution is not fully determined yet. $\langle \phi^2_\epsilon \rangle$ varies with different value of α. Hence, there exist an optimum value for α to minimize $\langle \phi^2_\epsilon \rangle$.

Let $\theta_0 = wx$ to describe the correlation line, then proton's emittance angle deviation is $\phi_\epsilon = \theta - \theta_0$, hence, the mean square emittance deviation of angle is:

$$\begin{aligned} \langle \phi^2_\epsilon \rangle &= \langle (\theta - wx)^2 \rangle \\ &= \epsilon(\frac{1}{\beta} + \frac{\alpha^2}{\beta} + w^2\beta + 2w\alpha) \end{aligned} \tag{4}$$

The minimum value of $\langle \phi^2_\epsilon \rangle$ is ϵ/β, obtained when $\alpha = -w\beta$. The sign of α is chosen for a beam focusing in x plane. The sign of α in y plane is opposite.

If we look at the phase ellipse in Fig. 1, we can note that the point on the phase ellipse of maxmum X has a slope of $-\alpha/\beta$, which is just the correlation coefficient w when the beam's phase space distribution has the optimized α value obtained above.

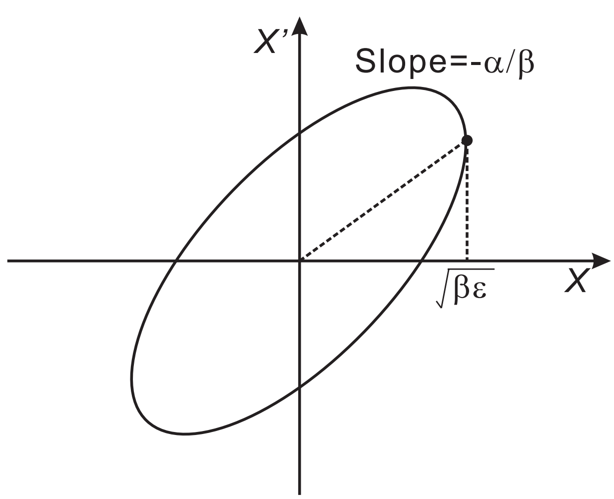

Figure 1: Phase space distribution.

SPATIAL RESOLUTION AND EMITTANCE

Besides the chromatic aberrations cause the image blurring, there is another factor that degrades the spatial resolution. When protons passing through the object of finite length, not only the moving direction is deflected by MCS, but the transverse position is also shifted for particles of deviated angle. For a zero transverse size incident proton beamlet, the MCS position and angle distribution after it passing through the object is [7]:

$$f(\theta_x, \theta_y, \rho_x, \rho_y) = \frac{3}{\pi^2\theta_0^4 L^2} F_x F_y \tag{5}$$

with

$$F_x = \exp\left[-\frac{2}{\theta_0^2}\left(\theta_x^2 - \frac{3\theta_x\rho_x}{L} + \frac{3\rho_x^2}{L^2}\right)\right] \tag{6}$$

$$F_y = \exp\left[-\frac{2}{\theta_0^2}\left(\theta_y^2 - \frac{3\theta_y\rho_y}{L} + \frac{3\rho_y^2}{L^2}\right)\right] \tag{7}$$

where, ρ is the transverse position deviation from the inject path, $\rho^2 = \rho_x^2 + \rho_y^2$; θ is the angle deviation, $\theta^2 = \theta_x^2 + \theta_y^2$;$l$ is the object length. After integration of Eq.5 over angle, we have:

$$\begin{aligned} \frac{dN}{d\rho} &= \int_{-\infty}^{\infty} f(\theta_x, \theta_y, r_x, r_y)\rho d\theta_x d\theta_y \\ &= \frac{3}{2\pi\theta_0^2 L^2} \exp\left(-\frac{3\rho^2}{2\theta_0^2 L^2}\right), \end{aligned} \tag{8}$$

and the RMS value of ρ is:

$$\sigma_\rho = \frac{\theta_0 L}{\sqrt{3}}. \tag{9}$$

This spatial resolution limit cannot be alleviated by downstream optics like chromatic aberration correction. Contrarily, the finite emittance degrades the spatial resolution further. In Fig. 1, we can see that there is an angular distribution for each specific X, this distribution exists at the particle impacting plane of the object. When protons come out the object, this angle distribution $\langle \phi^2_\epsilon \rangle$ would results in a position deviation like the MCS process mentioned above. These two process evolves together, see Fig. 2, and the total position deviation would be:

$$\sigma_\rho^2 = \sigma_{\rho_{MCS}}^2 + \sigma_{\rho_\epsilon}^2. \tag{10}$$

Hence, this emittance's influence to spatial resolution also requires the minimization of $\langle \phi^2_\epsilon \rangle$ like when we considering the emittance's influence to the chromatic aberration correction.

In a dynamic proton radiography case, a container is needed to shield the explosive object. The proton beam has MCS processes when it passes through the front and end windows of the container. People used to consider that only the MCS process of the end window contributes to the

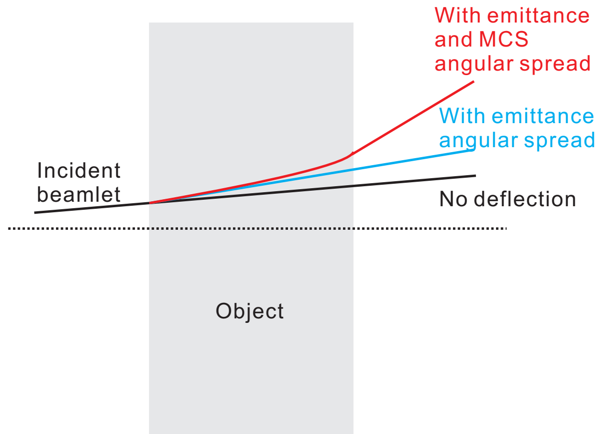

Figure 2: Transverse position deviations.

spatial resolution degrading, as illustrated in Fig. 3. When finite emittance is included in the picture, the contribution from the front window's MCS process also need to be considered because it increases the emittance of the incident proton beam.

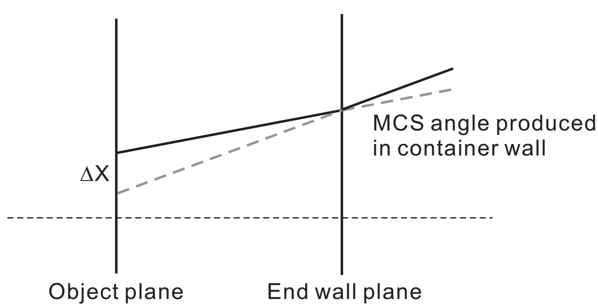

Figure 3: MCS created in end wall of the container produces spatial resolution enlarged.

MATERIAL IDENTIFICATION CAPABILITY AND EMITTANCE

In the middle of the Zumbro Lens, protons are sorted by their angles. An angular filter can be used to select protons falling into a certain angle range. By using two or more Zumbro Lens, multiple images can be obtained after each Zumbro Lens when the angle ranges are made smaller and smaller. Since the MCS radiation lengthes are different for different materials, images of different angle ranges for a single shot can be used to identify the material composition in the object through which protons pass [6]. This can be seen from the dependences of MCS angle ϕ_{MCS} in Eq. 1 to material length. Hence, the obtained images of different MCS angles can be fitted to identify the material composition.

While when finite emittance is considered, the emittance contributes to the exit beam angular distribution. Angle sorting is not uniquely determined by the MCS processes. When this emittance contributed angular noise is large enough, an extra set of lens is needed to distinguish it.

SUMMARY

We have presented an analysis of the finite emittance's influence to the proton radiography process using Zumbro Lens. Optimized incident beam's phase space distribution is found to minimize the emittance's influence to the chromatic abberations correction and spatial resolution.

An extra set of lens is suggested for material identification with beam of finite emittance beam.

REFERENCES

[1] N.S.P. King, et al, NIM A, 424, 1999, p84-91

[2] G.E. Hogan, et al, in Proceedings of the Particle Accelerator Conference, p579, 1999

[3] J.F. Amann, et al, LA-UR-97-1520

[4] H.J. Ziock, et al, LA-UR-98-1368

[5] C. Thomas Mottershead and John D. Zumbro, in Proceedings of the Particle Accelerator Conference, p1397, 1997

[6] Christopher L. Morris, LA-UR-00-5716

[7] Rossi B., High Energy Particles, New Jersey, Prentice-Hall, 1952, p66