# LINEAR OPTICS MEASUREMENT AND CORRECTION IN THE SNS ACCUMULATOR

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*Abstract*

In order to achieve a more robust and optimal performance, the difference between the real machine and its underlying model should be understood and eliminated. Discrepancies between the measured and predicted linear optics suggest possible errors of the focusing magnets and diagnostic devices. To find and correct those errors, a widely used method, orbit response matrix (ORM) approach is applied to the SNS storage ring, which successfully brings the tune deviation from $3\%$ to $0.1\%$, improves horizontal beta beating from $15\%$ to $3\%$, and perfectly flattens the orbit. In this article, we discussed the progress and possible future improvements with the SNS ring optics correction.

## INTRODUCTION

The accumulator of the spallation neutron source (SNS) is a ring of 248 meters circumference, with 44 BPMs and 52 steering correctors. Final transverse quasi-uniform distribution is achieved by injection painting, with the full intensity of $1.5 \times 10^{14}$ and power of 1.4 MW. The accumulator is four-fold symmetric with each super-period containing one FODO arc section and a doublet straight section. There are 32 arc dipole magnets and 52 quadrupole magnets, of which 16 located in the straight and 8 in high-dispersion areas of the arcs. Working point of the ring is at (6.23,6.20) but it was set empirically because the online model's prediction for the tune differs from the real value by about 0.2. To enhance the machine's reliability and capability, we need to have a better understanding to develop a precise model. Hence, linear optics correction becomes important and necessary, and the response matrix method (ORM), which has been proven to be able to correct linear optics, is applied on SNS accumulator to calibrate the machine.

Historically, the ORM approach has been used in electron rings to uncover malfunctioning BPMs, roll angles of magnets, mis-aligned higher order multipoles and calibration of power supplies. However its application to high intensity proton synchrotrons such as SNS accumulator is more difficult due to the reduced orbit stability and precision of orbit measurements.

## ORBIT RESPONSE MATRIX

The orbit response matrix (ORM) method is defined as the linear orbit response function to a small change of dipole kick angle (M in the equation below):

$$\begin{pmatrix} x \\ z \end{pmatrix}_{2m} = M \begin{pmatrix} \theta^x \\ \theta^z \end{pmatrix} = \begin{pmatrix} \mathbf{M}^{XX} & \mathbf{M}^{XZ} \\ \mathbf{M}^{ZX} & \mathbf{M}^{ZZ} \end{pmatrix}_{2m\times 2n} \begin{pmatrix} \theta^x \\ \theta^z \end{pmatrix}_{2n} \tag{1}$$

where $x$ and $z$ are the detected horizontal and vertical closed orbit shift at $m$ BPMs, $\theta^x$ and $\theta^z$ are the strength of the horizontal and vertical bumps. The off-diagonal submatrices, $M_{ij}^{ZX}$ and $M_{ij}^{XZ}$ are zero, if there is no coupling between the horizontal and the vertical planes.

Theoretically, the ORM matrix is given by the Green's function [4]:

$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi\nu} \cos(\pi\nu - |\phi_i - \phi_j|) \tag{2}$$

where i and j represent the ith BPM and jth dipole kicker.

Therefore, the ORM matrix can be both measured easily and calculated theoretically if the linear optics is known. That makes it an appropriate choice to judge the agreement between the model and the real machine, i.e. a merit-function $\chi^2(p)$ [5]:

$$\chi^2(p) = \sum_{ij} \left( \frac{\mathbf{M}_{ij}^{\text{meas}} - \mathbf{M}_{ij}^{\text{model}}}{\sigma_i} \right)^2 = \sum_k V_k^2 \tag{3}$$

where $\sigma_i$ is the measured noise level on the $i^{th}$ BPM.

The minimization of $\chi^2$ is therefore equivalent to making every $V_k$ as small as possible. In the minimization process, this can be achieved by varying some model parameters $x_n + \triangle x_n$ based on singular value decomposition (SVD) algorithm, where the changes $\triangle x_n$ suggests the error in the machine:

$$V_k(x_n + \triangle x_n) = V_k(x_n) + \sum_n \frac{\partial V_k}{\partial x_n} \triangle x_n = 0 \tag{4}$$

However, the assumption of linearity in equation 4 is usually invalid in real cases. The dependence is normally nonlinear. To tackle it, not only should we use specific algorithm, such as Levenberg-Marquadt algorithm when the problem is very nonlinear, but also we need to put constraints on those parameters. [3]

## LOCO WITH CONSTRAINTS

LOCO is a powerful analytical tool to compute the orbit response matrix and to fit it to the model. It was first written by James Safranek to correct the optics of the NSLS X-ray ring [1] and developed later by people at SLAC. It follows

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the similar minimization procedure as we described in the previous section. Not only the fitting parameters such as quadrupole strengths are included, but also the measured ORM's correction needs to be considered, e.g. gains and rolls of the BPMs and the correctors, as well as the momentum deviation due to the horizontal correctors.

Since coupling between LOCO fit parameters can cause excursions of the solution in unconstrained directions, it is natural to put a penalty on such excursions, which brings the $\chi^2$ to the following new format [3]:

$$\chi^2 = \sum_{ij} \left( \frac{\mathbf{M}_{ij}^{\text{meas}} - \mathbf{M}_{ij}^{\text{model}}}{\sigma_i} \right)^2 + \frac{1}{\sigma_{\triangle K}^2} \sum_k w_k^2 \triangle K_k^2 \tag{5}$$

where $\sigma_{\triangle K}$ is an overall normalization constant and $w_k^2$ is an individual weighting factor to constrain the corresponding quadrupole. This constraint played an important role in SNS to find out the best solution.

## FITTING RESULTS

It requires two stages to fit the ORM with LOCO:

First, to find the quadrupole gradient errors, we choose our fit parameters as 6 quadrupole power supplies, 42 BPM gain factors and 52 corrector strengthes (24 in horizontal and 28 in vertical). We will exclude dispersion from the fitting target because orbit error in quadrupoles and sextupoles leads to changes in dispersion which are not accounted for in the model. We also fit the energy changes at correctors with the Constant-Momentum method in LOCO [5]:

$$M_{model} = (\frac{\triangle p}{p})_{fit} D_{measured} + M_{ATmodel} \tag{6}$$

where D is the dispersion. In SNS accumulator, the horizontal dispersion is designed to be 4m on arcs and zero through the straight section, while the vertical is small within 0.3m variance.

Second, we add the skew quadrupoles' strengths and coupling factors of BPMs and correctors into the set of fit parameters and do the fitting again. This completes the first stage to reveal quadrupole errors and BPM/corrector calibration.

Then, to fit the beta and dispersion functions, we include the measured dispersion as an additional column in the response matrix, and choose the so-called Fixed-Path-Length method in LOCO [5]:

$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi\nu} \cos(\pi\nu - |\phi_i - \phi_j|) + \frac{D_i D_j}{\alpha L} \tag{7}$$

After two iterations, as shown in Fig 1, $\chi^2$ has been reduced from order of $10^{-1}$ to $10^{-3}$, with the beta change (statistical standard deviation of $\frac{\beta_{after}}{\beta_{before}}$ at 42 BPMs) between $8\%$ to $11\%$. It is worthwhile to mention that the orbit error in the experiment is small. Therefore, although

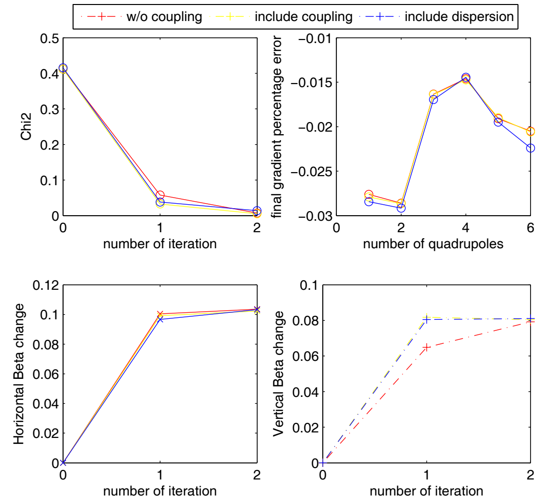
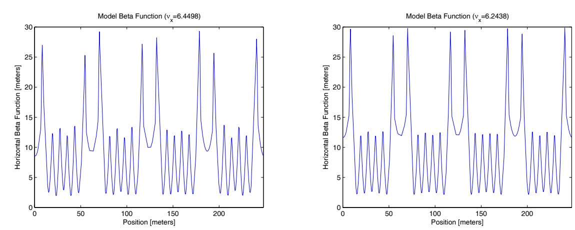



Figure 1: Comparison of the fitting for different stages.



(a) before (b) after

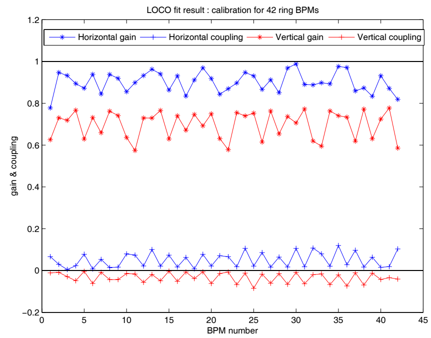
Figure 2: $\beta_x$ beating is reduced after corrections.

the model does not contain orbit errors, the results of the two stages do not differ much.

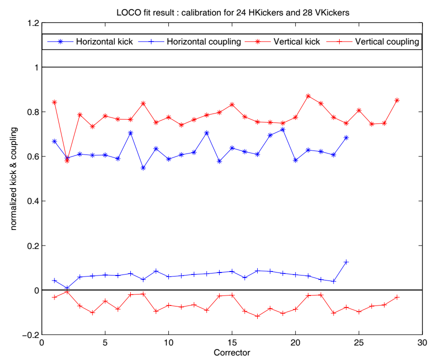
The convergence of the $\chi^2$ becomes much slower after two iterations while the corresponding beta change and gradient errors are relatively large. Since smaller gradient changes are preferred for optics control due to the linear field-current assumption, the present gradient corrections were dialed into the real machine:
$-2.76\%$ for QV11a12, $-2.86\%$ for QH10a13,
$-1.64\%$ for QV01a09, $-1.46\%$ for QH02a08,
$-1.91\%$ for QV03a05a07, $-2.05\%$ for QH04a06.

Results are exciting, the discrepancy between $\nu_{model}$ and $\nu_{meas}$ was reduced from 0.2 to 0.008, and it is now much easier to flatten the orbit. The LOCO fit also indicates that the gradient errors are the main source for the $\beta_x$ distortion, reducing the horizontal beta beating from $15\%$ to $3\%$ as shown in Fig.2.

LOCO fit also predicts the calibration for the BPM and corrector gains and coupling factors shown in Fig 3, from which we can see the BPM coupling appears to be $6\%$ on average and peaks at $10\%$ in some cases with a global pattern. Since coupling due to the electronics should be randomly distributed and small and the ratios of the two coupling (x and y) are not -1 in most locations, we can exclude BPM block rotation and electronics from the major contribution. Such a large coupling and global pattern should




(a) BPMs' calibration




(b) Correctors' calibration

Figure 3: Fitted BPM and corrector calibration factors.

be caused by the lattice coupling. Although we added the skew quadrupoles to the fit parameters, it is apparent that the settings of them did not kill the lattice coupling entirely. That's also a reasonable explanation for the large regular coupling of correctors, because the geometric variation should not contribute this much.

Those gain factors in Fig.3 are not absolute values but have an overall scaling factor between BPMs and correctors, which is inevitable due to the definition of response matrix. Absolute values cannot be obtained unless dispersion is large and included in the fitting parameters. Unfortunately, we have large horizontal dispersion on arcs but only small vertical dispersion. However, the relative pattern of the fitted gains is reliable.

By scaling the measured betatron function to the model value, it suggests the overall scaling factor to be 1.06. The measured betatron function is obtained by analyzing BPM's responses for two kicks with the formula 8.

$$\beta_i = \frac{4sin^2(\pi Q)}{sin^2\triangle}\left(\frac{x_{ia}^2}{\beta_a\theta_a^2} + \frac{x_{ib}^2}{\beta_b\theta_b^2} - \frac{2x_{ia}x_{ib}cos\triangle}{\sqrt{\beta_a\beta_b}\theta_a\theta_b}\right) \quad (8)$$

where $\triangle = |\phi_a - \phi_b|$ is the phase advance between the two correctors, and the quantities $\triangle$, $\phi_a$ and $\phi_b$ should be the corrected/fitted results. So, $\theta$ should be $\theta/s$ and $\beta$ should be $\beta/s$ where s is the overall scaling factor. It is varied to bring the "measured" beta close to the model value (Fig.4), and this procedure is repeated for several

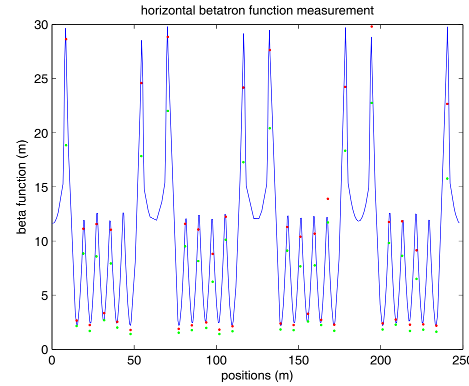



Figure 4: Red spots are the measured beta at BPMs after applying the overall scaling factors, while Green spots are not.

two-correctors sets to fit the factor.

## CONCLUSION

Orbit response matrix analysis has been used in SNS accumulator to find out the quadrupole gradient errors, correct the beta function and dispersion and figure out the calibrations of BPM and corrector. The correction of quadrupole gradient errors has already been implemented in the ring, which enhanced applications that were developed based on model optics, such as the tune-setting, local bump creation, and orbit correction. In the future, it is intended to figure out the skew quadrupole strengths to kill the lattice coupling thoroughly, and model all magnet rotations and translations including the dipoles and sextupoles. Also, betatron function measurement needs to be further investigated because both Model Independent Analysis (MIA) and the two-kicker response method are very sensitive and not so reliable.

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