

# AUTOMATING THE COMPUTATION OF QUADRUPOLE TRANSFER MAPS AND MATRICES UTILIZING ELECTROMAGNETIC FIELD SOLUTIONS

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## Abstract

An automated procedure for the calculation of particle transfer maps using computed magnetic field data has been developed for several types of magnetic quadrupoles. The Automated Transfer Map Generator (ATMG) software used for these calculations combines the Analyst<sup>TM</sup> program and specialized modules of the Particle Beam Optics Laboratory (PBO Lab<sup>TM</sup>). Analyst's scripted solids capability is used to develop models of different magnet concepts. The geometry and material attributes for a given magnet concept are encapsulated by a small number of magnet parameters. Quadrupoles of the same basic concept can be simulated by using different values for the magnet parameters. The three-dimensional magnetic field solver (MS3p) of the Analyst program is used to obtain the fields. New PBO Lab modules are used to automate the field computation, and then calculate the transfer maps and matrices through third-order using the Venturini-Dragt method. Examples for three different types of magnetic quadrupole lenses are presented: electromagnetic air-core, electromagnetic iron-core, and rare-earth permanent magnet quadrupoles.

## INTRODUCTION

The Automated Transfer Map Generator (ATMG) software addresses the computation of particle transfer maps for optics elements directly from detailed electromagnetic field data for the elements. The approach is based upon the method developed by Marco Venturini and Alex Dragt [1]. The field data is obtained from the Analyst program [2]. Parametric models of electromagnetic devices have been developed where the geometric and material attributes of a given device are determined by the values of parameters utilized. Special modules of PBO Lab [3] have been developed to provide an interactive, intuitive, graphic user interface (GUI) for selecting parameter values, and then using the values to create input data for Analyst. The PBO Lab modules are also used to run Analyst, which produces field data for the device, and then use that field data to compute the transfer maps. Examples for magnetic quadrupoles are discussed.

## ANALYST MODELS

Analyst is a software package used for computer-aided, physics-based design of accelerator and microwave components. It has support for geometric model creation/editing, automated 2D and 3D meshing, result

processing, and optimization. Analyst uses the finite-element method for solving the Maxwell equations in static, frequency, and time domains. Analyst also has comprehensive scripting support via the object-oriented Python programming language. Python scripts were used in this work to facilitate linking PBO Lab and Analyst.

Scripts were used to construct the basic geometry of each magnet, and each script contains parameters that determine the exact shape of the magnet. The script parameters are available to the Analyst system for use when the model is constructed. Each model script must contain a small set of functions with specified signatures, and once installed into the Analyst system they are available within the user interface or when Analyst is called by an external program such as PBO Lab. Different magnet topologies were captured as scripts during this work. Figure 1 illustrates Analyst wire frame views of the 3 quadrupole magnet concepts of this paper.

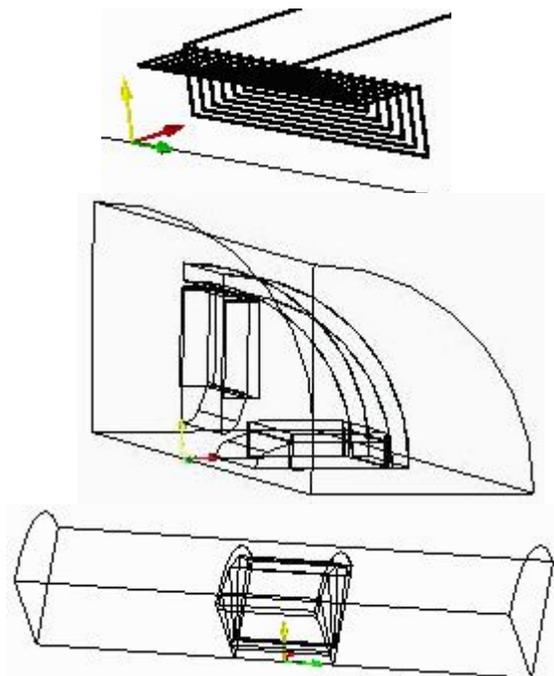


Figure 1: Analyst wire frame views for one quadrant of each quadrupole concept used in this work: air core (top), iron core (center) and permanent magnet (bottom).

Each view in Figure 1 shows one-quarter of the magnet, rotated to provide a perspective that displays interior elements. The beam axis lies along the lower edge of each image (green arrow). The upper image (air core

quad) is also zoomed in, while the two lower images (iron core and permanent magnet quads) show the outer most boundary of the field solution region ("air box").

## PBO LAB MODULES

Specialized modules for PBO Lab [3] have been developed to automate the field computation, and then calculate the transfer maps and matrices. The Open Architecture Software Integration System (OASIS™) was used to create the new modules. OASIS [4] is a specialized PBO Lab module that encapsulates a generic application template with methods to implement translations between a program's native data representation and the PBO Lab object model. OASIS-defined modules can work with existing executable or library programs, generally requiring no new source code. OASIS provides a point-and-click, drag-and-drop GUI for developers to define the specialized requirements of their particular program.

Three new modules were developed, one for each of the three magnet concepts. Each module is used to (a) select the magnet concept, (b) set up the magnet parameters, and (c) run Analyst to obtain the fields. A fourth new module is used to (d) compute particle transfer maps through third-order from the field data. Existing PBO Lab tools [5] are used to obtain third-order transfer matrices (i.e. aberration coefficients) from the map data.

## QUADRUPOLE MODEL PARAMETERS

Each of the quadrupole concepts are specified by a number of physical properties and computational parameters. The physical properties include the dimensions of the various magnet components, coil currents for electromagnets or remnant field values for permanent magnets, and similar magnet parameters. The computation parameters include the boundaries of the field region, mesh settings and the number and locations of field values to be used for map calculations.

Baseline concepts for each magnet type were obtained from the literature. The baseline concept for the air core electromagnetic quadrupole was the pulsed Panofsky magnet of Li, et al [6]. The baseline concept for the iron core electromagnetic quadrupole was a microbeam lens quadrupole constructed by Dyer Energy Systems [7]. The permanent magnet quadrupole baseline concept was a rare earth design of the type described by Halbach [8]. The methods and tools used for this work were applied to the baseline concepts. The results obtained were compared to published and other data for the baseline concepts in order to benchmark the ATMG software.

In order to compare the various magnet concepts, a hard edge (HE) quadrupole reference model was used in this work to define certain common properties. The HE reference specified the physical length (10 cm), the radial aperture (2 cm), and the integrated gradient-length product. The HE gradient was taken to be 234.73047 T/m (100 times that of the reference [5] example). This gradient is high and impractical for many concepts, but provides a useful test reference. In addition, the external

dimensions of each concept were specified to be 1.1 times the physical length (excluding the air core quadrupole). These physical properties differ significantly from the parameters of the baseline concepts described in the preceding paragraph and helped to establish the wide applicability of the ATMG software. Finally, the number of field data points (401) was the same for all models, and the air box field region was taken to extend longitudinally to  $\pm 10$  times the radial aperture and to enclose a volume that was transversely 2.3 times the physical length.

One physical property was adjusted for each magnet concept in order to obtain the HE reference quadrupole integrated field strength ( $G \bullet L$ ). The coil current was used for the air core and the iron core electromagnetic quadrupoles, while the remnant field was used for the permanent magnet quadrupole (PMQ). Table 1 gives the integrated field strength, as well as the peak magnetic gradient (G) and effective magnetic length (L), for the HE reference and the three magnet concepts considered here.

Table 1: Integrated gradient-length product ( $G \bullet L$ ), effective magnetic length (L) and peak gradient (G) for the 3 models of this work and hard edge (HE) reference.

Param.	HE	Air Core	Iron Core	PMQ
$G \bullet L$ (T)	23.473047	23.473042	23.473108	23.473044
L (cm)	10.000000	6.953465	10.784450	10.420860
G (T/m)	234.73047	337.57330	217.65690	225.25000

The Analyst field solver was used to compute the magnetic fields for all three concepts. For the PMQ concept the analytic model of Halbach [8] was also used. Figure 2 illustrates the longitudinal (z) dependence of the magnetic field magnitudes for the quadrupole component. Analyst fields are used for the electromagnetic concepts and the analytic field for the PMQ.

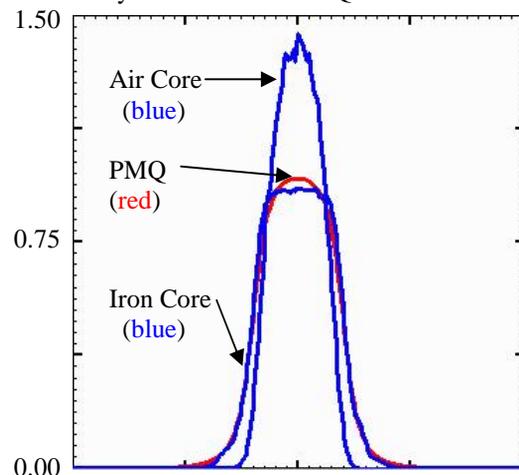


Figure 2: Magnitudes of the radial magnetic field for the quadrupole component as a function of longitudinal position  $z$  ( $\pm 0.25$  meters range) at one-half the radial aperture. Vertical units are multiples of the HE field.

## TRANSFER MAPS AND MATRICES

The transfer map computation uses the Venturini-Dragt method [1]. Their method involves the use of magnetic

field values on an imaginary cylinder that parallels the beam axis (longitudinal direction) inside the aperture of the magnet. The ATMG software uses a specialized PBO Lab module that automates the map computation using the Analyst produced magnetic fields. The resulting data is loaded into a PBO Lab element (Map Piece) for use in the MARYLIE [9] particle optics code. Third-order transfer matrices can also be computed from the Map Piece data [5] and stored in a Matrix Piece for use in the TRANSPORT [10] or TURTLE [11] optics codes.

It is useful to compare the transfer maps and matrices for the 3 quadrupole concepts with those for the hard edge (HE) reference. The HE reference contains all of the so-called intrinsic aberrations [12] which are present in all quadrupole magnets, independent of the fringe field details. Comparison to the HE reference thus permits one to evaluate the effects of the each quadrupole concept on aberrations. The PBO Lab Map and Matrix Pieces have a differencing tool for simplifying the comparisons [5].

Due to the equality of the integrated gradient-length products for each of the magnet concepts (Table 1) the 1<sup>st</sup> order (R-matrix) maps are very similar. Significant differences appear in the 2<sup>nd</sup> and 3<sup>rd</sup> order maps. The "spectrum plots" of the PBO Lab Map and Matrix Pieces can be used to quickly and visually isolate the important differences. For a 234.568077 MeV proton beam ( $\beta=0.6$ ), important differences from the HE intrinsic aberrations appear in both the chromatic and geometric matrix elements. Table 2 provides a summary of the 2<sup>nd</sup> and 3<sup>rd</sup> order chromatic aberration matrix elements, giving the HE reference value and the differences for the three quadrupole concepts considered.

Table 2: 2<sup>nd</sup> and 3<sup>rd</sup> order chromatic aberration matrix elements as a difference (Diff) from intrinsic (HE) values.

Aberration Coefficient	HE	Air Core Diff	Iron Core Diff	PMQ Diff
T <sub>216</sub>	6.90887	0.44469	-0.57689	-0.61788
% Diff	-	+6.44	-8.35	-8.94
T <sub>436</sub>	-13.5914	0.54921	-0.88457	-0.97220
% Diff	-	-4.04	+6.51	+7.15
U <sub>2166</sub>	-5.48057	-0.64281	0.79812	0.84992
% Diff	-	+11.7	-14.6	-15.5
U <sub>4366</sub>	15.5202	-0.85171	1.41410	1.55939
% Diff	-	-5.49	+9.11	+10.0

### SUMMARY

An automated procedure has been developed for the computation of magnetic quadrupole transfer maps and matrices from magnetic field data. The Automated Transfer Map Generator (ATMG) utilizes new scripted-solid Python scripts created for the Analyst program and custom PBO Lab Modules that support data setup, Analyst execution, and map computation. Three ATMG quadrupole examples have been presented that illustrate the broad range of applicability of the approach. The

most important nonlinear aberrations were identified. The procedure can be applied to any similar magnets, and readily extended to support other quadrupole concepts.

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### REFERENCES

- [1] M. Venturini and A. J. Dragt, "Accurate Computation of Transfer Maps from Magnetic Field Data," *Nuc. Instr. Meth. Phys. Res. A* 427, 387-392 (1999).
- [2] Analyst is available from Simulation Technology and Applied Research, Inc., Mequon, Wisconsin, <http://www.staarinc.com>.
- [3] PBO Lab is available from AccelSoft Inc., San Diego, California, <http://www.ghga.com/accelsoft>.
- [4] G. H. Gillespie and B. W. Hill, "Open Architecture Software Integration System (OASIS) for the Particle Beam Optics Laboratory (PBO Lab)," *Proceedings of PAC07*, paper THPAS037, 3579-3581 (2007).
- [5] G. H. Gillespie and B. W. Hill, "PBO Lab Tools for Comparing MARYLIE Lie Maps and TRANSPORT Transfer Matrices," *Proc. EPAC06*, 2275-77 (2006).
- [6] Y. Li, P. Chin, M. Reiser, M. Venturini, J. G. Wang, Y. Zou and T. F. Godlove "Design, Simulation and Test of Pulsed Panofsky Quadrupoles," *Proceedings of PAC99*, 3369-3371 (1999).
- [7] "Microbeam Lens," Dyer Energy Systems, Inc. technical report, 5 pages. Copy provided courtesy of B. A. Doyle, E. S. Bielejec and B. R. McWatters, personal communication (2008).
- [8] K. Halbach, "Design of Permanent Multipole Magnets with Oriented Rare Earth Cobalt Material," *Nuclear Instruments and Methods* 169, 1-10 (1980).
- [9] A. J. Dragt, et al, "MARYLIE 3.0 User's Manual, A Program for Charged Particle Beam Transport Based on Lie Algebraic Methods," 901 pp (2003).
- [10] D. C. Carey, K. L. Brown and F. Rothacker, "Third-Order TRANSPORT with MAD Input - A Computer Program for Designing Charged Particle Beam Transport Systems," *SLAC-R-530*, 316 pp (1998).
- [11] D. C. Carey, "TURTLE with MAD Input (Trace Unlimited Rays Through Lumped Elements), a Computer Program for Simulating Charged Particle Beam Transport Systems, and DECAY-TURTLE Including Decay Calculations," *Fermilab-Pub-99/232*, 196 pp (1999).
- [12] R. Baartman, "Intrinsic Third-Order Aberrations in Electrostatic and Magnetic Quadrupoles," *Proceedings PAC97*, 1415-1417 (1998).