EXPERIMENTAL FREQUENCY MAP ANALYSIS USING MULTIPLE BPMS*

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Abstract

Frequency map analysis is being widely used, nowadays, both in simulations to design or improve accelerator lattices, as well as in experiments to study the transverse nonlinear dynamics in accelerators. A significant challenge to the use of frequency map analysis in experiments is the usually very fast decoherence of transverse oscillations, caused by the large nonlinearities of state-of-the-art lattices. Due to the decoherence, the center of mass oscillations of bunches often disappear in less than 100 turns. A potential way to get around this limitation is the use of multiple BPMs (Beam Position Monitors) distributed (symmetrically) around the storage ring. This paper describes the challenges multi-BPM frequency map analysis poses as well as initial results using the ALS.

INTRODUCTION

Frequency Map Analysis (FMA) is being widely used in both simulations and experiments to design and improve accelerator lattices [1, 2]. It can provide a global view of the dynamics by studying the tune diffusion of transverse oscillations. The key numerical requirement for FMA is the precise measurement of the transverse tune in a small number of turns. The useful number of turns is often restricted by decoherence. This presentation shows how multiple BPMs (Beam Position Monitor) can help to gain higher precision on the tune measurement. We also show the result of a decoherence study during the first 100 turns after a horizontal/vertical kick. A FMA result using experiment data is also presented. If one would use only one BPM, it would not be easy to achieve the required resolution of transverse tunes.

DFT AND NAFF

The turn-by-turn closed orbit data recorded by BPMs is processed by DFT (Discrete Fourier Transformation) or more advanced algorithms such as Interpolated Fourier Transformation, or NAFF (Numerical Analysis of Fundamental Frequency) [3].

As an extension of DFT, NAFF searches for the maximum of

$$c_T(\omega) = \langle f, g\rangle = \frac{1}{T}\int_{-T}^{T} f e^{i\omega t}\chi \, dt$$

where $\chi(t) = 1 + \cos(\pi t/T)$ is the Hanning filter (window). The use of a window allows the determination of the frequencies with much greater accuracy. In the Appendix, Fig. 5 and Fig. 6 shows asymptotic properties of frequency precision depending on the number of data points. In Fig. 5 the signal has a white noise with σ about 1% of the signal amplitude. In Fig. 6, the signal has an exponetially damped amplitude $e^{-t/\tau}$ and $\tau = 1000$. From this benchmark, NAFF gives best performance, and for a 10^{-10} precision, it needs only about 1000 data points, which would only be a few turns in a modern storage ring, if we can use all the BPMs.

MULTIPLE BPMS

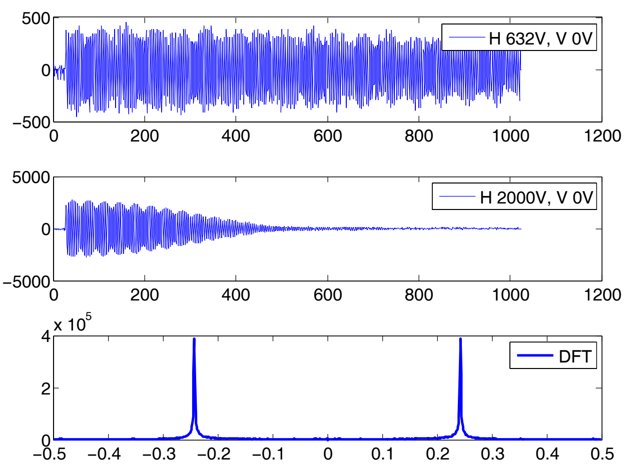

Figure 1: Raw turn-by-turn BPM data for a horizontal kick of 632 V and 2000 V. The frequency is shown in the third plot, where the horizontal tune is the large peak.

The frequency analysis, DFT or NAFF, relies on the fact that $\{1, \sin(nx)/\sqrt{\pi}, \cos(nx)/\sqrt{\pi}\}$, $n = 1, 2, ...$ forms an orthonormal set. The DFT of data points is defined as $H_n \equiv \sum_{k=0}^{N-1} h_k e^{2\pi i k n/N}$. One requirement for the data sampling is that the data points are equally spaced. i.e. $h_k = h(t) * \delta(t - kT)$, where T is the period. If the data is sampled non-equally spaced, it will introduce high frequency terms. Due to the aliasing effect, this will bring larger noise into frequency space and could reduce the frequency resolution. In a storage ring, different BPMs have different phase advance between them, and also different beta functions. Blindly grouped data may introduce extra artificial "periodicity", and could create additional spectral peaks in frequency space. They may dominate over the true transverse tune. The top plot in Fig. 2 shows the frequency of turn-by-turn data from 96 BPMs, the data are organized

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in a turn-wise way, i.e. $\{h_{00}, h_{10}, \ldots, h_{95,0}, h_{0,1}, \ldots\}$, where h_{ij} represents the j^{th} turn closed orbit data recorded by the i^{th} BPM. A different way of organizing the data may bring a better result.

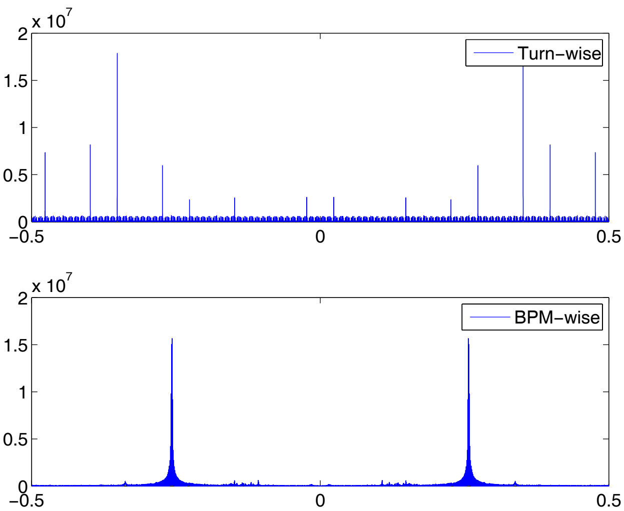

Figure 2: DFT of two multi-BPM data analysis. The top one groups data in the order of beam traveling time. The bottom one groups in the order of BPMs.

The closed orbit signal received by the ℓ^{th} BPM at the k^{th} turn is

$$h_{\ell k} = \sqrt{\beta_x^{(\ell)} \epsilon_x} e^{i(2\pi k \nu_x + \phi_x^{(\ell)})}$$

the superscript (i) is the index of BPM, ϵ_x horizontal emittance, β_x the horizontal beta function, and ϕ_x the phase at the indicated BPM location. For the data group $\{h_{11}, h_{12}, \ldots, h_{1K}, h_{21}, \ldots, h_{LK}\}$ the DFT is

$$H_n = \sum_{\ell,k} h_{\ell k} e^{\frac{2\pi i(\ell * K + k)n}{LK}}$$

where L is the total number of BPMs, K is the turn number.

$$\begin{aligned} H_n = &\sum_k h_{0k} e^{\frac{2\pi i k n}{(LK)}} + \sum_k h_{1k} e^{\frac{2\pi i (K+k)n}{(LK)}} + \cdots \\ &+ \sum_k h_{(L-1)k} e^{\frac{2\pi i((L-1)*K+k)n}{(LK)}} \end{aligned} \quad (1)$$

The signal received by a different BPM in the same turn have the same tune but different amplitude and phase: $h_{1k}/h_{0k} = \sqrt{\beta_x^{(1)}/\beta_x^{(1)}} e^{\phi_x^{(1)} - \phi_x^{(0)}}$. From the linearity and translation properties of the Fourier transform, the second part of H_n is only a phase shift of the first term and then a scaling factor of $\sqrt{\beta_x^{(1)}/\beta_x^{(1)}}$:

$$\begin{aligned} \sum_k h_{1k} e^{\frac{2\pi i(K+k)n}{(LK)}} &= \sum_k h_{0k} \sqrt{\frac{\beta_x^{(1)}}{\beta_x^{(1)}}} e^{\frac{2\pi i(K+k)n}{(LK)}} \\ &= e^{\frac{2\pi i n}{L}} \sum_k h_{0k} \sqrt{\frac{\beta_x^{(1)}}{\beta_x^{(0)}}} e^{\frac{2\pi i k n}{(LK)}} \end{aligned}$$

Therefore

$$H_n = (1 + \sqrt{\frac{\beta_x^{(1)}}{\beta_x^{(0)}}} e^{\frac{2\pi i n}{L}} + + \cdots + \sqrt{\frac{\beta_x^{(L)}}{\beta_x^{(0)}}} e^{\frac{2\pi (L-1) i n}{L}}) H_{n1}$$

where H_{n1} is the first term in Eq. (1). In most cases, β_x are not perfectly symmetric, and the sum part does not cancel. The final spectrum of H_n is defined by H_{n1}, which is the horizontal tune without any other side band. The DFT of real BPM data from the ALS is shown in the bottom plot in Fig. 2.

The tune measurement can be enhanced by grouping BPM turn-by-turn data in BPM-wise order. The comparison with the other method, where data is grouped in Turn-wise order is shown in Fig. 2. Since the number of points increased as we are using multiple BPMs, the tune measurement becomes more precise for a given short time period. This opens the door to fast tune diffusion experiments.

DECOHERENCE AFTER KICK

Transverse decoherence occurs because different particles have slightly different oscillation frequencies. For a Gaussian beam $\psi_0(J) = e^{-J/J_0}/\sqrt{2\pi J_0}$, the distribution after the kick $\beta\theta$ is given by [4]

$$\begin{aligned} \psi_2(\phi, J, t) &= \psi_1(\phi - \omega(J)t, J) \\ &= \psi_0(J + \theta\sqrt{2J\beta} sin(\phi - \omega(J)t) + \beta\theta^2/2) \end{aligned}$$

where $\psi_1(\phi, J)$ is the beam distribution immediately after the kick.

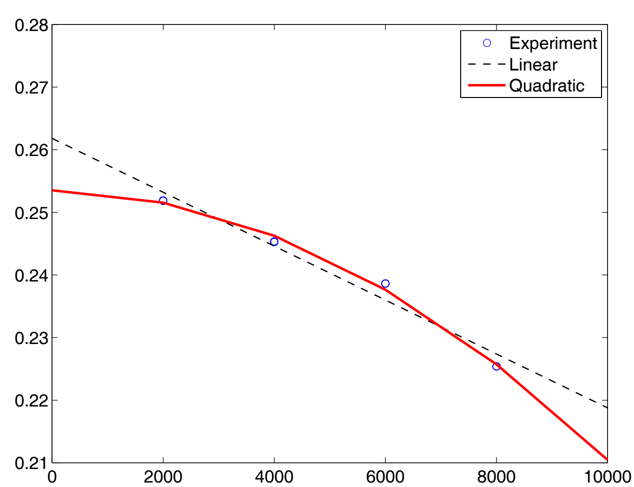

Figure 3: Transverse decoherence. The transverse tune ν_x dependence on a horizontal kick. ν_x are analyzed right after the kick and before damped out.

For a small transverse kick, assuming amplitude-dependent betatron frequency is given by $\omega(J) = \omega_0 + \omega' J$, the slowly varying oscillation amplitude is [4]

$$\langle x \rangle^{\text{ampl}}(t) = \frac{\beta\theta}{1 + \Theta^2} \exp\left[-\frac{\beta\theta^2\Theta^2}{2J_0(1 + \Theta^2)}\right]$$

where $\Theta \equiv \omega' J_0 t$. The linear dependence on J_0, therefore quadratic on kick strength observed at ALS is shown in Fig. 3.

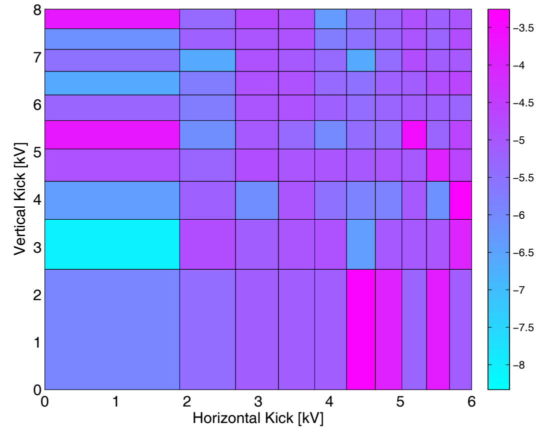

Figure 4: Frequency Map of a Multi-BPM measurement, the color and diffusion represents $\log_{10}(\Delta\nu_x/N_{\mathrm{Turn}})$ at different kick strength.

A FMA experiment was carried out by combining several horizontal and vertical kicks ranging up to 6 kV and 8 kV. There were 65 BPMs and each recorded 1024 turns of the beam centroid position. 10 turns of data starting from the 100^{th} and 120^{th} turn are used for frequency analysis with NAFF. The tune diffusion rate shown in Fig. 4 are $\log_{10}((\nu_{x,100} - \nu_{x,120})/20)$. The magnitude of the diffusion rate as well as the general features of the frequency map agree reasonably with simulated data.

CONCLUSION

We showed a method using multiple BPM data to improve the convergence of the transverse tune measurement, and proved this both mathematically and numerically. This method then was used for a transverse decoherence study and Frequency Map Analysis. It has advantages for the study of cases with fast decoherence using turn-by-turn data from multiple BPMs. We plan to continue to test the method in more detail in the future and carry out detailed comparisons of calculated and measured tune diffusion rates.

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APPENDIX

It is well known that DFT has a resolution proportional to the number of points in the time series. NAFF [3] has a much better precision, can be up to N^4 for analyzing the fundamental frequency. This section shows the comparison between DFT, interpolated DFT and NAFF.

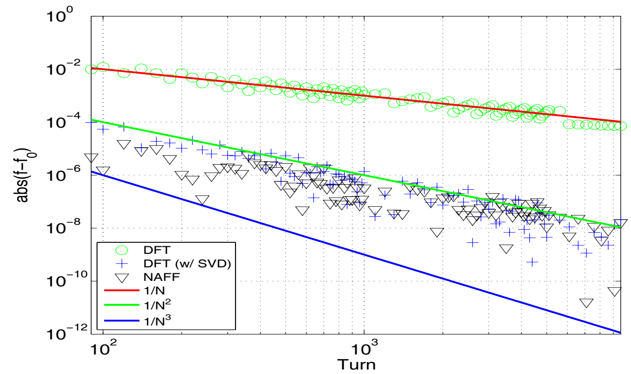

Figure 5: The frequency precision depends on number of points. The test signal has single frequency and a white noise, σ equals 1% signal amplitude.

Two types of noise are added to a simple harmonic oscillation. One is the noise of displacement, which can be from instruments, as shown in Fig. 5. The signal is $s(t) = \sin(2\pi\nu_x t) + AN(\sigma)$, where $N(\sigma)$ is a standard white noise and A is its amplitude. Due to the searching mechanism, NAFF could give better result when noise is smaller, while DFT could not.

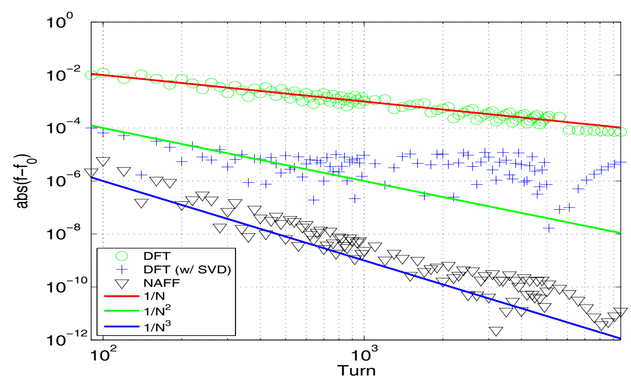

Figure 6: Comparison of different frequency analysis method. The test signal has an exponential amplitude damping, damping time $\tau = 1000$ turns.

The other type of comparison based on the fact that the beam may have decoherence. For the purpose of comparing the precision of three numerical method, we assume the amplitude decays exponentially, i.e. $s(t) = (1 + e^{-t/\tau})\sin(2\pi\nu_x t)$. The result is shown in Fig. 6 for $\tau = 1000$ turns. NAFF will have better asymptotic behavior if the damping time is shorter.