STUDIES OF THE ν_r = 3 / 2 RESONANCE IN THE TRIUMF CYCLOTRON*

Y.-N. Rao, R. Baartman, G. Dutto, L. Root, TRIUMF, Vancouver, Canada

Abstract

The TRIUMF cyclotron is 6-fold symmetric, but has a 3rd harmonic magnetic field gradient error. As well, there is a 3rd harmonic component generated from the beating of the primary harmonics with the 9^{th} harmonic. Both can contribute and drive the $\nu_r = 3/2$ resonance. As a consequence, the radial phase space ellipses become stretched and mismatched; this introduces a radial modulation of beam density and thereby causes a sensitivity of the extracted current to, for example, small changes in rf voltage. The cyclotron has "harmonic" correction coils, but these were designed to generate a first harmonic, not a third harmonic. Their 6-fold symmetric layout can only generate a 3rd harmonic at one particular phase and so can only partially compensate for this resonance. For a complete compensation, the 6 pairs of this harmonic coil would have to shift in azimuth by $\sim 30°$. This paper describes the simulations performed with COMA to study the effect of this resonance. Initial measurement results are also presented.

INTRODUCTION

The TRIUMF cyclotron operates such that the extraction stripper foils for beamline 1 and beamline 2A are configured in radial shadow mode at 500 MeV. Slight changes in the circulating beam orbit due to small changes in rf voltage can cause a large fluctuation of the split ratio of beam currents between BL1 and BL2A. This is because there exists a radial modulation in the beam density. This modulation starts at ~ 428 MeV and persists to 500 MeV extraction. Actual measurements with a high energy probe with a radial differential head have demonstrated such an oscillation in the beam density. This oscillation originates from the $\nu_r = 3/2$ resonance occurring at ~ 428 MeV. In order to understand this resonance, we performed Monte Carlo simulations using the linear motion code `COMA` [1]. `COMA` tracks particles using matrices obtained from the equilibrium orbit calculations and can simulate the probe scan and the interaction of beam with extraction foil, etc.

THIRD HARMONIC IN BASE FIELD

The standard magnetic field of the TRIUMF cyclotron is described in Fourier harmonics, among which there exists a third harmonic component. This residual third harmonic is plotted in Fig. 1 over a region of radius between 288 and 306 inch. This is the region in which we are interested, because the $\nu_r = 3/2$ resonance occurs at around 428 MeV at which the static equilibrium orbit scallops between $\sim$ 291.6 and $\sim$ 299.2 inch. Also plotted in Fig. 1 is the 3rd harmonic amplitude and phase angle.

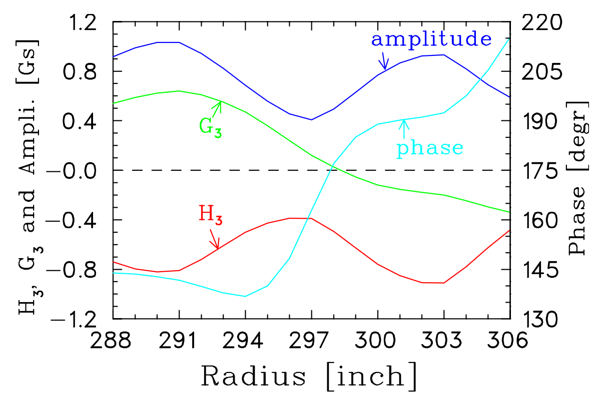

Figure 1: Plot showing the Fourier coefficients H_3 and G_3 of the residual third harmonic component in the base magnetic field, in a region of radius between 288 and 306 inch. Also shown is its amplitude and phase angle.

The $\nu_r = 3/2$ resonance is driven primarily by the radial gradient of the third harmonic amplitude. We calculated the gradient dA_3/dR from Fig. 1 and found it to be 0.1 G/inch at maximum. Can such a small gradient cause betatron amplitude growth, leading to a mismatch of the radial phase space?

MISMATCH CAUSED BY RESONANCE

We ran `COMA` to simulate the passage of beam through the resonance [2]. The transfer matrices used in the `COMA` run were obtained from the equilibrium orbit calculations. `COMA` simulations started at $\sim$300 MeV, safely below the $\nu_r = 3/2$ resonance energy of 428 MeV. Optimum centering conditions were chosen for the orbit such that the coherent oscillation amplitude was 100 times smaller than the typical incoherent betatron amplitude of $\sim$0.15 inch, and also a proper rf phase was selected for the central ray such that it took a minimum number of turns to travel from 300 MeV all the way to 500 MeV. Moreover, at start, the radial phase ellipse was exactly matched with cyclotron acceptance and the particles generated all had the same rf phase as the central ray. Tracking calculations were carried out and particles' radial phase space was recorded turn by turn. The results are shown in Fig. 2, over an energy range from 420 MeV to 452 MeV.

Clearly, the ellipse becomes stretched as the beam is accelerated approaching 428 MeV. As the beam is accelerated away from the resonance, the ellipse, which is now

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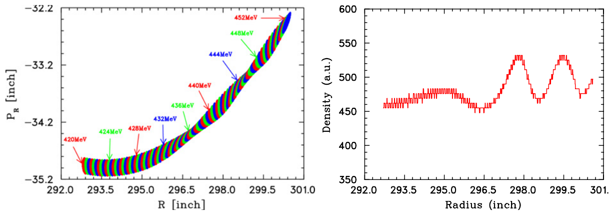

Figure 2: Left: turn-by-turn variation of the radial phase space simulated for passage through the $\nu_r = 3/2$ resonance. Clearly, one can see the rotation (precession) of the ellipse. Right: radial modulation of beam density simulated, caused by this precession of the radial ellipses. Here we only illustrate the first 2 periods of precession and modulation; in fact, the precession and density modulation persist to 500 MeV extraction.

mismatched with the cyclotron acceptance, starts to rotate. Such a rotation, or precession, leads to a modulation of beam radial density, as is shown in Fig. 2. Such a modulation in fact persists to 500 MeV, and the rate of precession increases as $(\nu_r - 3/2)$ increases (see Fig. 5).

One would guess that the resonance is completely driven by the third harmonic error as is shown in Fig. 1. If this is true, then the resonance should disappear after the third harmonic radial gradient is eliminated out of the field. For simplicity, in our studies we left out the third harmonic by bringing the Fourier harmonics H_3 and G_3 both to zero from radius 285 all the way to 318 inch, but keeping it as it was at elsewhere. Using this "new" field, new transfer matrices were generated, followed by a `COMA` run repeated with the starting conditions at 300 MeV. The results appear that the mismatch and therefore the density modulation amplitude are reduced by ∼40% only, but still remain ∼60% (see Fig. 3).

This turns out to be due to a beating effect between the harmonics in B_z, e.g. the 6th harmonic beating the 9th harmonic yields an additional 3rd harmonic. The radial motion, in the first order approximation, can be described with a differential equation [3]

$$\frac{d^2x}{d\theta^2} + \nu_x^2 x = x\left[\frac{r_{\rm eo}^2}{\overline{B}\,\overline{R}}\left(\frac{d\Delta B}{dr}\right)_{r=r_{\rm eo}} + \frac{2r_{\rm eo}\,\Delta B(r_{\rm eo})}{\overline{B}\,\overline{R}}\right] \tag{1}$$

where ΔB denotes the Fourier components except for the primary harmonics of $6N(N = 1, 2, \ldots)$, represented as

$$\Delta B = \sum_{n\neq 6N} C_n(r)\cos\left(n\theta + \phi_n\right) \tag{2}$$

and $r_{\rm eo}$ denotes the radial coordinate of the equilibrium orbit. In the first order approximation, $r_{\rm eo}$ is represented as

$$r_{\rm eo} = \overline{R} + \sum_{N=1}^{N_{max}} A_N \cos\left(6N\theta + \phi_{6N}\right) \tag{3}$$

Here $r_{\rm eo}^2$ contains a term of $\cos(12\theta)$. This term times the 9th harmonic term $\cos(9\theta)$ as contained in the $d\Delta B/dr$ gives an extra 3rd harmonic and a 21st harmonic.

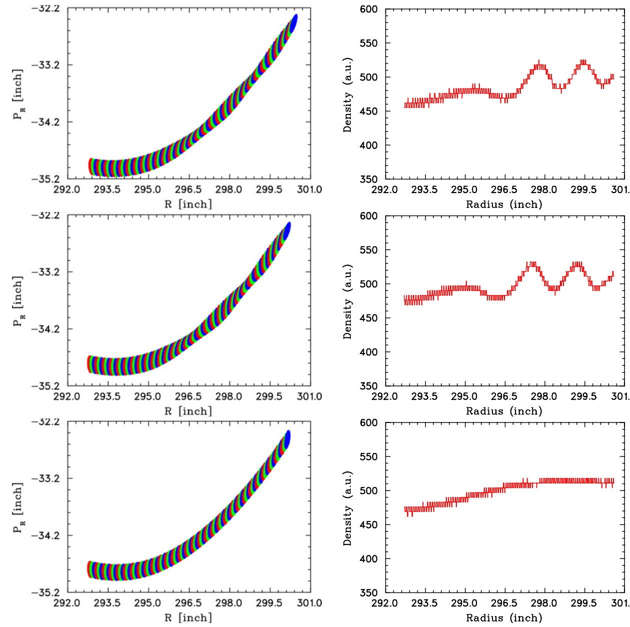

Figure 3: Top: The residual third harmonic shown in Fig. 1 was left out over radii from 285 to 318 inch but kept as it was at elsewhere. Compared to Fig. 2, the mismatch and therefore the density modulation are reduced by only ∼40%. Middle: The primary harmonics and the 9th harmonic were kept as they were, but the others were all brought to zero. Comparing with the top plots, one can see that the mismatch and density modulation are almost unchanged. Bottom: Only the primary harmonics were kept as they were, while all the others were brought to zero in the same radial region. As a result, the mismatch and density modulation disappear. This confirms that the beating effect between the primary harmonics and the 9th harmonic is responsible for the ∼60% mismatch and density modulation that remain after the 3rd harmonic error is left out.

The above theory was proved through two sets of simulations. One set is keeping the primary harmonics and the 9th harmonic as they were, but bring all the other harmonics to zero. Another run is keeping only the primary harmonics as they were, but leave out all the other harmonics. For both runs, the results are compared in Fig. 3.

CORRECTION OF RESONANCE

The TRIUMF cyclotron is equipped with 13 sets of harmonic coils, where the HC#13 can render a third harmonic gradient of up to 1.0 G/inch in the energy region of interest. We attempted to use HC#13 to produce a field bump and added it onto the base field, and then repeated the previous `COMA` runs, to see whether we could correct the resonance.

The field of HC#13 was surveyed on the median plane in such a pattern that coils on the sectors 5 and 2, 6 and 3, 4 and 1 were powered alone in B_z mode and excited with the same amount of current of 40 A. This survey covered a radial region from 162 to 324 inch in a step of 3 inch. Fig. 4

portrays the field strength vs azimuth at radius of 306 inch, where the coils excitation was 40 A. It also shows the amplitude and phase over radii between 288 and 312 inch. Using these coils, we can produce a third harmonic but we cannot change its phase except for a switch of $180°$. Simulations using this field show that we can only obtain a

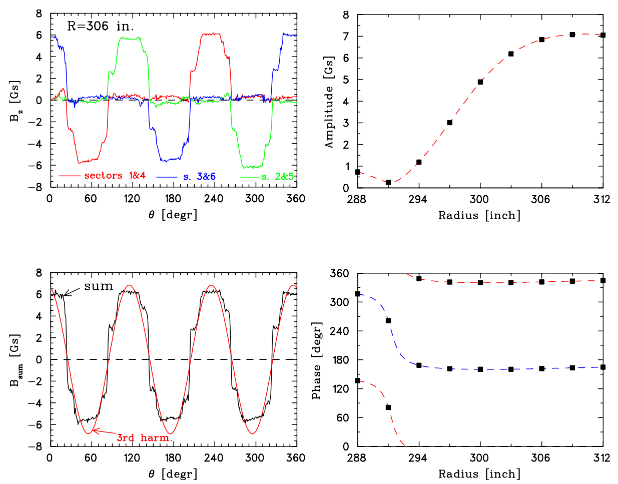

Figure 4: Top-Left: Surveyed magnetic field for HC#13 vs azimuth (at radius of 306 inch), where the coils on the sectors 1 & 4, 2& 5, 3& 6 were excited alone in B_z mode with 40 A. Bottom-Left: Sum of the whole 6-sector's excitations of 40 A (black) and the corresponding 3rd harmonic component obtained from Fourier transform (red). Amplitude (Top-Right) and phase (Bottom-Right) of the 3rd harmonic produced with HC#13 at 40 A excitation. The dots correspond to the surveyed data; the dash lines are meant to guide the eye. Note that there are only two possible phases $180°$ apart (the red and blue).

partial correction. This is achieved by using an excitation of ∼10 A and the red dash line depicted phase in Fig. 4, that is: coils on sectors 1 and 4 in normal polarity, 2 and 5 in reverse polarity, 3 and 6 in normal polarity. In this way, we could expect to reduce the density modulation by $\sim 40\%$ in its amplitude. See Fig. 5. This was demonstrated by the actual measurements using high energy probe. See Fig. 6. The reason why we cannot get a complete correction is because the 3rd harmonic produced does not have exact match in the phase. Further simulation shows that, for a complete correction (see Fig. 6), the existing 6 pairs of coils would have to shift by $30°$ in azimuth.

CONCLUSIONS

Simulation results show that the 3rd harmonic error remaining in the base field and the beating created 3rd harmonic component both can contribute and drive the $\nu_r = 3/2$ resonance. As a consequence of this resonance, the radial phase ellipses of beam become stretched and mismatched, so causing a radial modulation of beam density and thereby an instability of ratio of the intensities of the two simultaneously-extracted beams in radial shadow

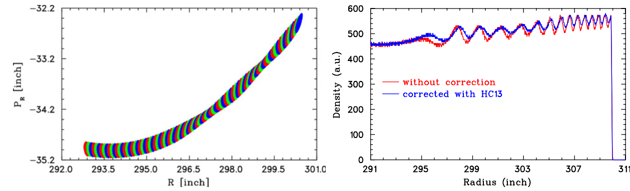

Figure 5: Results of simulation for correcting the resonance by using harmonic coil #13. This is the maximum correction that could be achieved by using the phase as shown in the red dash line in Fig. 4 and an excitation strength of ∼10 A. Compared with the case of no correction, the stretches of ellipses become less, and the density modulation gets reduced by ∼40% in its amplitude.

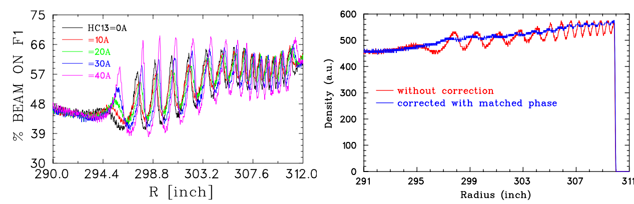

Figure 6: Left: Plot showing the high energy probe differential finger measured radial beam density modulations, under various excitations of HC#13. It's clear that a 10 or 20 A excitation reduces the oscillation amplitude by $\sim 40\%$ (the difference between 10 and 20 A is very small), whereas 30 A and above make the modulations even worse. This result exactly agrees with the simulation predicted. Right: Plot showing the result of simulation, where the existing 6 pairs of coils in HC#13 are shifted by $30°$ in azimuth, as a consequence, the resonance and therefore the density modulation get almost completely corrected.

mode. By using the existing harmonic coil #13, we can produce a 3rd harmonic that could only partially compensate for this resonance and therefore reduce the density oscillation by ∼40%. For a complete compensation, the 6 pairs of coils in HC#13 would have to be shifted by $\sim 30°$ in azimuth.

ACKNOWLEDGMENTS

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