RADIATION OF A CHARGE CROSSING A LEFT-HANDED MEDIUM BOUNDARY AND PROSPECTS FOR ITS APPLICATION TO BEAM DIAGNOSTICS*

A.V. Tyukhtin#, S.N. Galyamin, T.Yu. Alekhina, E.G. Doil'nitsina
Physical Dep. of St. Petersburg State University, St. Petersburg, 198504, Russia

Abstract

We analyze the radiation of a charge crossing the boundary between vacuum and a medium having a "left-handed" frequency band realized in some modern metamaterials. We consider both the case of a semi-infinite medium and the case of a circular waveguide and obtain exact and approximate expressions for the field components. We develop an algorithm for their computation. It is shown that two types of radiation can be generated in vacuum. One of them is an ordinary transition radiation having a relatively large magnitude. Another type of radiation, identified in this study, is referred to as "Cherenkov-transition" radiation. Conditions for generating this radiation are obtained. Properties of radiation can be used for diagnostics of beams. For example, a detector with two energy thresholds can be designed.

INTRODUCTION

In the sixties, Veselago predicted the existence of media having simultaneously negative permittivity ε and permeability μ [1,2]. Under these conditions, the electric field vector, magnetic field vector, and wave vector form a left-handed orthogonal set. Thus, such media are usually referred to as "left-handed media" (LHM). The direction of the energy flow and the direction of the phase velocity are opposite in LHM. Artificial metamaterials possessing "left-handed" properties in gigahertz frequency band were presented recently [3–6]. They are composed of distinct conducting elements having a size and spacing much less than the wavelengths of interest. Therefore such media can be described by the macroscopic parameters ε and μ being simultaneously negative in some limited frequency range only [2]. Cherenkov radiation (CR) in LHM was investigated in ref. [7,8]. Transition radiation (TR) at the interface between vacuum and LHM has not been considered until now.

In the present paper, we analyze the electromagnetic field (EMF) generated by a small bunch with a charge q passing through the interface ($z=0$) separating two homogeneous isotropic media described by permittivities and permeabilities ε_1,μ_1 ($z<0$) and ε_2,μ_2 ($z>0$). The bunch moves uniformly along the z-axis ($z=Vt=c\beta t$). The medium is assumed to be dispersive. It has both a "left-handed" frequency band and a "right-handed" one. We compare the spatial distribution of the spectral harmonics of the EMF in two cases:

(1) the medium is "right-handed" at the frequency under consideration;

(2) the medium is "left-handed" at the frequency under consideration.

SOME GENERAL RESULTS

The spectral harmonics of the scalar Φ_ω and vector $\vec{A}_\omega = A_\omega \vec{e}_z$ potentials are presented as the sum [9]

$$\begin{Bmatrix} A_{\omega 1,2} \\ \Phi_{\omega 1,2} \end{Bmatrix} = \begin{Bmatrix} A^b_{\omega 1,2} \\ \Phi^b_{\omega 1,2} \end{Bmatrix} + \begin{Bmatrix} A^q_{\omega 1,2} \\ \Phi^q_{\omega 1,2} \end{Bmatrix}. \quad (1)$$

The "forced" field A^q_ω, Φ^q_ω is the charge field in an unbounded medium:

$$\begin{Bmatrix} A^q_{\omega 1,2} \\ \Phi^q_{\omega 1,2} \end{Bmatrix} = \frac{iq}{2} \begin{Bmatrix} c^{-1}\mu_{1,2}(\omega) \\ V^{-1}\varepsilon^{-1}_{1,2}(\omega) \end{Bmatrix} \exp\left(i\frac{\omega}{V}z\right) H_0^{(1)}\left(s_{1,2}(\omega)\rho\right), \quad (2)$$

where $H_0^{(1)}(\xi)$ is the Hankel function, $\rho=\sqrt{x^2+y^2}$, $s_{1,2}(\omega)=\sqrt{\omega^2V^{-2}\left(n^2_{1,2}(\omega)\beta^2-1\right)}$, $\operatorname{Im} s_{1,2}>0$, and $n^2_{1,2}(\omega)=\varepsilon(\omega)_{1,2}\mu_{1,2}(\omega)$. The "forced" field contains CR if the charge velocity exceeds the Cherenkov threshold. "Free" field A^b_ω, Φ^b_ω is expressed as

$$\begin{Bmatrix} A^b_{\omega 1,2} \\ \Phi^b_{\omega 1,2} \end{Bmatrix} = \frac{q}{\pi V}\int_0^\infty \begin{Bmatrix} A^b_{\omega,k_\rho 1,2} \\ \Phi^b_{\omega,k_\rho 1,2} \end{Bmatrix} J_0(k_\rho \rho)\exp\left(ik_{z1,2}|z|\right)k_\rho dk_\rho, \quad (3)$$

where $k_{z1,2}=\sqrt{\omega^2c^{-2}n^2_{1,2}-k^2_\rho}$, $\operatorname{Im} k_{z1,2}>0$, and $A^b_{\omega,k_\rho 1,2}$, $\Phi^b_{\omega,k_\rho 1,2}$ are some functions of k_ρ and ω determined by the boundary conditions.

Two methods were applied to investigate integral (3). First, using the steepest descent technique, we obtained asymptotic expressions of (3). They are valid for $\omega c^{-1}|n_{1,2}|R \gg 1$, where $R=\sqrt{\rho^2+z^2}$. As was demonstrated, the most interesting novel effects were related to the saddle point contribution yielding TR and the contribution of one of the poles affecting the "forced" field. In the case of LHM, this effect can be interpreted as a new type of radiation; we refer to it as "Cherenkov-transition radiation" or CTR (see below). Second, using a certain transformation of the integration path in the complex plane, we produced an effective algorithm

*Work supported by the Education Agency of Russian Federation, RFBR (09-02-00921a, 09-02-08080z), SBIR DOE (DE-FG02-08ER85031).
#tyukhtin@bk.ru

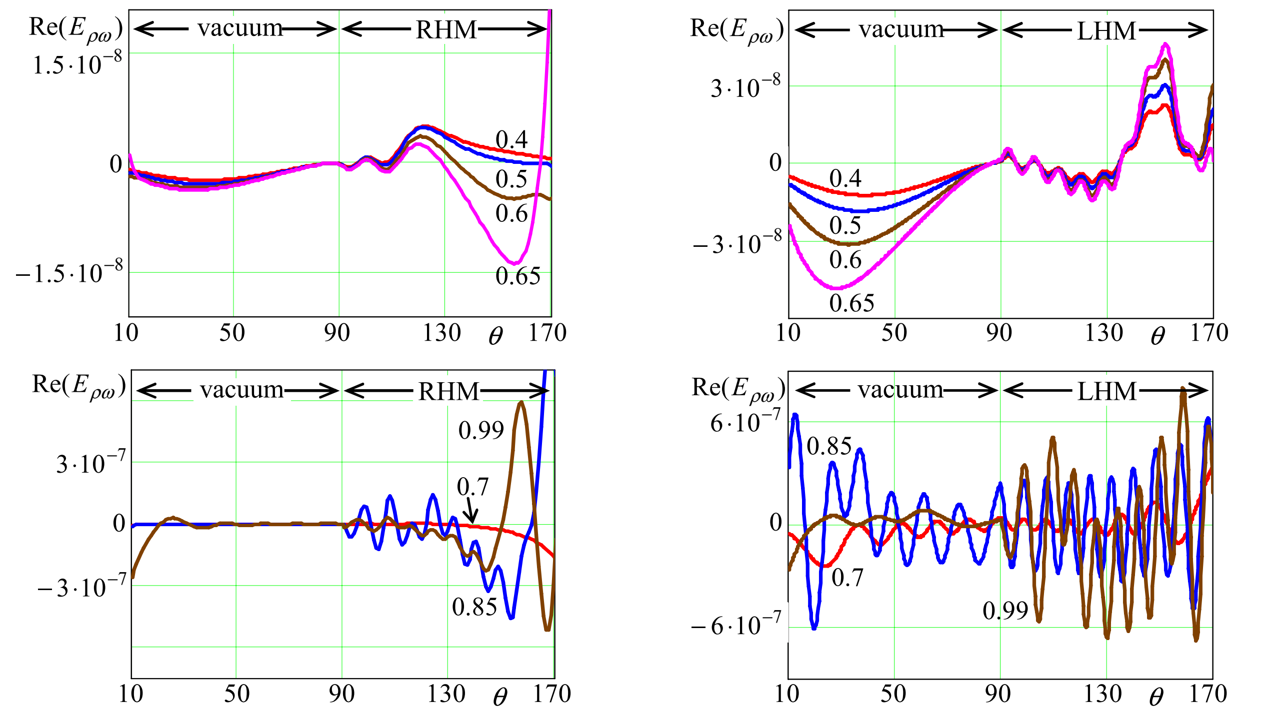

Figure 1: Dependence of $\mathrm{Re}\, E_{\rho\omega}\, (\mathrm{V \cdot m^{-1} Hz^{-1}})$ on the angle θ for vacuum–RHM (left) and vacuum–LHM (right) for $\beta < \beta_{CR} = 0.7$ (top) and $\beta > \beta_{CR}$ (bottom) ; $\varepsilon = 1.7$, $\mu = 1.2$ for RHM, $\varepsilon = -1.7$, $\mu = -1.2$ for LHM; $q = -1\,\mathrm{nC}$, $\nu = 10\,\mathrm{GHz}$, $R = 15\,\mathrm{cm}$. The magnitudes of β are indicated near the curves. The full internal reflection threshold corresponds to $\beta_{FIR} = 0.98$.

for computing integral (3) as well. The results of both methods were in a good agreement.

SPATIAL DISTRIBUTION OF THE FIELD HARMONICS

We considered the behavior of the spectral harmonics of the full field at the semicircle $R = const$ in the xz-plane. The angle θ increases from 0 (negative part of the z-axis) to π (positive part). Some results of the computations are presented in fig.1. The half-space $z < 0$ was assumed to be vacuum. Parameters of the half-space $z > 0$ are indicated in the figure's caption. The main effects were as follows:

First, we considered the situation where the charge velocity does not exceed the Cherenkov threshold for the medium ($\beta < \beta_{CR} = |n_2|^{-1}$). One can see (fig. 1, top) that in case (2) (vacuum–LHM) the TR is much larger than in case (1) (vacuum–RHM).

Second, we considered the situation where the charge velocity exceeds the Cherenkov threshold, and therefore CR is generated in the medium. In case (1) the EMF is relatively small both in vacuum and in the medium near the boundary (fig. 1, bottom). In case (2) the EMF is of the same order of magnitude in the whole half-space $z > 0$. The most important finding is that a significant EMF can be observed in vacuum at some velocities. This effect is explained by the backward direction of CR in case (2), i.e., the energy flux density of CR forms an obtuse angle with the direction of the charge motion (fig. 2, top). CR falls on the boundary, resulting in reflected and transmitted fields (they can be named “Cherenkov-

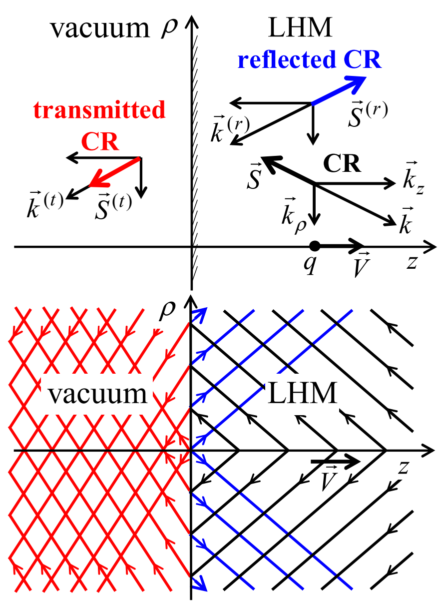

Figure 2: Top: Wave vectors and power flux densities of CR and CTR (reflected and transmitted waves). Bottom: The lines are parallel to the power flux density of CR (black), the reflected wave (blue) and the transmitted wave (red); one can see areas of interference.

transition" radiation, CTR). The reflected wave interferes with CR in the vicinity of the interface, where black lines intersect blue ones (fig. 2, bottom). The transmitted waves emerging from the upper and lower parts of the interface interfere in the vicinity of the negative z-semi-axis, where red lines intersect themselves (fig. 2, bottom). One can see an evident interference at angles $\theta < 50^\circ$ for $\beta = 0.85$ and at angles $90^\circ < \theta < 130^\circ$ for $\beta = 0.99$ (fig. 1, right, bottom).

Using the mentioned asymptotic representation, we obtained the following rigorous conditions: CTR in vacuum (transmitted wave) exists if

$$\beta_{CR} < \beta < \beta_{FIR}\,, \qquad (4)$$

where $\beta_{FIR} = (n_2^2 - 1)^{-1/2}$. The lower threshold of CTR in vacuum is the Cherenkov threshold and the upper one is connected with the full internal reflection at the interface. Result (4) is demonstrated in fig. 1, right, bottom. This effect may be useful for the creation of double-threshold detectors of charged particles.

THE CASE OF WAVEGUIDE

The radiation of a charge moving along the axis of a cylindrical waveguide of radius a through the boundary between vacuum and LHM was studied as well. It should be mentioned that the solution of such a problem in the case of RHM was obtained in ref. [10].

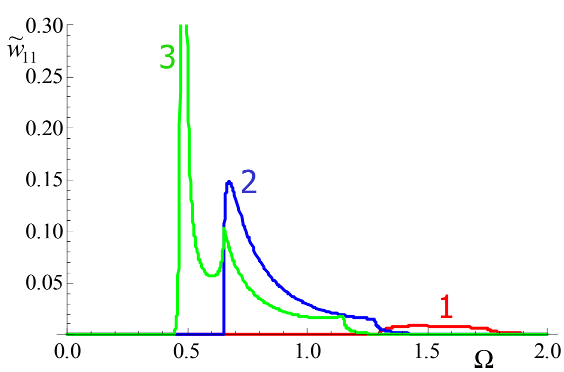

Figure 3: Dependence of the energy spectral density of the first mode in vacuum $\tilde{w}_{11}$ (normalized with multiplier $\pi c/4q^2$) as a function of the normalized frequency $\Omega = \omega/\omega_{pe}$ for different values of the radius: 1 – $a = 1$ cm, 2 – $a = 2$ cm, 3 – $a = 3$ cm; $\beta = 0.5$, $\omega_{pe} = 9$GHz.

As usual, the EMF was presented as a decomposition in an infinite series of normal modes with resonance frequencies $\omega_n = \chi_{0n} c a^{-1}$, where n is the mode number and χ_{0n} is the n^{th} zero of the Bessel function ($J_0(\chi_{0n}) = 0$). Next, the energy passing through the cross section of the waveguide in vacuum Σ_1 and in LHM Σ_2 for all time was calculated. These values were obtained as integrals with respect to ω and the sum of spectral energy densities of modes $w_{n1,2}$:

$$\Sigma_{1,2} = \int_0^\infty d\omega \sum_{n=1}^{\infty} w_{n1,2}(\omega)\,. \qquad (5)$$

If the charge velocity exceeds the Cherenkov threshold CR with frequencies ω_{0n} are generated in the medium. These frequencies depend on the charge velocity, parameters of the medium and the waveguide radius. The conditions for existence of CTR in vacuum are stricter than those in the case without the waveguide. For the mode with number n, they can be written as

$$\left|n_2(\omega_{0n})\right|^{-1} < \beta < \left|\left(n_2^2(\omega_{0n}) - 1\right)^{-1/2}\right|. \qquad (6)$$

The energy spectrum of the n^{th} mode of TR in the medium includes both a left-handed frequency band and a right-handed one. The energy spectrum of the n^{th} mode of TR in vacuum is limited from below by ω_n, as well as in the case of RHM. TR is excited if the radius of the waveguide exceeds some critical value. TCR is generated for larger values of radius. This effect is illustrated by fig. 3 in the case where the left-handed frequency band lies between 0 and $0.75\omega_{pe}$ (ω_{pe} is an electron plasma frequency). The energy spectrum of the first mode in vacuum $\tilde{w}_{11}$ is shown. One can see three cases:

- the 1st mode of TR exists in the right-handed range only (curve 1);
- the 1st mode of TR includes both the right-handed range and the left-handed one, but CTR is not generated (curve 2);
- the 1st mode of TR includes both the right-handed range and the left-handed one, and CTR is generated (curve 3).

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