# STUDY ON DEPOLARIZATION TIME OF RESONANT DEPOLARIZATION EXPERIMENT*

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*Abstract*

Radial alternating magnetic field is generated to act on polarized beam to give rise to resonant depolarization and calibrate the energy of electron by feeding power to a pair of vertical installed striplines in HLS (Hefei Light source). In the paper, the relationship between depolarization time and power fed into the striplines is investigated, and spin frequency spread is considered too. As a result, a depolarization time of 64s is acquired with an amplifier power of 15W fed into the striplines.

## INTRODUCTION

It was first mentioned by Ternov et al. [1] in the early sixties of last century that the electrons circulating in the storage ring naturally polarize with time due to emitting synchrotron radiation in the form of quantum of photons. With the so-called Sokolov-Ternov [2] effect, the maximum degree of polarization can be achieved in a ring without vertical bendings while the spin of electrons precesses around the direction of guiding field $H_z$ with frequency $\Omega_z$ in the laboratory coordinate system,

$$\Omega_z = (1 + a\gamma)\omega_o = (1 + \upsilon_s)\omega_o \qquad (1)$$

where $a$ is the anomalous magnetic moment of electron, γ the relativistic Lorentz factor, $\omega_o$ the revolution angular frequency of synchronous electrons in the ring, and $\upsilon_s$ the spin tune describing the times the spin vector precesses around the bending field for one turn electron cycles around the ring. In the meantime, if there is an alternating magnetic field applied to the horizontal direction, resonance phenomenon would occur when the condition below is met,

$$\omega_{dep} = \Omega_z \pm n\omega_o = (a\gamma \pm m)\omega_o,\quad n, m\text{=integer} \qquad (2)$$

and the polarization of beam would be destroyed. Since $\upsilon_s$ is determined by energy of electron only, the electron energy can be calculated accurately with the resonance frequency.

Once the polarized beam is acquired, resonant depolarization experiments can be carried out to calibrate the energy of electron. The polarization of beam was proved exists in HLS lately. Following that, we'll use a frequency generator and a power amplifier to feed high-frequency power into the striplines vertical installed in the storage ring and generate a radial alternating magnetic field to depolarize the polarized beam to calibrate the electron energy.

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## SPIN MOTION AND THE PROBLEMS OF SPIN FREQUENCY SPREAD

The spin motion of a charged particle moving in a static electromagnetic fields is given by the interaction of its intrinsic magnetic moment with the magnetic field $\vec{B}$:

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} = \vec{\Omega} \times \vec{S} \qquad (3)$$

For an electron, here $\vec{\mu} = \frac{-ge}{2m_e}\vec{S}$ is the intrinsic magnetic moment, e the elementary charge, $m_e$ the rest mass of electron, $\vec{S}$ the spin-vector of electron; and gyromagnetic ratio g=2 in Dirac theory while in actual experimental measurements it deviates a bit from 2 due to the radiative correction of quantum electrodynamics. This correction is represented by the gyromagnetic anomaly $a = (g-2)/2 = 0.0011596$.

$$\vec{\Omega} = \frac{ge}{2m_e}\vec{B} = \frac{e(a+1)}{m_e}\vec{B} \qquad (4)$$

is the rotation vector of spin-vector. In relativistic cases, implement Lorentz transformation in respect of the particle rest frame, $\vec{\Omega}$ turns to $\vec{\Omega}_{BMT}$:

$$\vec{\Omega}_{BMT} = \frac{e}{m_e\gamma}\left[(1+\gamma a)\vec{B}_\perp + (1+a)\vec{B}_{//} - (a + \frac{1}{1+\gamma})\gamma\vec{\beta} \times \frac{\vec{E}}{c}\right] \qquad (5)$$

and the spin motion of relativistic electron is described by the Thomas-BMT equation:

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{BMT} \times \vec{S} \qquad (6)$$

Here, $\vec{\Omega}_{BMT}$ is the precession frequency of spin with regard to the laboratory frame. $\vec{B}_\perp$ and $\vec{B}_{//}$ denote transverse and longitudinal component of magnetic field relative to the electron velocity respectively; $\vec{\beta}$ is the ratio of the velocity to the light velocity c. The angular frequency of spin-vector motion about the rotation vector $\vec{\Omega}_{BMT}$ is [3]

$$\frac{d\theta}{dt} = \left|\vec{\Omega}_{BMT}\right| \qquad (7)$$

From the equation (5), it shows that the magnetic term B and electric term $E/c$ have nearly the same effect, that is to say, a magnetic filed of 1 Tesla is comparable to an electric field of $3\times10^8 V/m$. Normally, the effect of the electric field in a storage ring can be neglected comparing with the magnetic fields. Besides, consider only the ideal particle cycling in a flat ring, the magnetic component $\vec{B}_{//}$ is neglected too. Finally, the rotation vector of spin-vector turns into, just the same as the formula (1),

$$\vec{\Omega}_{BMT} = \frac{e(1+\gamma a)}{m_e\gamma}\vec{B}_{\perp} = \vec{\omega}_o + \frac{ea}{m_e}\vec{B}_{\perp} \qquad (8)$$

where $\vec{\omega}_o = \frac{e\vec{B}_{\perp}}{m_e\gamma}$ is the cyclotron angular frequency. The precession frequency of the particle's spin in itself frame is $\frac{ea}{m_e}\vec{B}_{\perp}$. So, we derive in the particle frame that

$$\frac{d\theta}{dt} = \frac{ea}{m_e}B_{\perp} \quad \text{or} \quad \frac{d\theta}{ds} = \frac{ea}{m_e c}B_{\perp} \qquad (9)$$

the latter is expressed by the azimuth variable s instead of the time variable t.

Now, while passing the striplines for one time, the spin is rotated towards horizontal by the radial field with an angle

$$\Delta\theta = \int_0^l \frac{ea}{m_e c}B_x ds \qquad (10)$$

Here subscript x denotes the direction of radial magnetic field $B_x$, and $l$ is the length of striplines. Since the magnetic signal generated by the striplines is a sinusoidal waveform, the effective action of kicker should be averaged over half-cycle time [4] and one replaces $B_x$ by $\bar{B}_x = \frac{2}{\pi}B_x$. So, the spin is rotated by $\overline{\Delta\theta} = \int_0^l \frac{2ea}{\pi m_e c}B_x ds$.

When the precession cone angle charges $n\cdot\overline{\Delta\theta} = \pi/2$ for the ensemble of electrons after passing through the striplines n times, the polarization of beam is destroyed entirely. The time needed for this change is

$$\tau = \frac{n}{f_0} = \frac{\pi}{2}\left[\int_0^l \frac{2ea}{\pi m_e c}B_x ds\right]^{-1}\cdot\frac{1}{f_0} \qquad (11)$$

where $f_0$ is the cyclotron frequency.

If the energy and states are same for all electrons, the depolarization time can be described by the formula above. But in fact the states of the large number of electrons in the beam are diverse, and the energy of electrons has also some disparity. The actual distribution of electrons has a nominal energy E with a certain extent of spread. So the spin frequency and resonance frequency have a distribution too, and depolarization resonance occurs in a certain range of frequency sweep in the resonant depolarization experiments. Therefore, the factual depolarization time is related to the sweep interval $\Delta\omega_{dep}$ in view of spin frequency spread. For equilibrium particle, resonant frequency consists of a series of resonance points $\Omega_z \pm n\omega_o$. Due to the energy spread, spin frequency spread exists too. The resonance points originally turn into a series of resonance regions.

For a synchronous electron the spin precession frequency is described by formula (1), while for an asynchronous electron is described by

$$\Omega = \left(1+\frac{E_o+\Delta E}{mc^2}a\right)\cdot\left(1+\alpha\frac{\Delta E}{E_o}\right)\omega_o \qquad (12)$$

where $\alpha$ is momentum compaction factor. Moreover, the asynchronous electrons oscillate with the synchrotron frequency $\omega_s$ in longitudinal direction. With this modulation the precession frequency turns into a spectrum consisting of a central line and many synchrotron sidebands spaced by $\omega_s$. The width of the central line is [5]

$$\langle\delta\Omega\rangle_s = \frac{1}{2}\alpha\upsilon_s\left(\frac{\Delta E}{E_0}\right)^2\omega_o \qquad (13)$$

Furthermore, taking into account the effects of nonlinear magnetic field on the energy spread, the precession frequency spread is mainly caused by the quadratic nonlinearity $\partial^2 H_z/\partial x^2$. Betatron oscillations caused by this nonlinear effect, via transverse-longitudinal coupling, results in new energy spread. Ref. 5 indicates that the magnitude of spin frequency spread among various electron storage rings is normally not exceeding $10^{-5}\omega_o$, ensuring the precision of the resonant depolarization experiments. Nonetheless, the effective width of resonant region $\Delta\omega_{dep}$ should be measured by experiment because there are many other factors may cause frequency spread such as power supply ripple and so on.

If we assume the width of central line of frequency spread is $10^{-5}\omega_o$, i.e. 2π×45.33Hz, the uncertainty of energy in the experiment is

$$\Delta E = \frac{mc^2\Delta\omega_{dep}}{a\omega_o} = 4406.49eV$$

Assume that the central energy derived from central resonance frequency is 800MeV, then we can get the precision of the calibrating energy is $\frac{\Delta E}{E}\sim 10^{-6}$.

## DEPOLARIZATION TIME, DEPOLARIZATION FIELD AND INPUT POWER

The depolarization time can be calculated by [6]

$$\tau_{dep} = \frac{\Delta\omega_{dep}}{|\Omega_x|^2} \qquad (14)$$

where $\Delta\omega_{dep}$ is the sweep interval, and the nutation frequency $\Omega_x$ around the horizontal direction is expressed[6]:

$$\Omega_x = \frac{1}{2}\frac{H_x l}{\langle H_z\rangle L}\left|F(s_{dep})\right|\Omega_z \qquad (15)$$

with L the circumference of the ring, $\langle H_z\rangle = \frac{H_z\cdot 2\pi\rho}{L}$

the average bending field with respect to the whole ring, and $F(s_{dep})$ the spin response function which is relative to the lattice of the ring. $|F(s_{dep})|>1$ means the lattice of ring strengthens resonant depolarization while $|F(s_{dep})|<1$ hinders resonant depolarization.

Employ the computed result of finite wire, the peak value of radial magnetic field along the ideal orbit in free space generated by a pair of vertical installed striplines is

$$H_x = \frac{I_m}{2\pi r_0}\left(\frac{l/2+z}{\sqrt{r_0^2+(l/2+z)^2}}+\frac{l/2-z}{\sqrt{r_0^2+(l/2-z)^2}}\right) \quad (16)$$

where $I_m$ is the maximum instantaneous value of feed-in AC signal, $r_0$ the distance between the beam and striplines, z the longitudinal location with regard to the symmetric center position of the striplines. The factual value of magnetic field in the vacuum chamber calculated by numerical simulation has an approximate relationship as below shows with the value in free space [7],

$$H_{x(\text{vacuum chamber})} = \sigma H_{x(\text{free space})} \approx \frac{1}{2} H_{x(\text{free space})} \quad (17)$$

with a factor "$\sigma$" approx equals to 1/2. So the depolarization integral field in the vacuum chamber is approx

$$\int_{-\frac{l}{2}}^{\frac{l}{2}} H_x(z)dz = \frac{\sigma I_m}{\pi r_0}\left(\left(r_0^2+l^2\right)^{\frac{1}{2}}-r_0\right) \quad (18)$$

Apply the result of (18) to (15), and combine with (14) and the relationship $I_m=\sqrt{2P/Z_S}$, where P is the input power and $Z_S$ the matched impedance, the depolarization time can be obtained:

$$\tau_{dep} = \Delta\omega_{dep}\frac{2Z_S}{P}\cdot\left(\frac{\langle H_z\rangle L\pi r_0}{\sigma\Omega_z\left|F(s_{dep})\right|\left(\sqrt{r_0^2+l^2}-r_0\right)}\right)^2 \quad (19)$$

For HLS, we have $\mu_0\langle H_Z\rangle = 1.2T\times 2\pi\times 2.2221m/L$, $L = 66.1308m$, $\Delta\omega_{dep} = 2\pi\times 1.0kHz$, $r_0 = 0.04m$, $\Omega_z = 2\pi\times 12.7627MHz$, $l_{dep}$ =0.72m, and the value of spin response function where the striplines installed is 1.569. With commercial available 50Ω technology of matched impedance, we obtain the relationship between depolarization time, as well as radial depolarization integral field, and input power as shown in Fig. 1. The depolarization time is about 64s, while an integral field of 0.0264Gauss·m is obtained, with a 15W input power. They are marked out in the figure.

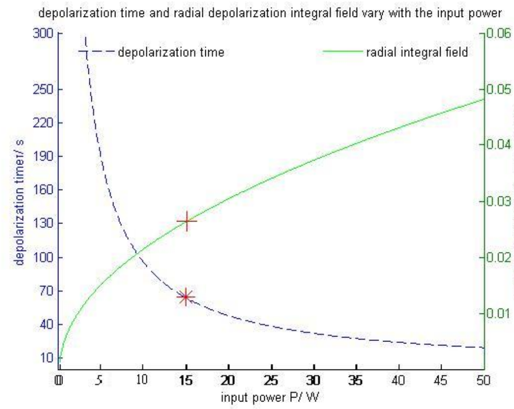


Figure 1: The depolarization time and the radial integral field vary with the input power.

## SUMMARY

The vertical installed striplines in storage ring are used to generate a radial alternating magnetic field for resonant depolarization experiments in HLS. The calculational depolarization time is about 64s, in consideration of spin frequency spread, with an amplifier power of 15W fed into the striplines.

## REFERENCES

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