TRANSVERSE SCHOTTKY NOISE WITH SPACE CHARGE

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Abstract

The effect of space charge on the transverse Schottky spectrum of coasting and bunched beams is studied using measurements and simulations together with analytic models. The measurements of transverse Schottky bands from heavy ion beams are performed in the SIS-18 synchrotron at GSI. In addition we analyze the noise spectrum from a particle tracking code with self-consistent space charge. The results are compared to analytic models for coasting and for bunched beams with space charge. For coasting beams an analytic model based on the transverse dispersion relation with linear space and chromaticity reproduces the characteristic deformation of Schottky bands with increasing space charge, observed in both measurement and simulation. For bunched beams we find good agreement between the shifts of synchrotron satellites observed in the simulations and a simplified model for head-tail modes with space charge.

INTRODUCTION

Schottky signals play an important role for the diagnostics of ion beams in circular machines. The Schottky noise spectrum, divided into separate bands, contains information on the incoherent and coherent frequencies in the beam [1]. In many machines the Schottky spectrum is used routinely to probe e.g. betatron tunes and tune spreads. For high intensity beams at medium energies, like in the FAIR synchrotron's [2], it is important to understand the effect of space charge on the Schottky spectrum. It is well known that in coasting beams space charge leads to longitudinal Schottky spectra with a double-peaked shape. The transverse Schottky bands, centered at the betatron sidebands, are deformed in a characteristic way as well [3]. From the deformation one can retrieve important information on collective beam properties, like the space charge tune shift or the Landau damping rate. This will be illustrated in the present contribution using transverse Schottky spectra with space charge obtained in the SIS-18 synchrotron at GSI. In bunched beams the Schottky bands split into synchrotron satellites. From the longitudinal Schottky spectrum the space charge induced frequency shifts of bunch modes can be obtained [4]. In this contribution we will present an analysis of the transverse Schottky spectrum from simulations of bunched beams affected by the transverse space charge force.

COASTING BEAMS

The transverse Schottky spectrum is centered around the betatron sidebands

$$Q_n = (n \pm Q_0) \tag{1}$$

with the bare tune Q_0 and the harmonic number n. In the absence of space charge the width of a band is determined by the chromatic tune spread

$$\delta Q_n = S\delta_{\rm rms} \tag{2}$$

with the rms momentum spread $\delta_{\rm rms}$, $S = [\pm\xi - \eta_0(n \pm Q)]$, the chromaticity ξ and the slip factor η_0. For a round beam space charge induced an incoherent tune shift $Q_0 - \Delta Q^{sc}$ with

$$\Delta Q^{sc} = \frac{qIR}{4\pi\varepsilon_0 c E_0 \beta_0^3 \gamma_0^2 \epsilon} \tag{3}$$

the beam current I, the particle charge q and energy E_0, the relativistic parameters γ_0 and β_0, the ring radius R and the emittance ϵ of the rms equivalent K-V distribution. The low intensity shape $P_0(Q_f)$ of a Schottky band (frequency $Q_f = f/f_0$, revolution frequency f_0) is modified by space charge according to

$$P(Q_f) = \frac{P^0(u)}{\mid \epsilon(u) \mid^2}. \tag{4}$$

with the normalized tune

$$u = \frac{Q_f - (n \pm Q_0) \pm \Delta Q_{sc}}{S\delta_{\rm rms}} \tag{5}$$

and the dielectric function $\epsilon(u) = 1 - D(u)$ (see e.g. Ref. [3]). For a Gaussian momentum distribution the dispersion function is

$$D(u) = i\sqrt{\frac{\pi}{2}} U_{sc} w\left(u/\sqrt{2}\right) \tag{6}$$

with the space charge parameter $U_{sc} = \Delta Q^{sc}/(S\delta_{\rm rms})$. It should be noted that the analytic expression for $P(Q_f)$ presented above results from a rigid beam approach assuming a linear space charge force. The validity of the rigid beam approximation for weak or moderate space charge parameters is questioned in [5]. However, here we will show that Eq. (4) reproduces very well the Schottky bands measured under moderate space charge conditions with $U_{sc} \lesssim 2$. The Schottky measurements were performed in the SIS-18 with coasting $^{40}\mathrm{Ar}^{18+}$ beams at injection energy (11.4 MeV/u) and for $N \lesssim 10^{10}$ ions. A lower sideband measured at 10.6 MHz for three different intensities is shown in Fig. 1. It can be observed very clearly that the asymmetry of the bands increases with intensity which we refer to the effect of space charge. From the Schottky data we obtain U_{sc} using a fit to Eq. (4) with the peak value, $\delta_{\rm rms}$ and the space charge parameter U_{sc} as free parameters. The result of the fits are shown in Fig. 1 together with the measured data. In Tab. 1 the space charge parameters obtained from the fit are

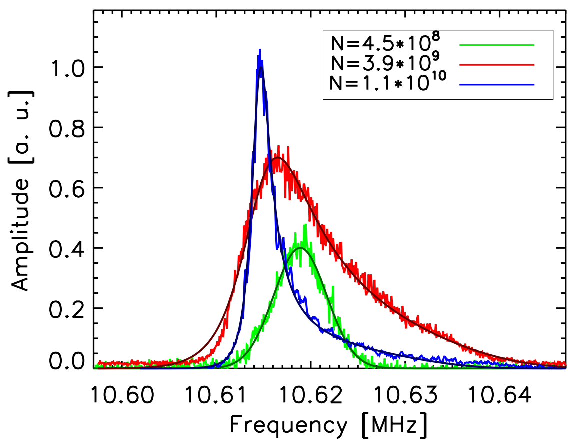

Figure 1: Measured transverse Schottky spectra (lower sideband) for different beam intensities. The solid curves represent the fit to Eq. 4.

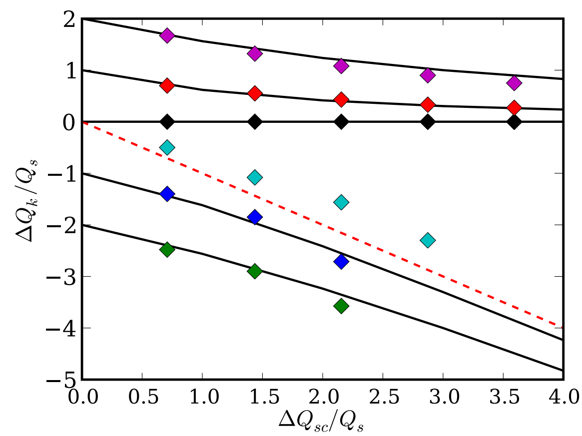

Figure 2: Head-tail mode frequencies ΔQ_k as a function of the space charge tune shift ΔQ_s. The symbols indicate the mode frequencies obtained from the simulation noise spectrum for an elliptic bunch distribution with $\Delta Q_{sc} = \Delta Q_{sc}^{\text{max}}$.

Table 1: Beam intensities, rms momentum spreads obtained from the longitudinal Schottky spectrum together together with the space charge parameters U_{sc} and the chromatic tune spread δQ obtained from the fit to a lower Schottky sideband at 10.62 MHz.

$N/10^9$	δ_{rms} $[10^{-4}]$	δQ	ΔQ_{sc}	U_{sc}
0.45	2.8	0.013	0.0	0.0
3.9	6.7	0.033	0.03	0.86
11.0	7.8	0.031	0.06	1.84

given. Space charge parameters were obtained also for upper sidebands and at different frequencies in order to confirm the validity of our measurement. Moreover we showed in [3] that for moderate space charge the noise spectrum obtained from a particle tracking code agrees very well with Eq. (4).

BUNCHED BEAMS

In the case of a low intensity, bunched beam the transverse Schottky band splits into equidistant synchrotron satellites

$$Q_{n,k} = (n \pm Q_0) + \Delta Q_k \tag{7}$$

with $\Delta Q_k = kQ_s$ and the synchrotron tune Q_s. In the absence of transverse nonlinear field components the width of each satellite is determined by the synchrotron tune spread [1]

$$\delta Q \approx kQ_s \frac{\phi_m^2}{16} \ll S\delta_{\text{rms}} \tag{8}$$

with the bunch length ϕ_m. In an intense bunch the transverse space charge force will shift the frequencies of head-tail modes and so the positions of the satellites in the Schottky spectrum. The mode frequencies can be obtained analytically only for the so called airbag distribution in a square potential well [7]

$$\Delta Q_k = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{(\Delta Q_{sc}/2)^2 + (kQ_s)^2} \tag{9}$$

For weak space charge $\Delta Q_{sc} \ll Q_s$ one obtains $\Delta Q_k = -\Delta Q_{sc}/2 + kQ_s$. For strong space charge the mode frequencies converge to $\Delta Q_k = 0$ for $k > 0$ and to $\Delta Q_k = -\Delta Q_{sc}$ for $k < 0$. The mode frequencies as a function of ΔQ_{sc} are plotted in Fig. 2. If $X_+(\phi, t)$ is the transverse offset of particles with positive momentum deviation one can write $X_+ = \exp(-iQ_0\omega_0 t)\exp(i\xi\phi/\eta_0)x_+(\phi)$ with $\omega_0 = 2\pi f_0$. The eigenfunctions for the square well airbag model are [7]

$$x_{\pm,k} = [\cos(k\pi\phi/\phi_m) \mp (i\Delta Q_k/kQ_s)\sin(k\pi\phi/\phi_m)] \tag{10}$$

The dipole moment seen by a Schottky pick-up is proportional to $\bar{x}_k = (x_{+,k} + x_{-,k})/2 = \cos(k\pi\phi/\phi_m)$. In addition the eigenfunction carry a quadrupole moment $\Delta x_k = (x_{+,k} - x_{-,k}) = (i\Delta Q_k/kQ_s)\sin(k\pi\phi/\phi_m)$. For strong space charge and for $k < 0$ the quadrupole moment is dominating. Therefore one can expect a decrease of the satellites with negative k. Also the eigenfrequencies of $k < 0$ modes should differ from the analytic solution Eq. 9 because the analytic model does not take into account the space charge effect on the envelope oscillations. However, for moderate space charge the shift of the eigenfrequencies obtained from Eq. 9 can serve as a very useful validation for simulation codes. In this study we use the particle tracking code PATRIC [6] which contains a 2.5D space charge solver. In the simulation $N_p = 10^6$ macro-particles were

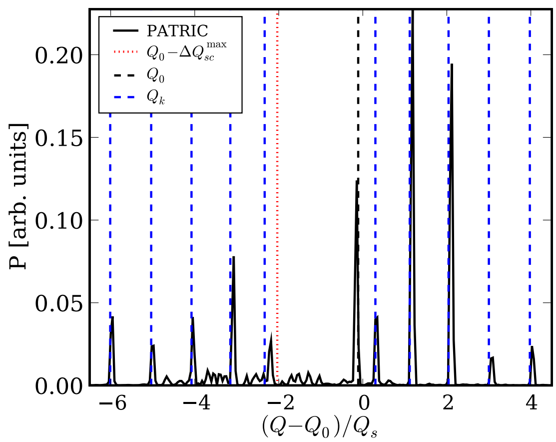

Figure 3: Simulation noise spectrum for an airbag distribution in a square well potential and $\Delta Q_{sc}/Q_s = 2$.

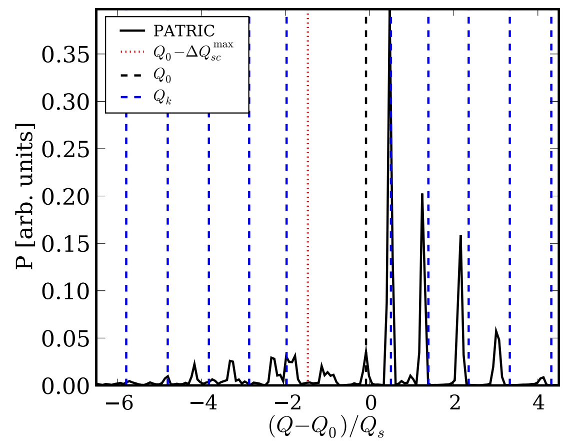

Figure 4: Simulation noise spectrum from a parabolic bunch for $\Delta Q_{sc}/Q_s = 1.5$ together with Q_k from Eq.9.

initially loaded in 3D phase space to reproduce a longitudinal elliptic bunch distribution in a rf bucket and transverse K-V distribution. If we include the self-consistent transverse space charge field we limit the simulation time to $T = 10T_s$ with $T_s = 1/(f_0 Q_s)$. The tune spread resulting from the finite simulation time is $\delta Q = Q_s T_s/T$, which is smaller than the expected tune spread in the rf bucket. One should keep in mind that the simulation of transverse Schottky spectra from bunched beam is more demanding than for coasting beams. Fig. 3 shows the noise spectrum obtained from the simulation for an airbag distribution in a square well potential and $\Delta Q_{sc}/Q_s = 2$. The satellites are in very good agreement with the ΔQ_k obtained from Eq. 9. However, for stronger space charge ($\Delta Q_{sc}/Q_s \gtrsim 4$) we find that the satellites with $k < 0$ disappear in the simulation noise spectrum. For realistic bunch profiles $\Delta Q_{sc}(\phi)$ varies along the bunch. At the bunch ends $\Delta Q_{sc}(\phi_m) = 0$ holds and $\Delta Q_{sc}(0) = \Delta Q_{sc}^{\max}$ in the bunch center. In our simulation we study the noise spectrum from different bunch distributions with the same rms bunch length and particle number. Here we show simulation results for an elliptic distribution with a parabolic bunch profile in a rf bucket. The bunching factor is chosen as $B_f = 0.35$. Fig. 4 shows the noise spectrum obtained from the simulation for $\Delta Q_{sc}/Q_s = 1.5$ together with the mode frequencies predicted from Eq. 9 for $\Delta Q_{sc} = \Delta Q_{sc}^{\max}$. The suppression and broadening of the negative satellites can be observed very clearly. Like for the airbag distribution in a square well potential, the negative modes disappear in the noise spectrum because of their vanishing dipole moment. Moreover, in a rf bucket the lines for $k < 0$ can be Landau damped very effectively because they overlap with incoherent spectrum centered around $Q_0 - \Delta Q_{sc}$. The lines for $k > 0$ are well pronounced. The position of the $k = 1$ satellite matches $\Delta Q_{k=1}$ from Eq. 9. With increasing k the observed frequency shifts induced by space charge are underestimated by Eq.9. In Fig. 2 the frequencies of the satellites in the simulation spectrum for different $\Delta Q_s c/Q_s$ are compared with Q_k from the airbag model (Eq. 9). For $\Delta Q_s c/Q_s \lesssim 3$ we observe an additional line below $Q_0 - \Delta Q_{sc}^{\max}$ in the simulation spectrum.

CONCLUSIONS

We demonstrated that in coasting beams the deformation of the transverse Schottky bands for moderate space charge can be described in the framework of a simple analytical approach. From a fit to the measured Schottky spectrum one can obtain the space charge tune shift and the Landau damping rate. For bunched beams in rf buckets we compared the space charge induced shift of the satellites in the simulation noise spectrum with the analytic eigenfrequencies of the airbag distribution in a square potential well. The negative satellites with $k < 0$ disappear with increasing space charge. For the positive satellites with $k > 0$ we found a good agreement between the two approaches. This suggest to use the positive Schottky satellites (e.g. $k = 1$) for the measurement of the space charge tune shift in bunches and for the validation of simulation codes.

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