

CIRCULARLY INCLINED SOLENOID CHANNEL FOR 6D IONIZATION COOLING OF MUONS*

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Abstract

Ionization cooling is essential for realization of Muon Collider [1], muons beam based neutrino factories and other experiments involving muons. The simplest structure - absorber(s) immersed in alternating solenoidal magnetic field - provides only transverse cooling since the longitudinal motion in the most suitable momentum range (2-300MeV/c) is naturally anti-damped. To overcome this difficulty it is proposed to periodically tilt solenoids so that a rotating transverse magnetic field was created. By choosing the phase advance per period above a multiple of 2π it is possible to ensure that muons with higher momentum make a longer path in the absorber (whether distributed or localized) thus providing longitudinal damping. Basic theory of such channel and results of tracking simulations are presented.

INTRODUCTION

Due to short lifetime of muons ($2.2\mu s$) the conventional methods of cooling are not applicable for muon beams, the only method fast enough is the ionization cooling (IC). Unfortunately there is no longitudinal cooling since the ionization loss decrease with momentum in the range of practical interest.

Several schemes were proposed [2-4] in order to achieve longitudinal cooling by forcing muons with higher momentum to make a longer path in the absorber so that they lose more energy. This can be realized by creating dispersion in particle positions (without significant path lengthening) and using wedge absorbers (so called RFOFO channel [2]) or by creating sufficiently large path lengthening with momentum and using a homogeneous absorber (Helical Cooling Channel [3]).

In both these schemes the dispersion is generated due to curvature of the reference orbit making these schemes selective w.r.t. the sign of muon charge. A different approach was proposed in [4] where dispersion is produced in the result of resonant perturbation of a straight channel comprised of block solenoids with alternating polarity and RF cavities between them.

This scheme – called “FOFO snake” - is especially attractive due to its technological simplicity and ability to cool both μ^+ and μ^- simultaneously.

In the initial proposal [4] the solenoids were inclined (or displaced) in one plane with a period of 4 solenoids. However, the subsequent analysis showed that such “planar snake” had insufficient transverse dynamic acceptance. It was possible to increase it by making the solenoid radius larger (hence the magnetic field smoother)

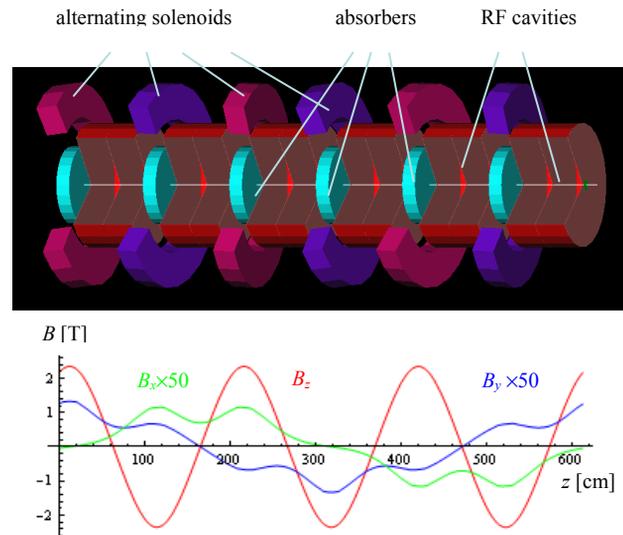


Figure 1: Schematic view and magnetic field distribution along the channel.

and by reducing the phase advance so that the resonance period was 6 or more solenoids.

These measures, still insufficient for dynamic acceptance, created problem with equalizing the damping rates of the transverse modes: with increase in the solenoid tilt one of the transverse modes gets more damping from the other one than goes into the longitudinal mode.

Two solutions to these problem were proposed, both aimed at providing more transverse mode mixing: adding a constant magnetic field component (R.Palmer) and making the snake helical. The first proposal worked remarkably well from the point of view of damping rate equalization but exacerbated problem with dynamic acceptance. Besides, it made the channel selective to the muon sign.

In the present report we consider the second proposal: helical FOFO snake.

ORBIT & LINEAR OPTICS

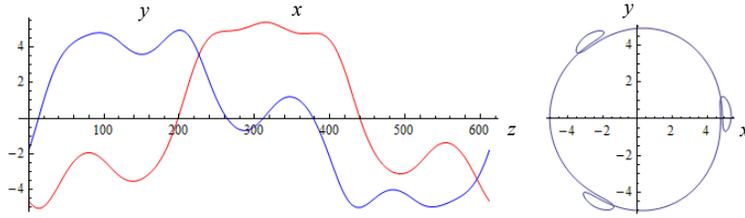
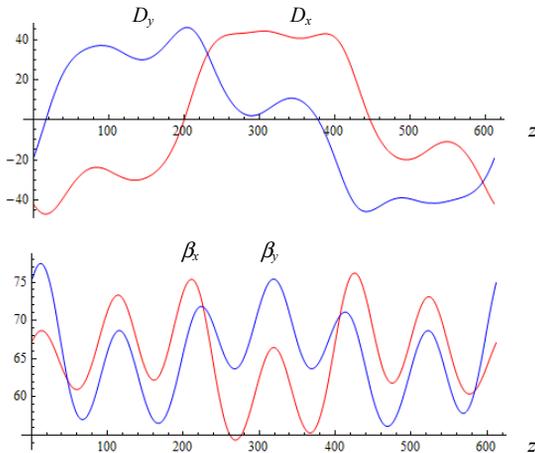
The basic idea is to create a rotating transverse magnetic field by inclining solenoids by angle ϑ in planes $x \cdot \cos(\phi_k) + y \cdot \sin(\phi_k) = 0$, $\phi_k = \pi(1-2/N_s)(k-1)$, $k = 1, 2, \dots, N_s$. With number of solenoids per period $N_s = 6$ the roll angles ϕ_k of these planes are: $0, 2\pi/3, 4\pi/3, 0, 2\pi/3, 4\pi/3$.

Schematic view of the channel for initial 6D cooling and magnetic field distribution are shown in Fig.1.

Parameters of the channel were chosen as follows:

- solenoids: $L=24\text{cm}$, $R_{in}=60\text{cm}$, $R_{out}=92\text{cm}$,
- RF cavities: $f_{RF}=200\text{MHz}$, $L=2 \times 36\text{cm}$, $E_{max}=16\text{MV/m}$,
- absorbers: planar, $L=15\text{cm}$, liquid H_2 .

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Figure 2: Periodic orbit for μ^+ and its projection on x - y plane.Figure 3: Dispersion (top) and β -functions (bottom)

A 3cm technological gap was left between solenoids and cavities so that the one cell length is 102cm and $L_{period} = 612$ cm. The cavity and absorber walls were not taken into account. RF cavities are timed at an imaginary particle moving along the axis with constant velocity v_0 .

The properties of the channel were studied using approach described in [5]. Fig. 2 shows periodic trajectory for positive muons in the case of solenoid pitch angle $\vartheta=7$ mrad, reference momentum $p_0=200$ MeV/c and magnetic field amplitude $B_{z,max}=2.35$ T. This set of parameters will be used as the baseline set.

Periodic trajectory for μ^- looks precisely the same with a shift by half period.

The shape of the dispersion function (Fig.3 top) almost exactly follows that of the periodic trajectory ensuring that higher energy muons make a longer path (momentum compaction factor $\alpha_c > 0$). Beta-functions (Fig.3 bottom) have minima in RF cavities and maxima inside absorbers, but the variation is not large.

The normal mode tunes and analytically calculated normalized equilibrium emittances for the cited set of parameters are given in Table 1. The tunes were computed from eigenvalues of a one-period transfer matrix, their imaginary parts describe oscillation damping due to regular part of ionization losses. Without stochastic

Table 1: Normal Mode Tunes and Emittances

mode	I	II	III
tune Q	$1.239+0.012i$	$1.279+0.007i$	$0.181+0.002i$
ε_N , mm	3.2	4.5	6.9

effects the emittances would have been damped as

$$\frac{d}{dz} \ln \varepsilon_j = 2 \times \frac{2\pi}{L_{period}} \text{Im} Q_j \quad (1)$$

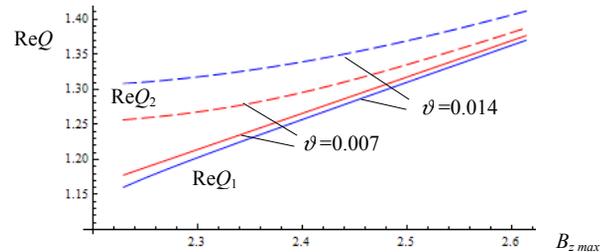
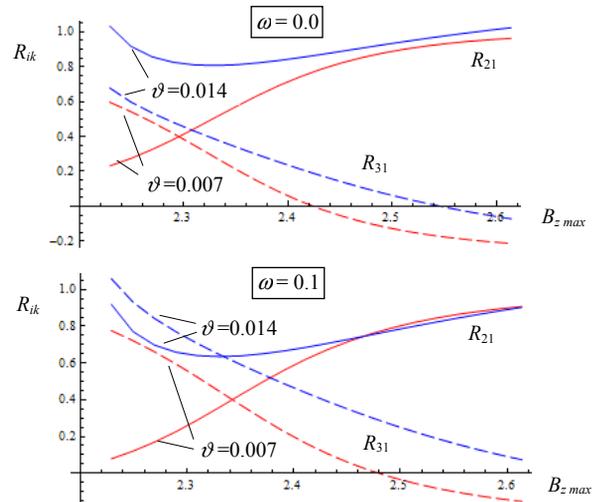
with damping lengths 41m, 72m and 196m.

Even with plane-parallel absorber surfaces there is an appreciable longitudinal damping due to large D' . Still, there is a significant imbalance in damping rates of the transverse modes. It can be reduced with larger pitch angle ϑ , introduction of (conical) wedge angle of the absorbers and/or tune change.

The transverse tunes (for a given momentum and channel geometry) can be varied with magnetic field strength. Fig.4 shows this dependence for two values of the pitch angle ϑ . Fig.5 shows the corresponding dependence of the ratios $R_{ik} = \text{Im} Q_i / \text{Im} Q_k$ for plane absorbers (top) and with wedge angle $\omega=0.1$ (bottom).

NONLINEAR DYNAMICS

Fig.5 shows that it is possible to achieve the desired distribution of damping decrements by adjusting solenoid inclination angle ϑ and the absorber wedge angle ω . However, precautions must be taken not to compromise the channel acceptance.

Figure 4: Transverse tunes vs $B_{z,max}$ for pitch angle $\vartheta=7$ mrad (red) and $\vartheta=14$ mrad (blue).Figure 5: Damping rates ratio vs $B_{z,max}$ for the absorber wedge angle $\omega=0$ (top) and $\omega=0.1$ (bottom).

Momentum Acceptance

Momentum acceptance is the most important characteristics for initial cooling channel since muons coming from the buncher/rotator have very large longitudinal emittance [6]. To get an insight into dynamics at large momentum deviations we can assume for a while the longitudinal motion to be slow and calculate betatron tunes and relative orbit length $\lambda=s(p)/L_{period}$ for a constant momentum (Fig.6).

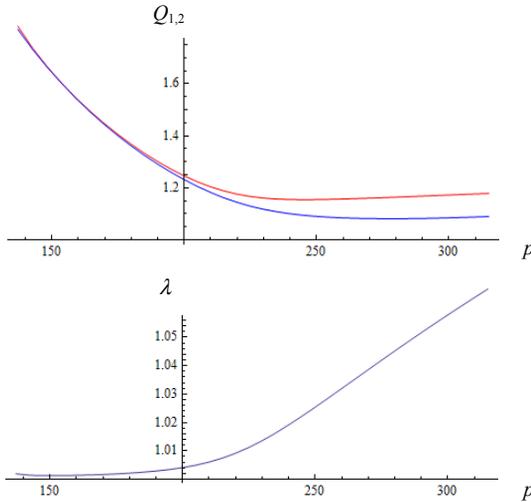


Figure 6: Betatron tunes (top) and relative orbit length (bottom) vs muon momentum (MeV/c) for the solenoid pitch angle $\vartheta=7\text{mrad}$.

Fig.6 reveals a remarkable fact of the transverse tunes “repulsion” from the integer resonance making the energy width of the betatron passband very large. The orbit length (and its maximum offset) increases just linearly with momentum.

To find the momentum width of the longitudinal separatrix let us introduce canonical variables

$$\delta_p = (\gamma - \gamma_0) / \beta_0^2 \gamma_0, \quad u = z - c\beta_0 t \quad (2)$$

Kinetic part of the longitudinal Hamiltonian in the considered quasi-stationary approximation is*

$$K(\delta_p) = \int [1 - \lambda(\xi) \frac{\beta_0}{\beta(\xi)}] d\xi \quad (3)$$

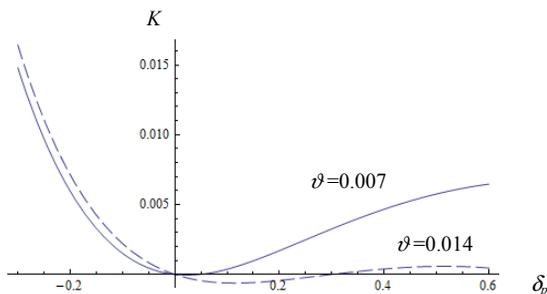


Figure 7: Effective longitudinal kinetic energy.

*) It can be easily verified that for small δ_p and $\lambda(0)=1$ eq.(3) yields standard expression for the slippage factor

Fig.7 shows large asymmetry in effective kinetic energy K which becomes a limiting factor for momentum acceptance at larger values of the pitch angle ϑ .

Transmission

Fully coupled nonlinear motion can be studied with the help of perturbation theory for non-Hamiltonian systems [5]. An important effect which this theory provides in the first order and which should be taken into account when doing tracking simulations is correlation between average particle momentum and amplitude of oscillations. For the baseline channel parameters

$$\delta_p = \delta_{p0} + 0.042J_1 + 0.013J_2 + 0.039J_3 \quad (4)$$

where δ_{p0} is the periodic orbit value and J_i is the action variable for the i -th normal mode.

This correlation has simple explanation: particles with larger oscillation amplitude make longer path and must have higher velocity to keep in phase with RF.

For tracking simulations 1771 particles were evenly distributed in tetrahedron $(J_1 + J_2)/2.6 + J_3/4 < 1$ (cm) with random phases. Correlation (4) was introduced into initial conditions. Without decays and stochastic processes the transmission over 25 periods was 97%.

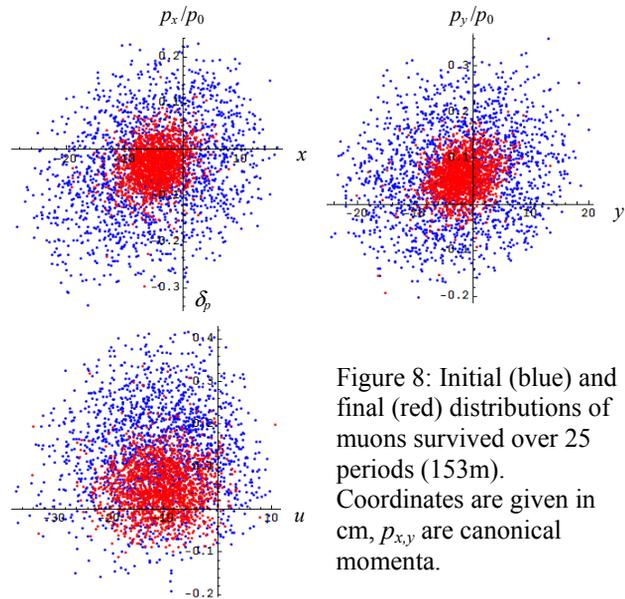


Figure 8: Initial (blue) and final (red) distributions of muons survived over 25 periods (153m). Coordinates are given in cm, $p_{x,y}$ are canonical momenta.

The proposed channel is a promising candidate for initial 6D ionization cooling. Its study using G4BL [7] is underway.

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