

# SPACE CHARGE SIMULATION ON HIGH INTENSITY CYCLOTRONS: CODE DEVELOPMENT AND APPLICATIONS \*

Jianjun Yang <sup>†</sup>, Tianjue Zhang, Shizhong An, Suming Wei, Yuanjie Bi, CIAE, Beijing, 102413, China  
 Andreas Adelman, PSI, Villigen, CH-5234, Switzerland  
 Yuzheng Lin, Department of Engineering Physics, TUB, Beijing, 100084, China

## Abstract

In high intensity cyclotrons with small turn separation, both the space charge effects of single bunch and the interaction of radially neighboring bunches play important roles. A PIC-based three-dimensional parallel code, OPAL-CYCL, is newly developed under OPAL framework which self-consistently covers these two collective effects. In this paper we also present the simulation results from the compact cyclotron CYCIAE-100 in the light of the on-going upgrade program of BRIF at CIAE, with the goal of 100 MeV, 200  $\mu$ A CW proton beam on target.

## INTRODUCTION

In high intensity cyclotrons, space charge effects play an important role for the following reasons. Firstly, the longitudinal space charge force causes additional acceleration for head particles and deceleration for tail particles, which can result in the growth of beam energy spread. Secondly, there is a strong radial-longitudinal coupling which is influenced by the non-linear radial and longitudinal space charge forces and the resultant “vortex” motion can change the charge distribution and turn separation seriously[1]. Lastly, the space charge force can reduce the axial tune and increase the axial beam size, which can cause beam losses when the axial size extends beyond the aperture of the accelerator [2]. In the past, a number of models have been developed to study its influences on the beam dynamics[1, 3, 4, 5, 6, 7].

For high intensity cyclotrons with small turn separation, single bunch space charge effects are not the only contribution. Along with the steady increase of beam current, the mutual interaction of neighboring bunches in radial direction becomes more and more important, especially at large radii where the distances between neighboring bunches diminished, and even overlap can occur. One example is the 100 MeV compact cyclotron CYCIAE-100 under construction at CIAE [8]. The aimed beam current is 200  $\mu$ A to 500  $\mu$ A. Because the average energy gain per turn is about 320 keV, the turn separation reduces to less than 10 mm in the first several revolutions as shown in Fig.1. The turn separation at out radii is less than 5 mm which is smaller than the typical radial size of beam, so multiple bunches will overlap.

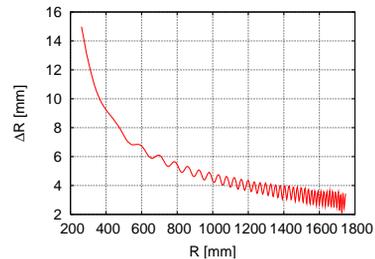


Figure 1: Turn separation for centroid particles in CYCIAE-100 cyclotron

## BASIC EQUATIONS AND PHYSICAL MODEL

In the cyclotrons and beam lines under consideration, the collision between particles can be neglected because the typical bunch densities are low. In time domain, the evolution of the beam’s distribution function  $f(\mathbf{x}, \mathbf{v}, t)$  can be expressed by a collisionless Vlasov equation:

$$\frac{df}{dt} = \partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = 0, \quad (1)$$

where  $\mathbf{E}$  and  $\mathbf{B}$  include both external applied fields, space charge fields and other collective effects such that wake fields

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_{\text{ext}} + \mathbf{E}_{\text{self}}, \\ \mathbf{B} &= \mathbf{B}_{\text{ext}} + \mathbf{B}_{\text{self}}. \end{aligned} \quad (2)$$

In order to model a cyclotron, the external electromagnetic fields are given by measurement or by numerical calculations. The space charge fields can be obtained by a quasi-static approximation. In this approach, the relative motion of the particles is non-relativistic in the beam rest frame, so the self-induced magnetic field is practically absent and the electric field can be computed by solving Poisson equation

$$\nabla^2 \phi(\mathbf{x}) = -\frac{\rho(\mathbf{x})}{\epsilon_0}, \quad (3)$$

where  $\phi$  and  $\rho$  are the electrostatic potential and the spatial charge density in the beam rest frame.

Eq.(3) can be solved numerically by Particle-Mesh methods using the Hockney’s FFT algorithm [10]. The electric field can then be calculated by

$$\mathbf{E} = -\nabla \phi, \quad (4)$$

and back transformed to yield space charge field  $\mathbf{E}_{\text{sc}}$  and  $\mathbf{B}_{\text{sc}}$  in the lab frame, required in Eq.(2) by means of a

### Beam Dynamics and Electromagnetic Fields

\* Work supported by the National Science Foundation of China, under contract 10775185,10125518

<sup>†</sup> yangjianjun00@mails.tsinghua.edu.cn

Lorentz transformation. Generally in cyclotrons the contribution of image charges and currents are small effects compared to space charges [2], hence it is a good approximation to use open boundary conditions.

After the space charge fields is obtained, the equations of motion of the particles are integrated using a 4<sup>th</sup> order Runge-Kutta integrator, in which the fields are evaluated for four times in each tracking step.

In order to include the contribution of neighboring bunches in the beam dynamics of those cyclotrons with small turn separation, we developed a new self-consistent physical model and numerical algorithm. In this model, initially a single bunch is injected and integrated. When the turn separation becomes smaller than the threshold value, new bunches are injected in every revolution period, and multiple bunches are integrated simultaneously. A detailed discussion of the model of neighboring bunch effects can be found in reference [9].

The above model and algorithm are implemented in the object-oriented parallel PIC code OPAL-CYCL, as one of the flavors of the OPAL (Object Oriented Parallel Accelerator Library) framework. A more detailed description of the hierarchical layout, the parallelization and the implementation issues of the OPAL framework and OPAL-CYCL code can be found in the User's Reference Guide [11].

In addition, apart from the multi-particle simulation mode, OPAL-CYCL also has two other serial tracking modes for conventional cyclotron machine design: single particle tracking mode and tune calculation mode.

The functionalities of the newly developed code have been thoroughly validated by performing different types of simulations which has been studied intensively before by other code in reference [9].

## SIMULATIONS AND DISCUSSIONS

### Applications on CYCIAE-100

In this section, we present the recent results of numerical simulation carried on CIAE 100MeV cyclotron CYCIAE-100 using OPAL-CYCL code.

First we double-check the focusing property of the designed magnetic field using the tune calculation mode of the code. The resultant betatron oscillation tune plot presented in Fig. 2, comparing with that of CYCLOP code[12]. As shown in this plot, the coupling resonance  $\nu_r = 2\nu_z$ , which can cause the growth of axial amplitude and destroy axial stability, only happens in the center region. However, duo to the large radial gain per turn in this region, the resonance crossing is not dangerous. So it is concluded that the magnetic field has a good focusing property.

Beam matching condition is another important issue in cyclotrons. A properly matched bunch should satisfy the conditions: the beam envelop must be periodic with the period of magnetic field. We obtained the shape of phase

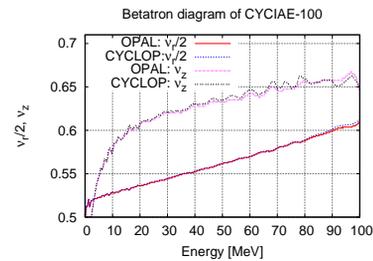


Figure 2: Tune of betatron oscillation of CYCIAE-100

space eigen-ellipse on the axial directions by single particle tracking, and generate the initial matched particle distribution accordingly. On the radial and longitudinal direction, it is not straightforward because there is a coupling between these two directions [13, 14] and for a matched beam the particle distribution is correlated in this two direction. Therefore the matched particle distribution could only be properly defined in a 4D space. By carrying out intensive simulation and comparison, finally we obtained a well matched initial distribution in 6D phase space for CYCIAE-100.

For a initially matched beam, space charge can distort the matching conditions, result in the blow up of the beam and cause beam loss. To evaluate its effects on CYCIAE-100, we carried out the single-bunch multi-particle simulation. We start with a well matched initial particle distribution at  $200\mu\text{A}$ ,  $500\mu\text{A}$  (the design goal) and  $1\text{mA}$  (goal of future upgrade) beam current respectively. The initial beam energy is  $1.49\text{MeV}$ , which is the energy at the time the beam leaves the central region of machine.

Fig. 3 shows the axial rms beam size in 3 revolutions for the coasting beam without acceleration. The rms envelop keeps the same periodicity with the magnetic field (four identical hill and valley) at low beam intensity, and it is the modulated along with the beam current increases. However, the modulation of envelop is acceptable, because up to  $1\text{mA}$  the increase of the rms size is less than 8%.

Fig. 4 shows the changes of the axial and longitudinal rms beam size during acceleration from  $1.49\text{MeV}$  to  $100\text{MeV}$ . From the upper plot of Fig. 4 we can see the that on the axial direction the beam sizes oscillate strongly during the first several turns because of the mismatching caused by space charge. After that the beam sizes change smoothly. Up to  $1\text{mA}$ , space charge does not cause blow up on the axial direction. From the below plot of Fig. 4 we can see that the beam size is decreasing along with beam current increase. This phenomenon can be physically explained by the "vortex" motion. The head particles of bunch gain energy from space charge and move toward larger radius. Because of the larger circumference it has to take, they move backward toward the bunch center. Similarly, the tail particles move forward toward the bunch center. As a result, the total length of bunch becomes shorter. This effects become stronger along with beam current increase. So it is concluded that space charge is helpful to reduce the lon-

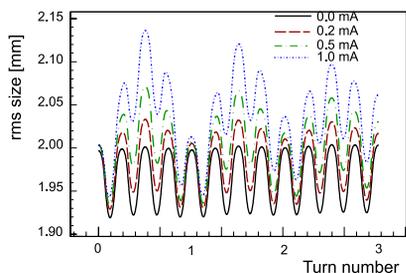


Figure 3: Comparison of the axial rms size of initially matched beams for 0mA, 0.2mA, 0.5mA and 1.0 mA at 1.49MeV without acceleration

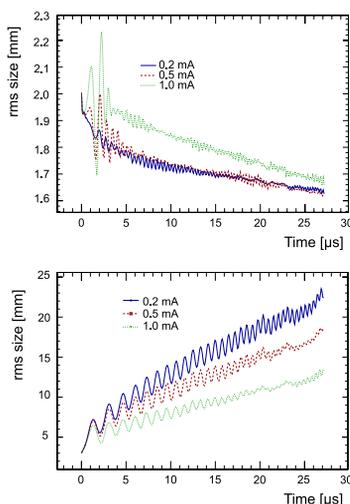


Figure 4: Comparison of the axial(upper) and longitudinal(lower) rms size of beams for 0.2mA, 0.5mA and 1.0 mA with acceleration at  $0^\circ$  azimuthal angle after each revolution.

gitudinal beam phase width and hence the energy spread. However, the spatial stationary spherical beam distribution, as observed on the PSI Injector II[4], is not formed on CYCIAE-100.

### Neighboring bunch effects in PSI 590MeV Ring

Neighboring bunch effects can be evaluated by comparing the difference of single bunches simulation and multiple bunches simulation. We tested for 3, 5, 7 and 9 bunches and found that the difference between the center bunch of 7 scenarios and that of 9 bunches scenarios is trivial. As we can see from Fig. 5, the FWHM of beam transverse profile from multi-bunch simulation is 1 mm narrower than that of single bunch and that the FWHM of energy spread is reduced slightly. This is caused by squeezing space charge force from the bunches at the smaller radius and those at the bigger radius.

From the comparison, we conclude that neighboring bunch effects impose visible impacts on the beam dynamics

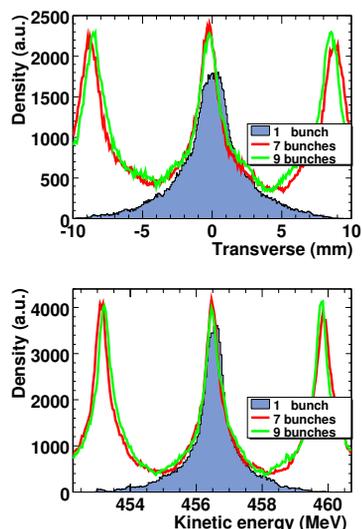


Figure 5: Comparison of the histograms along the transverse direction in the local frame (left) and the energy spectra (right) of 1 mA beam at  $112^\circ$  azimuthal position of turn 130 in PSI Ring cyclotron.

when the beam current get beyond 1 mA in PSI Ring. The bunch becomes more compact in the transverse direction and the energy spread is reduced slightly. Therefore neighboring bunch effects have positive influence on reducing beam loss in high intensity operation.

## ACKNOWLEDGMENTS

The authors thank M. Humbel, M. Seidel, W. Joho and AMAS group members at PSI for many useful discussions regarding high intensity beam dynamics in cyclotrons and programming.

## REFERENCES

- [1] M. M. Gordon, ICC'68, Oxford, 1968, p.305.
- [2] R. Baartman, ICC'95, Capetown, 1995, p.440.
- [3] W. Joho, ICC'81, Caen, 1981, p.337.
- [4] S. Adam, IEEE Trans. on Nuclear Science, 32(1985), p.2507.
- [5] S. Koscielniak et al., PAC'93, Washington, 1993, p.3639.
- [6] A. Adelman, PhD. thesis, (2002), ETHZ.
- [7] E. Pozdeyev, PhD. thesis, (2003), MSU.
- [8] T. J. Zhang, et al., ICC'07, Catania, 2007, p.33.
- [9] J. J. Yang et al., HB'08, Nashville, 2008.
- [10] R. W. Hockney and J. W. Eastwood, "Computer Simulation Using Particles", Hilger, New York, 1988.
- [11] A. Adelman, et al., Tech. Rep. PSI-PR-08-02, PSI, 2008.
- [12] M. M. Gordon, Particle Accelerators, 16(1984), p. 39.
- [13] W. Schulte and H. Hagedoorn, NIM 137 (1976), p.15.
- [14] P. Lapostolle, ICC'81, Caen, 1981, p.317.