

TRANSFER MATRIX METHOD USED IN RF TUNING ON DTL FOR CSNS

Z.R. Sun, J. Peng, S.N. Fu, IHEP, Beijing 100049, China

Abstract

China Spallation Neutron Source (CSNS) uses a conventional 324 MHz DTL with 4 tanks to accelerate H-ion from 3MeV to 81MeV. Currently the R&D on the first module of tank-1 is under proceeding. In design, the Tank-1 has a tilt field distributed partially in order to obtain most effective energy gain and low Kilpatrick parameter. In order to decrease the difficulty of tuning the partial tilt field distribution, a new analysis named transfer matrix method is introduced. Verification of the calculation and simulation of the transfer matrix has been finished with MDTFISH code, using parameters from CSNS.

INTRODUCTION

China Spallation Neutron Source (CSNS) has an H⁺ linac with a drift tube linac (DTL) structure between 3 and 81MeV in the first phase[3]. R&D program of the first 3-meter module of tank-1 is under proceeding. This module is most difficult section in the whole DTL linac. Usually, the specified electrical field distribution is flat or linear tilted. In this case, the perturbation of local frequency shift is calculated by Slater's theorem. In CSNS DTL-1, the field was designed more complex, combining a tilt part and a flat part so as to get a large accelerating gradient. It is believed that the field tuning becomes much more difficult. We have developed the transfer matrix method trying to deal with the problem.

TRADITIONAL ALGORITHM OF PERTURBATION THEOREM

In a RF cavity the local frequency perturbation along the longitudinal direction is described as function of z as $\omega(z) = \omega_0 + \delta\omega(z)$, where ω_0 is the resonant angular frequency of the cavity, and $\delta\omega(z)$ is the local frequency shift. The electrical field perturbation is calculated by the equation

$$\frac{d^2 E_z}{dz^2} + \mu\epsilon[\omega_0^2 - \omega^2(z)]E_z = 0.$$

Applying the small shift approximation, we obtain the Slater's Equation

$$\frac{d^2}{dz^2} \left[\frac{\delta E_z}{E_0} \right] = \frac{8\pi^2}{\lambda_0^2} \frac{\delta f}{f_0},$$

where $f_0 = \omega_0 / 2\pi = c / \lambda_0$. The boundary condition for a resonant cavity is $\frac{d}{dz} \left[\frac{\delta E_z}{E_0} \right] = 0$ for $z = 0$ and $z = L$, which represents metal end plates.

Then the perturbation is obtained by the equation $f_0 = \frac{1}{L_T} \int_0^{L_T} f(z) dz$. This is the Slater's theorem used in calculating perturbation for RF cavity design.

In CSNS DTL-1 we designed a complex electrical field, combining a tilt field part from 2.2MV/m to 3.1MV/m and a flat field part. When simulating the RF field distribution, we found the result different from the designed distribution from PARMILA specification, as shown in Figure 1.

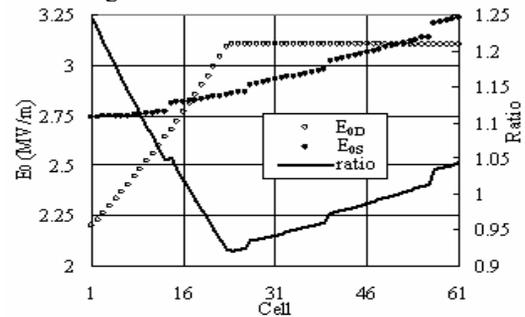


Figure 1: The electrical field distribution simulated by MDTFISH with structure from PARMILA.

PARTICLE TRANSPORT EQUATIONS

It is well-known that the particle transported in a quadrupole channel is described by the equation

$$\frac{d^2 u}{dz^2} + \Omega_U(z)u = 0$$

Here we have $u = (\beta_s \gamma_s)^{1/2} U$, where U means the transverse direction either x or y , and

$$\Omega_U(z) = \begin{cases} k_q^2 = \frac{eG_m}{m_0 c \beta_s \gamma_s}, (x: focus, y: defocus) \\ -k_q^2 = -\frac{eG_m}{m_0 c \beta_s \gamma_s}, (x: defocus, y: focus) \\ 0, (x, y: drifting) \end{cases}$$

The coefficient $\Omega_U(z)$ is a piecewise constant in the transport channel. For this kind of equation, the transfer matrix is chosen to express the solution. For example, the transfer matrix in a focusing magnet is

$$\begin{pmatrix} x \\ x' \end{pmatrix} = M_f \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

With the coefficient M

$$M = \begin{bmatrix} \cos(k_q z) & \frac{1}{k_q} \sin(k_q z) \\ -k_q \sin(k_q z) & \cos(k_q z) \end{bmatrix}$$

Here k_q is the focusing coefficient.

For a transport channel with N elements, the particle transverse movement is calculated by the equation

$$\begin{pmatrix} x \\ x' \end{pmatrix} = M \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} = \prod_{j=N}^{j=1} M_j \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

TRANSFER MATRIX USED IN PERTURBATION CALCULATION

It is obvious that the transfer matrix method is much easier than solving differential equations, especially when there are many elements. So the electrical field perturbation is easier to calculate if the equation and boundary condition have similar pattern.

From the basic electrical field distribution equation

$$\frac{d^2 E_z}{dz^2} + \mu\epsilon[\omega_0^2 - \omega^2(z)]E_z = 0$$

We can transform it by introducing frequency shift

$$\frac{d^2 E_z}{dz^2} - \frac{4\pi^2 \delta f}{c^2} [2f_0 - \delta f] E_z = 0$$

Here δf is a function of z. We define a coefficient

$R_E(z)$ as following

$$R_E(z) = \begin{cases} -k^2 = -\left| \frac{4\pi^2 \delta f}{c^2} [2f_0 - \delta f] \right|, (\delta f > 0) \\ k^2 = \left| \frac{4\pi^2 \delta f}{c^2} [2f_0 - \delta f] \right|, (\delta f < 0) \\ 0, (\delta f = 0) \end{cases}$$

with the parameter $k = \left| \frac{4\pi^2 \delta f}{c^2} [2f_0 - \delta f] \right|^{1/2}$

It represents a RF cavity composed of multi-cells which have different frequency shifts due to various errors. Comparing with the particle transverse movement equation, we can find the coefficient is also a piecewise constant along the longitudinal direction. Consequently we suppose the equation of the electrical field has also a transfer matrix solution.

Corresponding to the positive, negative and zero coefficients, the particle transverse movement has a focusing matrix, a defocusing matrix and a drift matrix respectively, while in the perturbation of electrical field they correspond to a frequency down shift, a frequency up shift and to no shift.

The electrical field perturbation equation is transformed to be

$$\begin{pmatrix} E \\ E' \end{pmatrix} = M(z) \begin{pmatrix} E_0 \\ E'_0 \end{pmatrix}$$

Here the suffix 0 stands for the initial electrical field and for the transfer matrix of the “frequency down shift” section, it is

$$M(z) = \begin{bmatrix} \cos(kz) & \frac{1}{k} \sin(kz) \\ -k \sin(kz) & \cos(kz) \end{bmatrix},$$

and for transfer matrix of “frequency up shift” section it is

$$M(z) = \begin{bmatrix} ch(kz) & \frac{1}{k} sh(kz) \\ ksh(kz) & ch(kz) \end{bmatrix},$$

while for transfer matrix of “frequency no shift” section it is

$$M_o(z) = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix}.$$

For a cavity with multi-cells, the electrical field distribution in the N-th cell can be calculated by multiplying the transfer matrices of the N sections as following

$$\begin{pmatrix} E \\ E' \end{pmatrix} = M(z) \begin{pmatrix} E_0 \\ E'_0 \end{pmatrix} = M_N(z) \prod_{j=N-1}^{j=1} M_j(L_j) \cdot \begin{pmatrix} E_0 \\ E'_0 \end{pmatrix}$$

Successive calculation for each cell leads to the whole distribution of the field in the cavity.

Concerning the boundary condition, we can see there is an obvious difference between the field perturbation in a RF cavity and particle movement in a quadrupole channel. For the particle, the initial phase space coordinates x_0 and x'_0 , and the focusing coefficient k_q are given parameters. But for RF cavity the constants E_0, E'_0 and k depend on the field distribution in the cavity. And because of metal end plates, we have $E'_0 = 0, E'(L_T) = 0$, as boundary conditions.

This means that the transfer element of the total matrix for the cavity $M_{21}(L_T) = 0$. This relation can be used to get parameter k .

Another important initial parameter is E_0 which needs to be determined before perturbation calculation. Here we make an assumption that the stored energy in a RF cavity keeps constant after introducing the local frequency shifts. Then we have the equation

$$\int_0^{L_T} E^2 dz = \int_0^{L_T} E_n^2 dz \quad (3-1)$$

Here the E_n stands for the no perturbation electrical field which is a given parameter and E is a function of E_0 . By this equation, we obtain the initial electrical field. Multiplying the matrix of each cell, we get the field plot along longitudinal direction. The tuning of the field can be primarily operated by the calculation comparing with simulation with codes.

APPLYING TRANSFER MATRIX METHOD IN CSNS DTL DESIGN

In CSNS DTL-1, the electrical field is designed as shown in Figure 1. All the cells are generated by PARMILA code. The simulation with DTLFISH shows that they have the same frequency, which is supposed to respond to a flat field distribution. By the transfer matrix method, we need to make two local frequency shifts to generate the tilt. In general, adjusting structures on several cells, instead of only two cells, has the least disadvantages. For this reason, the first 3 cells and the cells number 22, 23 and 24 are tuned to make necessary frequency shift. Then we have 5 discrete electrical field points from E_0 to E_4 which is shown in Figure 2.

Applying the transfer matrix method to calculate the electrical field, we get the final answer

$$(E_1 \ E_2 \ E_3 \ E_4) = (1.022 \ 1.344 \ 1.377 \ 1.377)E_0$$

Getting the initial electrical field E_0 from Equation 3-1, we plot the tuned field in Figure 2, which is almost coincident with the simulated result with MDTFISH as shown in Figure 3. It can be seen in Figure 3 that the tilt is generated as design required, but the last few cells have some difference. That is because of the frequency shift introduced by structural change of drift tubes. According to the difference shown in Figure 3, we apply again the transfer matrix method on the cells number 55, 56 and 57. The new structures are simulated with MDTFISH shown in Figure 4.

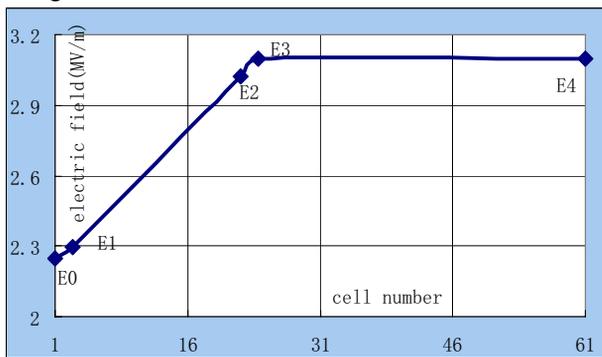


Figure 2: Calculated field pattern with matrix method.

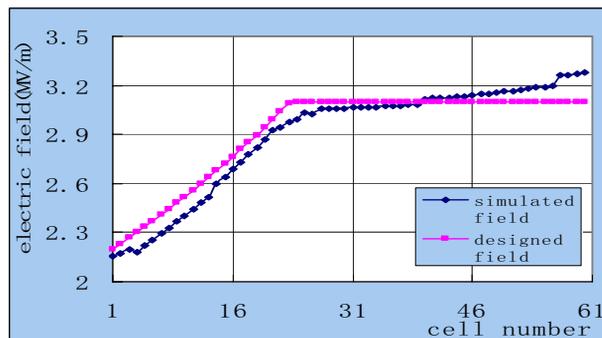


Figure 3: The simulation result with MDTFISH using modified structure from matrix method.

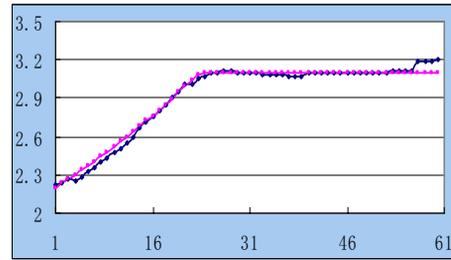


Figure 4: The simulation result of structure generated by using transfer matrix for a second time.

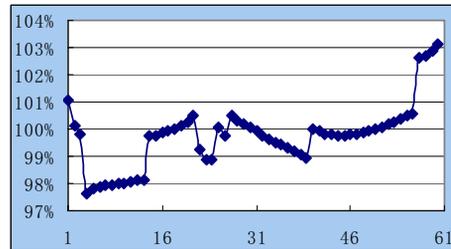


Figure 5: The ratio of simulation result to designed one.

Through the difference comparison of simulation result and designed field distribution shown in Figure 5, we can see the transfer matrix method can tune the field effectively. That will greatly decrease the difficulty on tuning the RF cavity.

MODEL MEASUREMENT

Now we have already finished the preparation for measuring CSNS DTL-1 RF cavity and test the transfer matrix method. More detailed data and the verification of the method will be given in a few months when the complete of the tank manufacture.

CONCLUSIONS

The equations for the electrical field distribution in a RF cavity have the same form as the particle movement in a quadrupole channel. So the transfer matrix should be able to solve both of them. By calculation and simulation, we have confirmed the transfer matrix method. The method is helpful both in RF cavity designing and tuning.

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