MODEL FOR ADDRESSING NSLS II LATTICE RESPONSE TO RANDOM, STATIONARY VIBRATION*

N. Simos#
NSLS II Project, BNL, Upton, NY 11973, USA

Abstract
An analytical model is being developed in an attempt to quantify the variances of lattice displacement and beam orbit jitter by adopting the model to NSLS II conditions. Relations governing stochastic processes as well as numerical analysis of the interaction of the accelerator ring structure with the randomized substrate and ground motion are used to construct appropriate transfer functions linking the field with the accelerator lattice distortions.

INTRODUCTION

The extremely tight photon beam parameters of the NSLS II impose challenging requirements onto the electron beam and its orbit stability in the 6-dimensional phase space. To achieve the demanding photon beam parameters, the electron beam orbit stability at the photon beam source locations must remain below a few hundred nanometers for a wide frequency band. The orbit stability at the photon extraction locations is directly coupled to the movement of the magnetic elements in the lattice itself coupled to the ring structure. While the vibration environment exciting the ring structure that in turn excites the lattice and eventually the electron beam is a combination of deterministic and stochastic or random noise, it is the random, uncorrelated part that may induce the largest orbit instabilities due to the inability for feed-back correction. Central to the orbit stability is the estimation of the variance of the position of magnetic elements in the lattice expressed in the form of a spatial cross power spectrum. Subsequently, the variance in the magnetic element position will translate into variance of the beam "jitter" and photon beam instability.

Several studies have addressed the beam orbit distortion resulting from lattice magnet movement due to ground vibration [1,2] for linear and circular machines. In all the studies, however, the quantification or the variance of the beam orbit distortion based on site-specific parameters and randomization has been challenging. The potential of numerically simulating the dynamic response of the overall accelerator structure and of the lattice opens the possibility of quantifying the processes involved.

In this study, a model combining analytical and numerical processes governing the interaction of stochastic ground motion with the light source ring is formulated in an attempt to quantify the variances of lattice displacement and beam orbit jitter for the specific conditions of the NSLS II. Specifically, the dynamic interaction of the NSLS II ring structure supporting the lattice with the stationary ground vibration field is addressed using a comprehensive 3-dimensional analytic model of wave propagation and wave-structure interaction. Cross transfer functions linking ground vibration with the ring and lattice are deduced from the numerical analysis leading to the formation of a multi-degree of freedom cross-spectral density of the lattice. The variance of the lattice spectral response at any of its DOF can subsequently be estimated as a function of the ground motion stochastic parameters.

OVERALL RANDOM MODEL

Ground motion at a gives site consists of a deterministic and stochastic (random) part. Multiple measurements at the site (offset in space and time) can help delineate the two contributions. It is assumed that the ground motion of interest is described by a stationary process which can be represented by power spectra that reflect the stochastic nature. Therefore, any "measurement" recorded at the site represents a "realization" of the stochastic process. Consider the green-field ground vibration as a cross correlated power spectrum

$$\left[S^{FF}{}_{ij}(\omega)\right] \quad i, j = 1,2,3 \tag{1}$$

Spatial variability over the accelerator site should be considered to be the result of fluctuations around a mean value. Such fluctuations can be randomized and incorporated into the process of transferring the stochastic motion of Eqn. 1 to the accelerator structure. Consider the fluctuating component $\boldsymbol{\varphi(x,y)}$ of a property describing the site (i.e. wave propagation velocity) and exhibiting spatial variability over the domain of interest (accelerator interaction with the site vibration) to have zero mean

$$E[\phi(x)] = 0 \tag{2}$$

and auto-correlation function

$$R_{\varphi\varphi}(\xi) = E[\varphi(x)\varphi(x+\xi)] \tag{3}$$

$\boldsymbol{x}$ is the position vector and ξ is the separation vector between two locations. Equation 4 represents an assumed correlation of the random property between two locations separated by a vector ξ while δ defines the distance of strong correlation. Geophysical site studies will provide information that will allow for the best estimate of δ.

$$R_{\varphi\varphi}(\xi) = \exp\left[-\frac{\xi}{\delta}\right]\left[1+\frac{\xi}{\delta}\right] \tag{4}$$

* Work supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886.
# simos@bnl.gov

The correlation between x_i and x_j is established from the covariance matrix and the randomized field φ of size N. From Eqn. 4 a random vector λ (also of size N) is generated.

$$Cov[\varphi_i \varphi_j] = R_{\varphi\varphi}\{\xi_{ij}\} \quad (5)$$

$$\{\lambda\}^T = [L]\{\varphi\}^T \quad (6)$$

$$LL^T = C_{\varphi\varphi}{}^{v} \quad (7)$$

Where L is the lower triangular matrix resulting from the Cholesky decomposition of the covariance matrix $C_{\varphi\varphi}$ satisfying the following relationship,

$$LL^T = C_{\varphi\varphi} \quad (8)$$

Thus, starting from a vector consisting of N independent random realizations of the parameter fluctuation and using a decomposition technique of the covariance matrix, a vector that contains both the randomness and the correlation between locations is formed. This vector of values is added to the expected value of the parameter for the zone leading to a distribution of values that are correlated. Using a Monte Carlo approach, M random vectors φ are generated leading (through the covariance matrix and its decomposition) to M profiles expressed in the form of λ vectors. The computational cost of generating random correlated fields is minimized because decomposition of the covariance matrix takes place once.

In addition to the stochastic ground motion that interacts with the ring supporting the lattice, random, uncorrelated noise it is assumed to exist on the ring floor supporting the lattice elements. This noise can be assumed to exhibit correlation over the floor or to be described by a white spectrum and be spatially and temporarily uncorrelated.

LATTICE SPECTRAL RESPONSE

The transfer of the random free-field motion expressed in terms of a cross-spectral function (Eqn. 1), through a site that has been randomized to reflect spatial variability, to the accelerator ring is achieved with transfer functions which capture the dynamic interaction of the ring structure with the spatially varying ground vibration. Fig.1 depicts time snap-shots of the interaction of surface waves (Rayleigh mode) propagating on the site and interacting with the ring structure. By relying on a large-scale numerical model the relation between the free-field motion which represent a "realization" of a random field or in other words it is a member of a family represented by the same power spectra the transfer functions **[$H_{Fr}(\omega)$]** between the free field and N locations on the ring (i.e. lattice magnetic element supports) can be established. In the complete case of a [**3x3**] cross-power spectrum in the free-field a [**NxN**] transfer matrix can be deduced that includes all the cross-terms.

$$[S_{rr}(\omega)] = [H_{Fr}(\omega)][S_{FF}(\omega)][H'_{Fr}{}^{*}(\omega)] \quad (9)$$

Where S_{rr} is the [**NxN**[cross-spectral density of the N locations on the ring (i.e. lattice supports), S_{FF} is the free-field power spectrum and $[H^*]$ is the transpose of the conjugate transfer function from the free-field to the ring locations. With the cross-spectrum at the support of each lattice magnetic element the power spectrum at the magnet location can be deduced from a similar relation defined by Eqn. 9 while considering a linear transfer function $H_{rm}(\omega)$ between the ring floor and the magnet,

$$[S_m(\omega)] = [H_{rm}(\omega)][S_{rr}(\omega)][H^*{}_{rm}(\omega)]^T \quad (10)$$

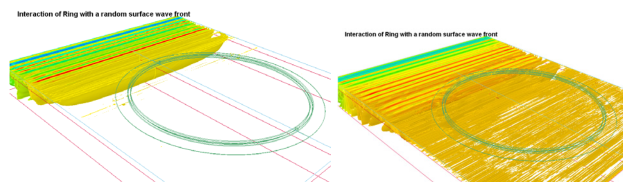

Figure 1: Simulated interaction of the NSLS II ring structure with ground motion (Rayleigh wave mode).

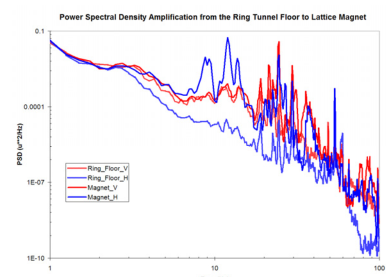

Figure 2: Measured floor vibration transfer to lattice.

The floor-to-magnet transfer function can either be analytically established for each independent lattice magnetic element or can be measured. Fig. 2 reflects measured transfer functions at the BNL NSLS. Therefore the variance associated with the position of the lattice magnetic elements, provided that there is no added random noise, can be deduced from

$$\sigma_m{}^2 = \int_{-\pi}^{\pi} S_m(\omega) d\omega \quad (11)$$

Assume, however, that over the lattice support there exist additional random noise N_{rr} that is uncorrelated with the ground motion but could exhibit spatial and temporary correlation over the support floor. The cross-spectral matrix now takes the form

$$[S^T{}_{rr}(\omega)] = [S_{rr}(\omega)] + S_{NN}(\omega) \quad (12)$$

For an existing ring supporting the accelerator lattice the total power spectrum containing the random noise can be established from measurements. Based on the assumption that the ground motion and the noise disturbance are uncorrelated, the spectral matrix of the noise can be deduced based on the relation

$$S_{NN}(\omega) = S^T{}_{rr}(\omega) - S_{Fr}(\omega) S_{FF}{}^{-1}(\omega) S_{Fr}{}^{*}(\omega) \quad (13)$$

The 'noise' spectral matrix S_{NN} may assume different forms. It can be described by a white noise spectrum causing movement of the N lattice supports to be uncorrelated. In the event that the only uncorrelated movement on the lattice ***N*** supports results from this noise disturbance, then the magnetic element uncorrelated movement can be deduced.

BEAM JITTER EVALUATION

To quantify the effect that stochastic motion has on the beam jitter, the analytical procedure described in the previous sections and numerical or simulation models of the interaction need to be employed. In particular, the understanding of how the realistic, multi-DOF structure, responds to the random motion can be deduced from a numerical model designed to mimic the spatially varying site conditions and the structure. Considerations of deterministic accelerator ring structure and lattice are considered valid and thus enable simplification of the problem. The power cross-spectrum representing the family of ground motion realizations can be deduced from measurements or defined according to formulae that have been deduced from studies [2] and have estimated the delineation of the correlated and uncorrelated parts of the power spectra. Measured coherence or correlation properties of a site ground motion (Fig. 3) will lead to site specific parameters relevant to the spectra cross terms.

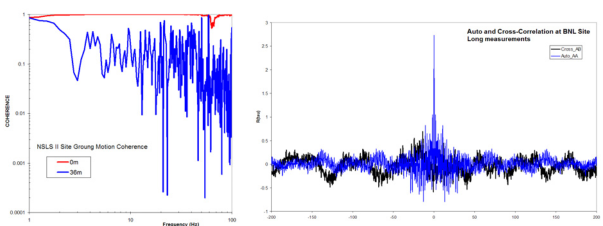

Figure 3: Coherence and correlation of ground motion recorded at the NSLS II site.

Based on the simulation of waves that are part of the ensemble represented by the power spectra and their interaction with the structure, transfer functions linking the random free-field with the accelerator lattice support are constructed. These transfer functions reflect the response based on the wave propagation mode that is considered (i.e. surface or body waves). With the cross-spectral matrix between the free-field and the ring floor $\boldsymbol{H_{Fr}(\omega)}$ either with added floor noise $\boldsymbol{N_{rr}}$ or not the cross-power spectra of the N lattice support locations is deduced. The cross-spectrum of the movement of the lattice elements about the reference position as a result of the floor movement is deduced from Eqns. 10 and 12. The dynamic response of the ring structure, however, plays a significant role in determining the cross-correlation terms. Correlated and uncorrelated ground motion will excite the structural modes making the floor movement at the N locations strongly correlated at the modal frequencies. Figure 4 shows some of the structural modes. Modes within the frequency band of interest, therefore, need to be identified and their influence on the cross-spectra terms are accounted for.

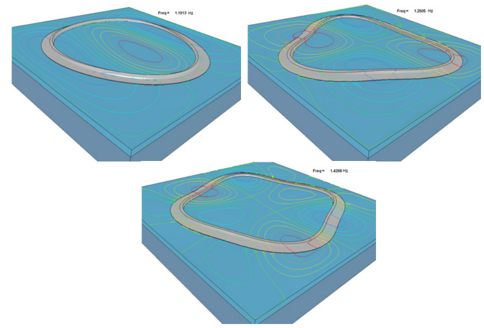

Figure 4: Mode ensemble of ring resting on the site.

The cross-spectral matrix over the N magnetic elements of the lattice which has accounted for (a) the stochastic nature of the site ground motion, (b) the spatial variability at the site, and (c) added, uncorrelated noise on the floor can be used to compute the variance of the beam jitter. Studies [1] have established relations that link movement in the magnetic element with the beam kick at the magnet location. The beam orbit distortion will be an integrated effect of all magnet motions. A key parameter in assessing the accumulated effect is the correlation length pertinent to the floor spectrum since correlation lengths greater than the betatron wavelength will have almost no effect. Assuming that the response function $\Phi_{ij}(\omega)$ describing the beam jitter at a magnetic element as a function of every magnet of the lattice (that accounts for the correlation length discussed above) is known, as it is the case for several machines, then the variance of the beam jitter can be deduced from

$$\sigma_{jitter}^{2} = \int_{-\pi}^{\pi} [\Phi_{ij}(\omega)][S_m(\omega)]d\omega \qquad (14)$$

SUMMARY

The framework of a comprehensive process for estimating beam orbit distortion variances due to ground motion, spatial site variability while combined with uncorrelated random noise on the ring has been formulated. Large-scale simulations of the interaction of the ring/lattice structure with the randomized fields are implemented to construct cross-spectral matrices. These are augmented with site measurements revealing correlation and spectral properties.

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