IMPEDANCE BUDGET DATABASE FOR THE EUROPEAN XFEL

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Abstract

The European XFEL contains hundreds of sources of wakefields. To judge their impact on the machine performance, an impedance budget database is developed. It contains wake functions of the point charge (Green functions) and allows to calculate the wake potentials for arbitrary bunch shapes.

INTRODUCTION

The European XFEL [1] requires a beam of a high intensity with small emittance and energy spread. The longitudinal wake field [2] introduces an energy losses and an energy chirp along the bunch. It modifies significantly the saturation length, the energy gain and the radiation bandwidth.

The wake field is a linear response of the system to an external excitation produced by a bunch. This response can be expressed in terms of a Green's function, which we will call a wake function. In the paper only the longitudinal component of the wake function will be considered.

In the following we do not take into account the Coulumb repulsion between the particles [3]. The coupling wake functions are properties of the beam environment, but not of the beam itself. Hence, if the wake function $w_{\|}(s)$ is known then the bunch wake potential $W_{\|}(s)$ for arbitrary bunch shape $\lambda(s)$ can be determined by a convolution.

$$W_{\|}(s) = -\int_{-\infty}^{s} w_{\|}(s-s')\lambda(s')ds' .$$

The European XFEL [1] contains hundreds of sources of the coupled impedances. To obtain the wake functions of different elements we use analytical and numerical methods. The numerical codes allow to calculate only wake potentials (usually for the Gaussian bunch shape). Hence, the wake potentials have to be fitted to the theoretical models to estimate the analytical forms of the short-range wake functions [4, 5].

The wake functions have usually singularities and can be described only in the terms of distributions (generalized functions). In the next section we introduce an approach to tabulate such functions and use them later to obtain wake potentials for different bunch shapes.

To have an overview of all elements and their geometries and properties, we have developed an impedance database. It allows to obtain an impedance for arbitrary bunch shapes, which are found by start to end simulations with beam dynamics codes.

Finally, we calculate an impedance budget for the current design of the European FEL with three bunch compressors [6].

WAKE FUNCTION MODEL

The longitudinal wake function $w(s)$ can be split in regular $\alpha(s)$ and singular $\chi(s)$ parts,

$$w(s) = \theta(s)\left[\alpha(s) + \chi(s)\right], \tag{1}$$

where $\theta(s)$ is the Heaviside step function and

$$\alpha(s) = w^{(0)}(s) + \frac{1}{C},$$

$$\chi(s) = Rc\delta(s) - c\frac{\partial}{\partial s}\left[Lc\partial(s) + w^{(-1)}(s)\right],$$

The regular function $\alpha(s)$ is tabulated easily. The singular part is described with the help of the constants R, L and the regular function $w^{(-1)}$ which has the asymptotic

$$\frac{\partial}{\partial s} w^{(-1)}(s) = o(s^{-1}) , \quad s \to 0 .$$

The Fourier transform of wake function (1) reads

$$Z(\omega) = Z^{(0)}(\omega) - \frac{1}{i\omega C} + R + i\omega\left[L + Z^{(-1)}(\omega)\right],$$

and constants R, L, C have meaning of resistivity, inductance and capacitance, correspondingly.

The wake potential for arbitrary bunch shape $\lambda(s)$ can be found by formula

$$W(s) = -\int_{-\infty}^{s} w^{(0)}(s-s')\lambda(s')ds' - \frac{1}{C}\int_{-\infty}^{s} \lambda(s')ds' -$$
$$-Rc\lambda(s) - c^2 L\lambda'(s) - c\int_{-\infty}^{s} w^{(-1)}(s-s')\lambda'(s)ds' ,$$

where $\lambda'(s)$ is a derivative of the bunch shape.

For example, the known analytical results for simple geometries fit in our model as follows:

a) diffractive model of the pillbox cavity [7]

$$w^{(-1)}(s) = -\frac{Z_0}{\sqrt{2\pi^2 a}}\sqrt{sg} ; \tag{2}$$

b) optical model of the stepped collimator [8]

$$R = \frac{Z_0}{\pi}\ln\left(\frac{b}{a}\right); \tag{3}$$

c) short range wake of the slowly tapered collimator [9]

$$L = \frac{Z_0}{4\pi c}\int r' dr . \tag{4}$$

The longitudinal bunch shape changes along the accelerator. The final energy spread is not a direct sum of the partial energy spreads in different sections. Hence, we need a procedure to add wake potentials obtained for different bunch shapes.

In order to add wake potentials we present them in normalized coordinates. We have N normalized charge distributions λ_i

$$\int \lambda_i(s)ds = 1, \quad i = 1,...,N ,$$

with central first and second moments $\{\mu_i, \sigma_i\}$.

The wake potential $W_i(s)$ obtained for the distribution λ_i can be written in the normalized coordinate $\hat{s} = (s-\mu_i)/\sigma_i$ as $\hat{W}(\hat{s}) = W_i(\hat{s}\sigma_i + \mu_i)$.

Hence, the total wake potential and the averaged bunch shape can be defined as

$$\hat{W}(\hat{s}) = \sum_{i=1}^{N} \hat{W}_i(\hat{s}) = \sum_{i=1}^{N} W_i(\hat{s}\sigma_i + \mu_i)$$

$$\hat{\lambda}(\hat{s}) = \frac{1}{N}\sum_{i=1}^{N} \sigma_i \lambda_i(\hat{s}\sigma_i + \mu_i).$$

WAKE FUNCTIONS IN ACCELERATOR

The current design of the European XFEL contains 812 TESLA accelerating cavities. Their wake function is [5]

$$w^{(0)}(s) = 43\exp(-24\sqrt{s}) \ \text{[kV/nC/cavity]}.$$

To linearize the longitudinal phase space a third harmonic module with 8 cavities will be installed after the booster. The wake function of the third harmonic cavity in kV/nC reads [10]

$$w^{(0)}(s) = 79.5e^{-34.5\sqrt{s}} + 0.225\left(\frac{\cos(5830 s^{0.83})}{\sqrt{s}+195s} - \frac{1}{\sqrt{s}}\right),$$

$$w^{(-1)}(s) = 0.45c^{-1}\sqrt{s}.$$

The reduction of the pipe radius from 39 mm to 20mm can be described by constant $R = 80[\Omega]$ (see Eq.(3)).

The diagnostic sections after the bunch compressors contain 6 transverse deflecting structures (TDS) with wake function in kV/nC [10]

$$w^{(0)}(s) = 257.6e^{-15.9\sqrt{s}} + 1.16\left(\frac{\cos(1760 s^{0.72})}{\sqrt{s}+1600 s^{1.23}} - \frac{1}{\sqrt{s}}\right),$$

$$w^{(-1)}(s) = 2.32c^{-1}\sqrt{s}.$$

The collimation section contains 4 collimators. The tapering is not helpful for very short bunches and therefore we can estimate the geometrical longitudinal wake by a simple Eq. (3). For the resistive wake a steady-state solution in round pipe is used.

For a round pipe of radius a the steady-state longitudinal impedance is given by [11, 12]

$$Z(\omega) = \frac{Z_s(\omega)}{2\pi a}\cdot\left[1 + i\frac{\omega}{c}\frac{a}{2}\frac{Z_s(\omega)}{Z_0}\right]^{-1},$$

$$Z_s(\omega) \approx Z_s^L(\omega) + Z_s^\kappa(\omega), \quad Z_s^\kappa(\omega) = \sqrt{\frac{i\omega\mu_0}{\kappa(\omega)}},$$

$$\kappa(\omega) = \frac{\kappa_0}{1+i\omega\tau}, \quad Z_s^L(\omega) = i\omega L,$$

$$L = \mu_0\left(\Delta_{ox}\frac{\varepsilon_r - 1}{\varepsilon_r} + 0.01\Delta_{rough}\right). \qquad (5)$$

To estimate a steady-state resistive wake of the elliptical pipe in the undulator we used a numerical code of M. Dohlus, which solve the integral equation by method of auxiliary sources.

The effect of the oxide layer and the roughness for the elliptical pipe are included through the inductance L in the same way as it was done in Eq.(5) for the round pipe.

UNDULATOR WAKES

The European XFEL contains a lot of undulators separated by intersections. The main source of the longitudinal wake in the undulator is a resistive wake. However the geometrical interruptions in the intersections produce up to 20 % of the energy spread. The geometry of the one intersection is shown in Fig.1 and the elements are listed in Table 1.

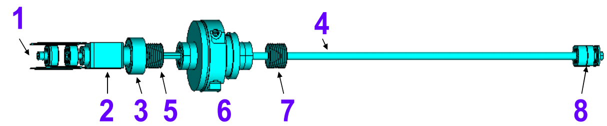

Figure 1: The undulator intersection geometry.

The wake functions of the elements are obtained through numerical and analytical calculations. The table 1 shows an impedance budget of the transition for a Gaussian bunch with RMS length $\sigma = 25\mu m$.

Table 1: Impedance budget in kV/nC

N	Element	Geom. Loss	Geom. Spread	Loss	Spread
1	Eliptical pipe	0	0	239	274
2	Pump	4,4	4,5	9	10
3	Absorber	69	27	70	28
4	Round pipe	0	0	22	32
5	Below	24	9	25	10
6	BPM	42	17	70	34
7	Below	24	9	25	10
8	Round/ell. transition	36	14	36	14
Total		**199**	**80**	**496**	**365**

The real bunch shape changes along the undulator and differs considerably from the Gaussian one.

IMPEDANCE DATABASE

The bunch shapes along the accelerator are shown in Fig.2. During acceleration and compression in three bunch compressors the bunch length reduces from 1.8 mm to 0.026 mm. The bunch shapes are obtained through beam dynamics simulations by M. Dohlus. We use these shapes in our database to estimate the wakes.

The impedance database is a Microsoft Access application. It contains a list of the main elements of the accelerator with their wake functions tabulated in agreement with Eq. (1). The database tables contain a list of element types, characteristics of elements, geometry descriptions, parameters and links to input files for the wake calculations, a list of sections, a list of bunch shapes for XFEL and other information. With help of different forms and reports, designed in the database, it is possible to calculate the wake potential for selected elements with selected bunch shapes. Different reports are available. All forms afford an opportunity to plot the wake potential for one element or group of selected elements.

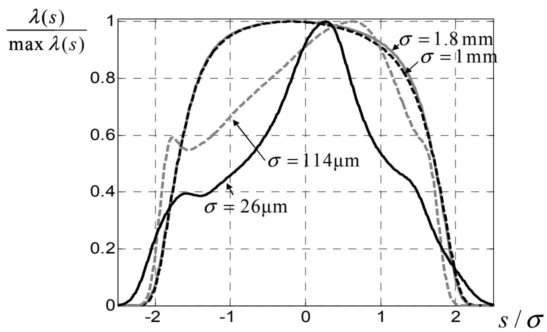

Figure 2: Bunch shapes in three stage bunch compressor scheme.

Table 2: Loss and spread parameters for sections

Name of section	Loss, MV/nC	%	Spread, MV/nC	%
Injector 1	0.16	0.3	0.06	0.03
Linac 1	0.34	1	0.16	1
Compressor 1	0.25	0.3	0.12	1
Linac 2	1.61	3	0.87	4
Compressor 2	1.28	2	0.56	2
Linac 3	12.8	22	7.18	32
Collimation	23.5	41	9.64	43
SASE2	18.9	33	14.1	62
Total	**58**	**100**	**22.6**	**100**

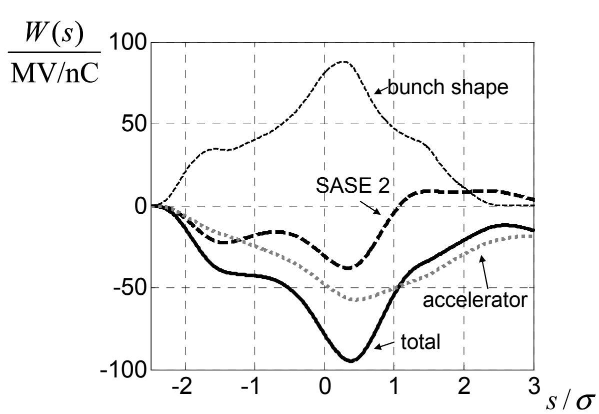

Figure 3: The longitudinal wakes.

As example we present an impedance budget for XFEL with bunch shapes shown in Fig.2. The loss and the spread parameters from the injector up to the end of the undulator SASE 2 are shown in Table 2 for each section. The same budget for each element type is shown in Table 3.

Table 3: Loss and spread parameters for elements

Name of element type	Num.	Loss MV/nC	%	Spread MV/nC	%
BPM CL	12	7.2	12	2.73	12
Collimator	4	5.2	9	2.29	10
Kicker	4	4.6	8	1.83	8
Pipe CL	1	4.5	8	2.89	13
Pump CL	78	0.6	1	0.24	1
Ell. pipe	42	8.96	16	10.45	46
Round pipe	41	0.9	2	1.7	8
Pump	41	0.3	1	0.34	2
Absorber	41	2.7	5	1.07	5
Bellow	82	2	3	0.75	3
BPM	41	2.7	5	1.06	5
Round/ Ell.	41	1.4	2	0.54	2
TESLA cav.	812	14.8	26	8.33	37
Harm. cav.	8	0.09	0	0.03	0
Flange	500	1.4	2	0.57	3
TDS	8	1.5	3	0.67	3
Total		**58**	**100**	**22.6**	**100**

Fig.3 presents plot of the total longitudinal wake potential from injector up to the end of SASE 2, the wake potential up to the undulators and the wake potential in SASE 2.

At the moment the database contains the main sources of the impedances. Meanwhile we continue to fill it with other elements. At the same time geometry and material properties of some elements are under development and could be changed in the future. Hence, an additional effort is required to bring the contents of the database up to date.

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