ANALYTICAL STUDIES OF COHERENT ELECTRON COOLING*

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Abstract

Under certain assumptions and simplifications, we studied a few physics processes of Coherent Electron Cooling using analytical approach. In the modulation process, the effect due to merging the ion beam with the electron beam is studied under single kick approximation. In the FEL amplifier, we studied the amplification of the electron density modulation using 1D analytical approach. Both the electron charge density and the phase space density are derived in the frequency domain. The solutions are then transformed into the space domain through Fast Fourier Transformation (FFT).

INTRODUCTION

Possibility of enhancing electron cooling using various collective instabilities - called either Coherent Electron Cooling or Stochastic Electron Cooling - was discussed by Ya. S. Derbenev since 1980 [1]. First specific mechanism of Coherent Electron Cooling, recently developed by V. N. Litvinenko and Ya. S. Derbenev [2, 3], is based on high gain free electron laser (FEL) . This method promises to be an effective method for significantly increases in the luminosity of high energy hadron colliders. Both analytical approach and macro particle simulation approach were developed to study the detailed modulation process [4, 5]. For infinite anisotropic plasma with κ velocity distribution for $\kappa = 2$, a closed form solution was derived for electron density modulation due to an ion moving with constant velocity. However, the effect due to merging the ion beam with electron beam involves changing of ion velocity. The physical process in FEL has been relatively well studied in simulation and theoretical approach. As most theoretical studies were focused on the emitted radiation, evolutions of the electron density and the phase space density in the FEL have to be derived.

In this paper, we present some analytical studies of these issues. By treating the merger as a thin kicker, we generalized the closed form solution in [4] to include the merging process. For the FEL amplification, we adopted the frame work of 1D theory[6]. Using the electron density modulation derived in [4] as initial condition, both the electron density amplification and the phase space density evolution inside FEL are derived in the frequency domain with space charge and energy spread effects being taken into account. The solutions in the time domain are obtained numerically by FFT.

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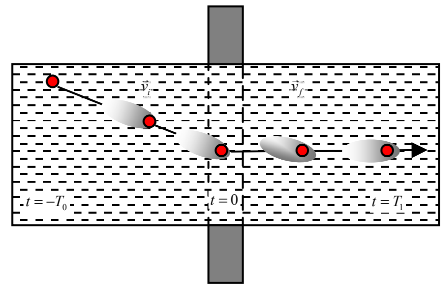

Figure 1: Schematic drawing of the merging process through a thin kicker. The red solid spot represents a moving ion and the grey ellipse around it represents the modulated electron cloud. The kick takes place in $t=0$ as the ion crossing the merger which is represented by the dark grey squares in the middle of the graph. The ion meets the electrons at $t=-T_0$ and separates from them at $t=T_1$.

MERGING EFFECTS IN ELECTRON DENSITY MODULATION

As the electron beam approaching the circulating ion beam with an angle, its velocity has to be changed in order to moving with the same velocity as the ion beam. The simplest model to describe the merging process is to assume the velocity change of the electron beam happens instantaneously, i.e. caused by a thin kicker. As shown in Figure 1, if we stay in the reference frame of the electrons, the process can be described by the shielding process of electrons to an ion which abruptly changes its velocity from $\vec{v}_i$ to $\vec{v}_f$ at a given time $t=0$.

For the system described above, the charge density of the moving ion can be expressed as

$$\rho(\vec{x},t) = Z_i e\left(1-H(t)\right)\delta(\vec{x}-\vec{v}_i t) + Z_i e H(t)\delta(\vec{x}-\vec{v}_f t) \qquad (1)$$

, where

$$H(t) \equiv \begin{cases} 0 & t<0 \\ 1 & t\geq 0 \end{cases}$$

is the Heaviside step function. We assume the velocity distribution is

$$f_0(\vec{v}) = \frac{1}{\pi^2 \beta_x \beta_y \beta_z}\left(1+\frac{v_x^2}{\beta_x^2}+\frac{v_y^2}{\beta_y^2}+\frac{v_z^2}{\beta_z^2}\right)^{-2} \qquad (2)$$

, where β_x, β_y and β_z are parameters to describe the 3D electron temperatures. Following similar derivation in [4],

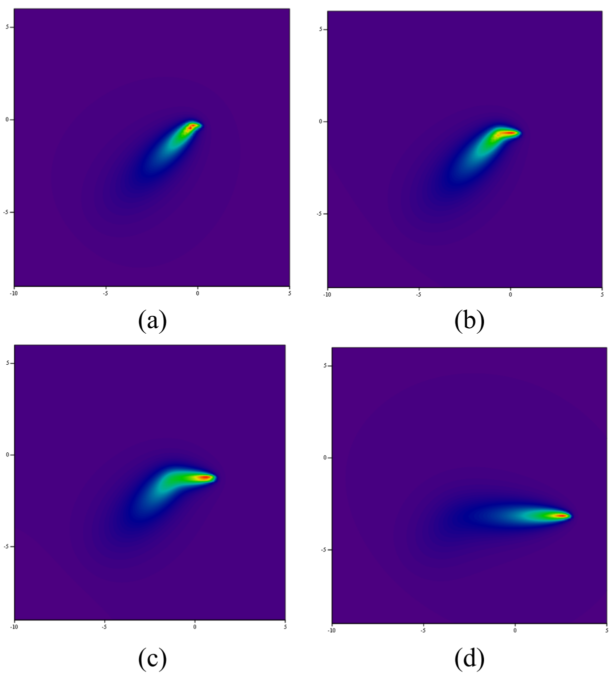

Figure 2: Example of electron density modulation in presence of an instantaneous kick at $t=0$ as calculated from eq. (3). The abscissa is the longitudinal position and the coordinate is the transverse position, both in units of Debye radius. In this example, $\vec{\bar v}_i=(2,0,2)$, $\vec{\bar v}_f=(0,0,4)$ and $\psi_0=2\pi$ *. (a) Snapshot at $\psi_1=0.05\pi$; (b) Snapshot at $\psi_1=0.1\pi$; (c) Snapshot at $\psi_1=0.2\pi$; (d) Snapshot at $\psi_1=0.5\pi$.

we obtained the following result for the electron density evolution,

$$\tilde n_1(\vec x,t)=\frac{Z_i}{\pi^2 r_{Dx}r_{Dy}r_{Dz}}\times \left\{\int_{\psi_1}^{\psi_1+\psi_0}\frac{\psi\sin(\psi)d\psi}{\left(\psi^2+(\bar x+\bar v_{ix}\psi)^2+(\bar y+\bar v_{iy}\psi)^2+(\bar z+\bar v_{iz}\psi)^2\right)^2} + \int_0^{\psi_1}\frac{\psi\sin(\psi)d\psi}{\left(\psi^2+(\bar x'+\bar v_{fx}\psi)^2+(\bar y'+\bar v_{fy}\psi)^2+(\bar z'+\bar v_{fz}\psi)^2\right)^2}\right\} \quad (3)$$

, where r_{Dx}, r_{Dy} and r_{Dz} are Debye radius of the electron plasma, ω_p is the plasma frequency of the electrons $\psi_{0,1}=\omega_p T_{0,1}$, $\bar x = x/r_{Dx}$, $\bar v_{ix}=v_{ix}/\beta_x$ and

$$\vec{\bar x}'=\vec{\bar x}-\left(\vec{\bar v}_f-\vec{\bar v}_i\right)\psi_1$$

* In reality, an ion typically spends very short time while merging the electron beam (i.e. $\psi_0<<1$) and its transverse velocity component is very large (i.e. $\vec{\bar v}_i=(\bar v_\perp,0,2)$, $\bar v_\perp>>1$). There, the parameters we used in this example correspond to the worst case scenario.

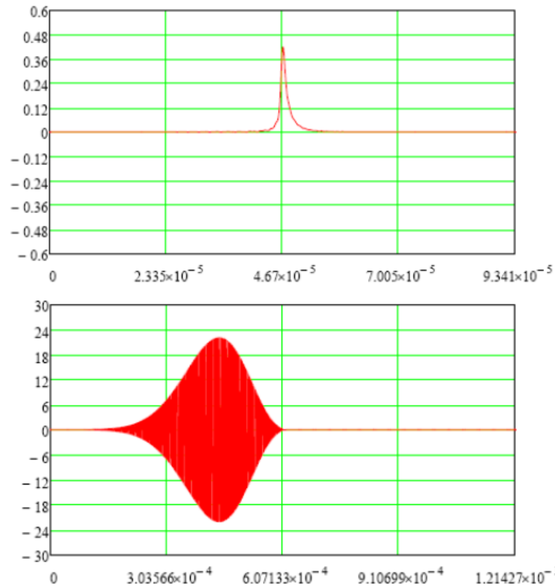

Figure 3: Electron density amplification in 1D FEL. The abscissa is time in units of nanosecond and the coordinate is the electron charge density with units $Z_i e(r_{Dx}r_{Dy}r_{Dz})^{-1}$. The top graph is the electron density modulation at 1 gain length and the bottom graph is the electron density modulation at 13 gain length. The electron beam going leftwards.

As shown in Figure 2, direct numerical integration in equation (3) is straightforward and calculation shows that the modulation before merging becomes negligible after $1/4$ of plasma oscillations.

CHARGE DENSITY AND PHASE SPACE IN 1D FEL

We adopted the formalism in [6] to study the amplification process in an 1D FEL. If we assume the longitudinal energy distribution of the electrons is Lorentzian and the energy spread is small, the electron current density in the frequency domain is given by [7]

$$\tilde j_1(\hat z,\hat C)=\sum_{i=1}^{3}A_i(\hat C)\lambda_i(\hat C)e^{\lambda_i(\hat C)\hat z}\tilde j_1(0,\hat C) \quad (4)$$

, where λ_i are three eigenvalues solved from equation

$$\lambda^3+2(\hat q+i\hat C)\lambda^2+\left[\hat\Lambda_p^2+(\hat q+i\hat C)^2\right]\lambda-i=0,$$

$\hat q$ is parameter to describe the electron energy spread, $\hat\Lambda_p$ is the space charge parameter, $\hat C$ is the reduced detune which is defined as

$$\hat C=(k_w-\frac{\omega}{2c\gamma_z^2})l_{gain},$$

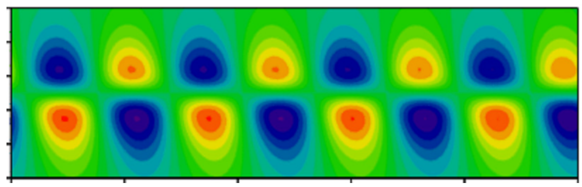

Figure 4: Phase space density modulation in FEL. The abscissa is time for 4 radiation period and the coordinate is the energy deviation from $-2\hat{q}$ to $2\hat{q}$. The graph shows the phase space density at 7 gain lengths. The red areas are with higher electron densities and blue areas with lower electron densities. The electron beam going leftwards.

where k_w is the undulator wave number, l_{gain} is the FEL gain length, $\hat{z}$ is the longitudinal location along the FEL in units of gain length and A_i are coefficients determined by initial modulation. The time domain solution is thus given by Fourier transformation of equation (4).

Apart from the density modulation, the energy modulation of the electrons in FEL is also important for the effectiveness of the kicker section. For example, preliminary simulation shows that the modulation tends to grow at the beginning of the kicker which is related to the energy modulation in FEL. For Lorentzian energy distribution, the phase space density in the frequency domain is

$$\tilde{f}_1\left(\hat{C},\hat{P},\hat{z}\right) = -\frac{e\theta_s n_0}{\pi\Gamma\varepsilon_0\rho}\frac{\hat{P}\hat{q}}{\left(\hat{q}^2+\hat{P}^2\right)^2}\sum_{i=1}^{3}A_i\frac{1+i\hat{\Lambda}_p^2\lambda_i}{\lambda_i+i\left(\hat{C}+\hat{P}\right)}e^{\lambda_i\hat{z}}, \quad (5)$$

where $\hat{P}$ is the energy deviation from the average energy of the electron beam, $\Gamma = l_{gain}^{-1}$ is the gain parameter and θ_s is the electron rotation angle in FEL [6]. The time domain solution is obtained by Fourier transformation of equation (5) with respect to $\hat{C}$, i.e.

$$\tilde{f}_1\left(\hat{z},\hat{P},t\right) = -\frac{c\Gamma\gamma_z^2}{\pi}e^{i\hat{k}_w\hat{z}}e^{i2\gamma_z^2\hat{k}_w(\hat{z}-c\hat{t})}\int_{-\infty}^{\infty}\tilde{f}_1\left(\hat{z},\hat{P},\hat{C}\right)e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}}d\hat{C},$$

where $\hat{k}_w = k_w/\Gamma$. As shown in Figure 4, there are more particles losing energy than gaining energy and the net energy lost by the electrons transfers to the radiation field. The whole pattern tilts to the left with a small angle, which is due to the dispersion effects. In the kicker section, the dense area (red) with higer energy and lower energy will approach each other due to the dispersion effect and electron density modulation is thus increased at the beginning of the kicker.

SUMMARY

The analytical solution of electron modulation is obtained for the simplified single kick merger. The result shows that the effects before the kick dissipated in $1/4$ of plasma oscillation. The electron density and phase space density within FEL is derived analytically in the frequency domain. Time domain solutions are obtained by FFT. The results can be used to benchmark simulation codes and serves as a tool for fast estimation. Further studies involve the analytical study of the kicker section, the diffraction effects in the FEL and the velocity modulation at the modulator.

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