INVESTIGATION OF BEAM - RF INTERACTIONS IN TWISTED WAVEGUIDE ACCELERATING STRUCTURES USING BEAM TRACKING CODES*

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Abstract

Investigations of the RF properties of certain twisted waveguide structures show that they support favorable accelerating fields. This makes them potential candidates for accelerating cavities. Using the particle tracking code, ORBIT, We examine the beam - RF interaction in the twisted cavity structures to understand their beam transport and acceleration properties. The results will show the distinctive properties of these new structures for particle transport and acceleration, which have not been previously analyzed.

INTRODUCTION

It is well known that twisted waveguides, in which a uniform cross section is rotated at a constant rate along a straight axis, can support waves at or below the velocity of light. Because of this, and the potential for easy fabrication, these helical structures are under consideration for applications in traveling wave tubes and particle accelerators. Analytic solutions of Maxwell's equations do not exist for twisted structures, and initial studies of these geometries made use of perturbation methods [1-3]. Perturbation techniques have been shown to work well when the twist rate is slow, but do not give accurate answers for rapidly twisting structures. In order to address this shortcoming, 2D and 3D numerical finite difference methods have been developed and applied to structures with arbitrary twist rates [4-6]. These studies show that twisted cavities may have real potential to serve as accelerating cavities. Accordingly, we have begun particle-tracking studies for beams in actual time-dependent 3D field configurations generated using the techniques developed in Refs. [4-6]. The tracking is carried out using the time-dependent Lorentz force equation tracker developed as a module in the ORBIT code [7]. This paper presents initial tracking results, while the latest work on designing, fabricating, and tuning twisted cavities and waveguides is presented in Ref. [8].

Because this work was just recently begun, present results are shown for only a single twisted cavity. This cavity has an axial period 6.6 cm, frequency $\nu = 2.74$ GHz, and $\beta = 0.6$, where β is the velocity divided by the velocity of light. The cavity is driven with a standing wave. The cross section of this cavity has a dumbbell shape. The acceleration is studied for protons. The results of this initial study will be used to iteratively improve the cavity design. Future work will also include the study of the accelerating properties of cavities with other axial periods, frequencies, βs, and cross sections. For example, we have begun to study a $\beta = 1.0$ cavity for the acceleration of electrons.

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SINGLE PARTICLE ACCELERATION

In scoping out the accelerating cavity, our first study was to determine the acceleration of on-axis particles with varying cavity length, beam β, and phase. For the calculations, the numerical field solutions were scaled such that the maximum electric field was equal to 24 MV/m. We varied the cavity lengths by taking different integer numbers of field periods. In all the calculations, end field effects were ignored.

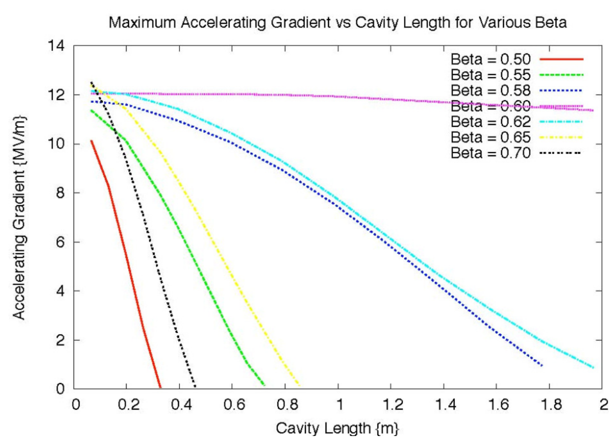

Figure 1. Maximum accelerating gradient versus cavity length for particles of different β.

Figure 1 shows the maximum accelerating gradient seen by an on-axis particle versus cavity length for different particle velocities, β. Except for the cavity $\beta = 0.6$, the accelerating gradient falls as the cavity length increases until, for a length of 1 m, even particles with β differing by only 0.02 from the cavity β see 1/3 less gradient. This is caused by the phase slippage of the particles relative to the cavity, which occurs increasingly fast as the particle β differences from the cavity β increase. In order to accelerate beams over a significant range of β values, it is necessary to restrict the cavity length to about 0.2 m, which corresponds to three complete rotations of the cross section or to six cells, with each cell being half a complete rotation of the dumbbell shape. Figure 2 illustrates the effective range of beam velocities that can be accelerated. It plots the maximum accelerating gradient for on-axis protons versus β, both for 0.2 m and 1.0 m cavity lengths. The figure shows that the cavity of length 0.2 m can deliver at least 2/3 of the maximum accelerating gradient to particles with $0.52 < \beta < 0.72$. The cavity design could be improved by lengthening the field period and reducing the frequency, so that phase slippage and the corresponding loss of accelerating gradient occurs over greater distances.

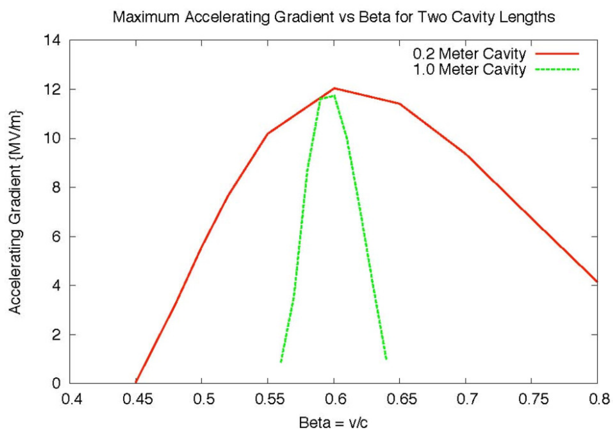

Figure 2. Maximum accelerating gradient versus β for cavities of length 0.2 m and 1.0 m.

Thus far, the discussion has concerned the maximum acceleration, and the effects of the relative phases between the cavity and the particles have only been mentioned regarding phase slippage. Figure 3 shows the energy gain in a cavity of length 0.2 m for particles of different velocities as functions of the relative phase between the particles and the cavity.

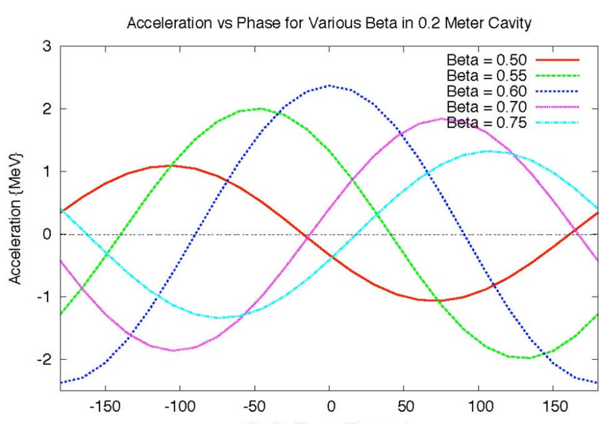

Figure 3. Energy gain versus relative phase for a cavity of length 0.2 m.

The figure illustrates a number of points. Maximum energy gain depends on β, as discussed above. Acceleration depends on phase, as any cavity can decelerate particles as effectively as it accelerates. The phase for maximum energy gain is a function of the particle velocity. In addition, longitudinal beam dynamics dictates that we generally don't operate at the phase for maximum acceleration, but rather at some phase offset in order to provide effective focusing. We now consider the properties of the 0.2 m six cell cavity in accelerating bunches of particles.

ACCELERATING BUNCHES

In order to further study the accelerating properties of the 0.2 m, $\beta = 0.6$, twisted cavity, we accelerate a 6D Gaussian bunch for various values of reference particle velocity and phase. We define the bunch coordinates with respect to the on-line reference particle. The values of the bunch parameters are $\sigma_x = 1.4$ mm, $\sigma_y = 1.4$ mm, $\sigma_{x'} = 0.14$ mr, $\sigma_{y'} = 0.14$ mr, $\sigma_\phi = 9°$, and $\sigma_E = 0.23$ MeV. In all cases, the transport of the reference particle is described by the results above for single particle acceleration, and we concentrate on the bunch transport with respect to that of the reference particle.

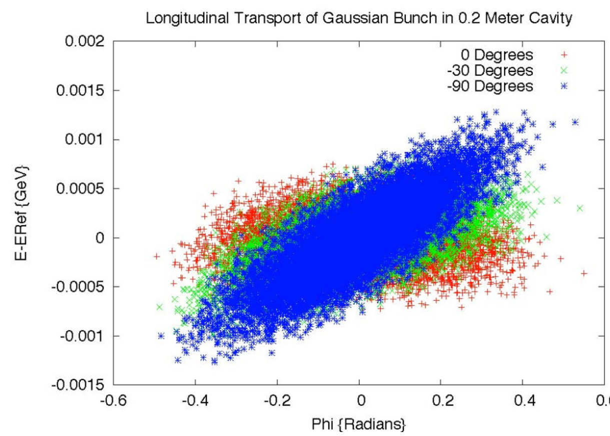

Figure 4. Longitudinal phase space distributions of 6D Gaussian bunch for different phases.

Figure 4 shows the longitudinal phase space distributions of the Gaussian bunch at velocity $\beta = 0.6$ after transport through the cavity at phases of 0°, -30°, and -90°. As the dark blue curve in Fig. 3 shows, the 0° case is maximally accelerated, but Fig. 4 shows that there is no longitudinal focusing. The case of -90° is maximally focused, as shown by the bunch rotation in Fig. 4, but Fig.3 shows that there is no net acceleration. The intermediate case of phase -30° is both accelerated and bunched. In this case the reference energy gain is 2.04 MeV, compared with 2.37 MeV for the 0° case, and the bunch rotation is midway between those of the 0° and -90° cases. It is somewhere near this last case that the cavity would likely operate.

In the transverse directions, the coordinates, x and y, of the bunch particles change only slightly in passing through the cavity. The transverse momenta, x′ and y′, undergo more significant changes, and these changes demonstrate coupling between the x and y phase planes. Figure 5 illustrates this coupling by plotting the vertical transverse momentum y′ as a function of the horizontal coordinate x for the Gaussian bunch at $\beta = 0.6$ and phases of 0°, -60°, and -90°. Even though the strength of the coupling varies with phase, Fig. 5 shows that it is likely to be significant at the operating phase of the cavity.

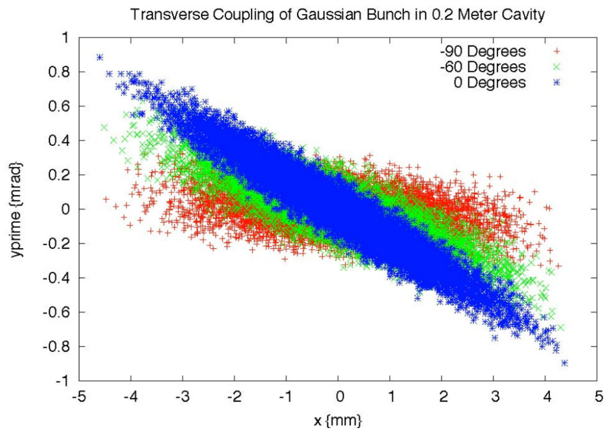

Figure 5. Coupling between x and y′ for $\beta = 0.6$ and three different phases.

Examination of the fields experienced by the particles in the cavities shows that the coupling is due to transverse electric fields. The transverse magnetic fields seen by the particles tend to oscillate in sign and thus cancel over the length of the cavity. Figure 6 shows, however, that the transverse electric fields seen by a particle at x = 4 mm are significant and of one sign. The horizontal field, E_x, is defocusing, and this can be corrected with quadrupole magnets. However, the vertical field, E_y, introduces the coupling, and correction would require the introduction of skew quadrupoles. While the multiparticle tracking results shown here are all for beams having $\beta = 0.6$, they apply equally to the other velocities studied ($0.52 < \beta < 0.72$), except that the phases must be taken relative to the phase of maximum acceleration for the particular reference velocity.

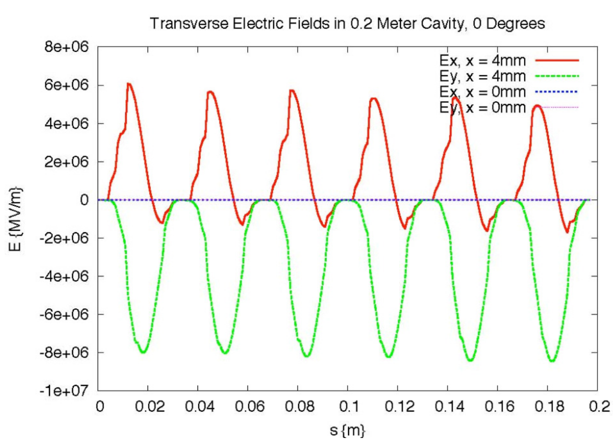

Figure 6. Vertical and horizontal electric fields seen by particle at $\beta = 0.6$ and 0° phase for x = 0 mm and for x = 4 mm.

CONCLUSIONS

Twisted cavities are potentially attractive structures for particle acceleration. Computational study is progressing further to investigate their electromagnetic properties and to optimize the structure. Preliminary RF measurements have been made on a few bench models, and sample prototype structures are being fabricated. This paper presents initial particle tracking results using numerically calculated time-dependent 3D electromagnetic fields. Specifically, protons are tracked through a $\beta = 0.6$ cavity with dumbbell-shaped cross section. For a peak electric field of 24 MV/m, the maximum accelerating gradient was found to be about 12 MV/m, and the maximum cavity length for effective acceleration of a substantial range of velocities was found to be about six field periods, or 0.2 m. When acceleration of bunches was considered, the most significant finding was coupling between the transverse planes brought about by E_y contributions to particles displaced in x, and vice versa. All of these results will be taken into account in the program to design and construct useful twisted accelerating cavities.

In addition to providing guidance to cavity design, future work on tracking in twisted cavities will involve additional physics considerations, such as space charge and end-field effects. Lattice design studies using twisted cavities will also be conducted, and the results will be compared with similar studies for existing cavities.

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