# ADVANCED SIMULATION AND OPTIMIZATION TOOLS FOR DYNAMIC APERTURE OF NON-SCALING FFAGS AND RELATED ACCELERATORS INCLUDING MODERN USER INTERFACES\*

C. Johnstone<sup>†</sup> and V. Kashikhin, Fermilab, Batavia, IL 60510, USA M. Berz and K. Makino, MSU and Particle Accelerator Corp., Batavia, IL 60510, USA P. Snopok, U. of CA, Riverside, CA and Particle Accelerator Corp., Batavia, IL 60510, USA

Abstract

The drive for higher beam power, high duty cycle, high reliability, and precisely controlled beams at reasonable cost has generated world-wide interest in Fixed-Field Alternating Gradient accelerators (FFAGs) [1]. FFAGs have the potential to combine both the energy variability of the synchrotron with the high duty cycle of the cyclotron. A new concept in non-scaling FFAGs has been invented in which the machine tune is stable over an extended acceleration cycle, a factor of a 3-6, or more, in momentum. To develop the new fixed-field accelerators, sophisticated simulation tools were required and implemented within the advanced accelerator simulation code, COSY INFINITY [2], in order to accurately describe the FFAG's complex electromagnetic fields including realistic edge-field effects and high-order dynamics. This new generation of accelerator design tools was essential in optimizing and verifying machine performance of not only this new FFAG concept, but FFAGs in general. The results of the new non-scaling FFAG simulation with COSY are presented here.

#### INTRODUCTION

The drive for higher beam power, high duty cycle, reliability, and precisely controlled beams at reasonable cost has generated world-wide interest in Fixed-Field Alternating Gradient accelerators (FFAGs) [1]. FFAGs have the potential to combine both the energy variability of the synchrotron with the high duty cycle of the cyclotron. A new concept in non-scaling FFAGs has been invented in which the machine tune is stable over an extended acceleration cycle, a factor of a 3-6, or more, in momentum (over an order of magnitude has been achieved). Fermilab Research Association (FRA) has elected to patent this concept, and a strong collaborative design effort to optimize, simulate, and demonstrate the technical feasibility of this accelerator approach is underway to be followed by a commercial engineering design.

To fully and accurately describe the FFAG's complex electromagnetic fields, new tools were developed for the study and analysis of FFAG dynamics based on transfer map techniques unique to the code COSY INFINITY including realistic edge-field effects and high-order dynam-

ics. With these new tools, closed orbits, transverse amplitude dependencies, and dynamic aperture are determined inclusive of full nonlinear fields and kinematics to arbitrary order. Various methods of describing complex fields and components are now supported including representation in radius-dependent Fourier modes, complex magnet edge contours, as well as the capability to interject calculated or measured field data from a magnet design code or actual components, respectively, thereby smoothly and efficiently integrating component design with simulation. These new advanced tools fill a critical need in advanced accelerator design. Conventional accelerator codes provide too little flexibility in field description and/or are limited to low order in the dynamics; as such they cannot adequately formulate and predict FFAG accelerators, especially in the presence of the strong nonlinearities from edge contours and fields along with other high-order effects. Outside of COSY, present public codes include only the cyclotron code CYCLOPS [3] and the field-map code ZGOUBI [4]. The former, which utilizes fields and their geometry expanded in polar coordinates, has limited accuracy in this application primarily due to lack of out-of-plane expansion order and in handling of edge-field effects; this has been particularly true for the case of rapid azimuthal field fall-off at magnet edges (as in the FFAG field profile of Fig. 1), an effect not present in cyclotrons (although there have been recent efforts to improve the edge description accuracy). The latter code, ZGOUBI, is presently being used successfully in FFAG development but requires dedicated effort and expertise to implement an FFAG design.

These new tools, combined with powerful internal optimizers, were critical in achieving the required (and design) performance of this new generation of accelerators: successfully demonstrating both the promised tune stability and a sustainable slow acceleration rate by a modest acceleration system. A specific example, simulation results for a compact, non-scaling, 8-cell design that achieves a 250-MeV extraction energy for protons, is presented here.

## FIELD DESCRIPTIONS

The newly developed tools for the study and analysis of the dynamics in FFAG accelerators are based on differential algebra and transfer map methods unique to the code COSY INFINITY. With these new tools, closed orbits, transverse amplitude dependencies and dynamic aper-

<sup>\*</sup>Work supported by the United States Department of Energy under Contract No. DE-AC02-07-CH11359.

<sup>†</sup> cjj@fnal.gov

ture are determined inclusive of full nonlinear fields and kinematics to arbitrary order. The dynamics are studied at discrete energies via a high-order energy-dependent transfer map.

The order-dependent convergence in the calculated maps allows precise determination of dynamic aperture and detailed particle dynamics. Using normal form methods and minimal impact symplectic tracking, amplitude- and energy-dependent tune shifts and resonance strengths are extracted. Optimization by constrained global optimization methods further refine and promote robust machine attributes.

Various methods of describing the fields will be presented, including representation of fields in radius-dependent Fourier modes, which include complex magnet edge contours and superimposed fringe fields, as well as the capability to interject calculated or measured field data from a magnet design code or actual components, respectively.

## General Combined-Function Magnets in COSY

In a number of specific cases it is possible to describe an FFAG lattice in terms of standard beamline elements. For these machines an extended set of combined function magnets have been implemented both with tilted and curved entrance edges and a choice of conventional fringe field models. Alternatively, actual data, either measured or from a magnet design code, can be entered for a calculation using exact fringe fields. The description of these elements allow a rather sophisticated level of design as long as the fields of individual elements do not strongly overlap (implying interaction between components and unnatural fringe fields) and as long as the field profiles are not unusual or extreme functions of radius or azimuthal angle.

The disadvantage of this approach is that it is based on a description relative to a reference orbit and therefore relies on the deflection properties about this orbit. Studying multiple reference energies, as must be done for an FFAG, makes for a rapidly expanding problem dimensionally and one that quickly becomes difficult to analyze. To counter this expansion, in the next sections, field descriptions were instead provided in terms of a laboratory-based coordinate system that applies to all possible reference orbits and reduces the descriptive magnitude of the problem.

#### Generalized FFAG Magnets

An enhancement of the above approach that provides greater flexibility and control, entails the superimposition of combined-function (CF) magnets. A "universal" FFAG magnet can be effectively described in terms of superimposed CF magnets with arbitrary, high-order (individual multipole) fields. Each overlay retains the required complex edge curves and associated high-order dynamics. This approach is new in that the effective centers of the constituent multipoles do not have to coincide physically in the CF magnet. This approach produces a truly arbitrary field

profile which is difficult or impossible to reproduce in other codes. Further, since FFAGs have completely periodic lattices, it is sufficient to define a "half-cell" for a repetitive simulation; that is, the structure is defined from one reflective symmetry point to another. Geometric closure of an orbit requires that all orbits, even off-reference ones, must be parallel at such points, or "reflection" does not hold; i.e. all derivatives must be zero for stable orbits. Hence it is sufficient to construct only a half cell map from which the full cell map is automatically generated. The full FFAG can be constructed from 2n sector-shaped half cells, each of which has a sector angle of  $\pi/n$ . As a consequence of this symmetry the lines normal to the orbits at these symmetry points form a radial line to the geometric center of the FFAG. Within each full cell, there are either two or three magnets, depending on the FFAG base unit cell, a FODO, doublet, or triplet structure, and each magnet has a radial field profile  $B_{y,i}$  given by  $B_{y,i} = B_{0,i} \cdot P_{B,i}(r)$ , where  $B_{0,i}$  is a reference field value and  $P_{B,i}$ , is a dimensionless polynomial in the polar radius r.

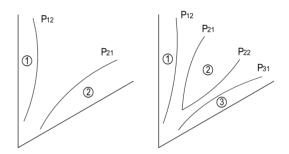


Figure 1: The curves describing the effective field boundaries of the FFAG magnets in a cell of angle  $\pi/n$  in the two-bend (left) and three-bend (right) case, with n representing the number of machine sectors.

The magnetic fields are bounded by curves as shown in Fig. 1. All curves in the (x,z) midplane are represented in terms of a parameter t in the form  $(x,z)_{ij}(t)=\vec{P}_{ij}(t)$ , where  $t\in[0,1]$  and the origin of the coordinate system is at the center of the accelerator. At the edge, the fringe field of the magnet bounded by that curve is described by an Enge Function

$$B_y(x,z) = \bar{B} \cdot \frac{1}{1 + \exp(P_i(d/D_i(r)))}$$

where  $\bar{B}$  is the main field acting at the point closest to (x,z) on the effective field boundary and  $P_i$  is a polynomial. Note that  $\bar{B}$  is the sum of the contributions from the individual multipoles. The quantity d is the distance of the point (x,z) to the effective field boundary, and  $D_i$  is the aperture of magnet i, which is allowed to vary with radius in polynomial form. Enge functions provide significant flexibility for the description of most types of realistic field fall-offs by appropriately choosing the coefficients of the polynomial in the exponential function. Thus, fringe field profiles based on other explicit functions are usually not

**Beam Dynamics and Electromagnetic Fields** 

necessary, at least in early stages of design. Fringe fields of neighboring curves are allowed to overlap. Overall, the fringe field description is very similar to the approach that has been followed very successfully in the study of high-resolution particle spectrographs.

The advantage of this particular field model is that it allows a relatively simple adjustment of the main parameters commonly studied in the design of FFAG magnets while providing a fully Maxwellian and realistic field description. Specifically, the radial field variation appearing in  $P_{B,i}(r)$  affects both the horizontal and vertical focusing for quadrupole and higher fields and only the horizontal focusing for dipole, and the edge curves  $\vec{P}_{ij}(t)$  affect both horizontal and vertical focusing. The Enge fall off is well known to represent realistic field profiles which can be adjusted to accurately describe most magnets and can even approximate a rapid, hard edge fall-off which is useful for comparison with codes without fringing fields.

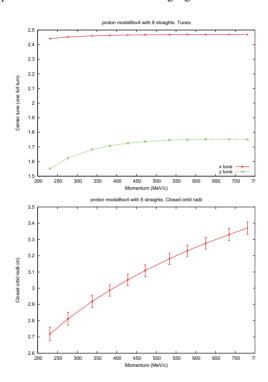


Figure 2: Machine tunes (red-horizontal, green-vertical) and radial orbit dependence for 250 MeV non-scaling proton FFAG.

## 30-250 MEV PROTON THERAPY FFAG

A working design has been completed based on an 8-cell, compact FDF triplet structure with a single, long insertion in each cell; i.e. 8 long insertions total. The energy reach is 30–250 MeV, limited by technical complications associated with the large horizontal aperture. The specific approach developed here has achieved ultracompactness without compromising practical and inexpensive magnetic field components, the attractive feature of the

Table 1: Parameters of the triplet 30–250 MeV non-scaling FFAG design as optimized and modeled in COSY.

Parameter	Unit	Injection	Extraction
Energy range	MeV	30	250
Cell $\nu_x/\nu_y$	$2\pi$ rad	0.31/0.16	0.31/0.21
Ring $\nu_x/\nu_y$	$2\pi$ rad	2.48/1.28	2.48/1.66
Average Radius	m	2.75	3.39
No. cells		8	
Long Straight	m	1.17	1.17
Peak Field F/D	T	1.21/-1.37	3.13/-3.41
Magnet Size F/D	m	0.65/0.13	0.803/0.176
Apertures F/D	m	0.63/0.55	

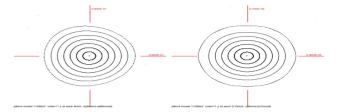


Figure 3: Dynamic aperture at midpoint, 112 MeV, in 0.8 cm (hor., left) and 0.75 mm (vert., right) steps. DA for both planes are large, particularly in the horizontal.

non-scaling FFAG. This working machine lattice utilized advanced optimization features in COSY to stabilize tune parameters further from the starting design lattice. This first stable working lattice, described by the parameters of Table 1, clearly demonstrates feasible technical and dynamical properties—as documented by the presentation of COSY simulation results in Figs. 2–3, using the new FFAG tools. Phase space portraits from full simulation tracking in COSY at an intermediate energy are shown in Fig. 3.

#### REFERENCES

- [1] C. Prior (Ed.), ICFA Beam Dynamics Newsletter No. 43, Aug 2007.
  - M. K. Craddock, New Concepts in FFAG Design for Secondary Beam Facilities and Other Applications, *Proc. of PAC'05*, Knoxville, TN, USA, May 2005, p. 261.
  - S. Machida, FFAG designs: http://hadron.kek.jp/ ~machida/mirror/misl/publications20090415.pdf. RACCAM project: http://lpsc.in2p3.fr/service\_ accelerateurs/raccam.htm.
  - E. Keil: http://keil.home.cern.ch/keil/keil.bib. CONFORM project, U.K. (EMMA and PAMELA). See also Proceedings of International Workshops on FFAG Accelerators, FFAG00-FFAG08.
- [2] M. Berz and K. Makino. COSY INFINITY Version 9.0 beam physics manual. Tech. Rep. MSUHEP-060804, Dept. of Phys. and Ast., Mich. State U., E. Lansing, MI 48824, 2006.
- [3] R. Baartman et al. CYCLOPS Tech. Rep.
- [4] F. Meot. The ray-tracing code ZGOUBI. Nucl. Inst. and Meth. A, 427:353-356, 1999, and F. Lemuet and F. Meot. Developments in the ray-tracing code ZGOUBI for 6-d multiturn tracking in FFAG rings, 2005.