

# MEDIUM FIELD Q-SLOPE STUDIES IN QUARTER WAVE CAVITIES

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## Abstract

The quality factor of superconducting radio-frequency niobium cavities decreases with the applied RF field in the medium field range. Medium field Q slope has been investigated by many authors mainly as a thermal feedback effect, but models do not fully explain experimental evidence. In this contribution we analyze medium field Q slope data measured on ISAC II low beta quarter wave cavities operating at 4.2K. The data is compared with a model that combines thermal feedback of the surface temperature and reduction of the critical temperature with the applied magnetic field. In an experimental investigation two surface heaters are added on the LHe side of the cavity in the high magnetic field region. A model to explain the experimental results is then presented and compared to the data.

## INTRODUCTION

The quality factor of a cavity,  $Q_o = \omega U / P = G / R_s$ , is defined as the number of RF cycles it takes to dissipate all the energy stored in the cavity, or, equivalently, as the ratio of the geometry factor  $G$  and the surface resistance  $R_s$  of the niobium wall. A typical plot of the cavity quality factor as a function of the peak magnetic field  $B_p$  shows a degradation in the range 20 – 100 mT known as ‘Medium field Q slope’. Understanding the origin of this phenomenon is important, in particular to future CW applications where cryogenic costs dictate the permissible cavity power and the Q-value determines the gradient. Previous studies on medium field Q slope conducted by Halbritter [1], Ciovati [2], Vines [3] mainly discuss a ‘global thermal instability’. Here the heat created at the interior surface of the cavity, if not efficiently conducted to the low temperature bath, can increase the temperature of the rf surface. The surface resistance, which depends exponentially on the temperature, will rise increasing the power deposition leading to a further increase of the surface temperature. This effect strongly depends on the properties of the niobium and of the interface to the He bath, usually modelled through the RF surface resistance  $R_s$ , thermal conductivity  $k$  and kapitza conductance  $H_k$ . Medium field Q slope is usually represented by a dimensionless parameter  $\gamma$  introduced by Halbritter [1], defined via an expansion of the surface resistance  $R_s$  in even powers of the peak surface magnetic field  $B$ :

$$R_s(B) = R_{s0} \left[ 1 + \gamma \left( \frac{B}{B_c} \right)^2 + O(B)^4 \right] \quad (1)$$

Here  $B_c$  is the thermodynamic critical field of niobium and  $R_{s0}$  is the surface resistance at small magnetic fields. However, for our (and many other real) cavities a power series of  $R_s$  also contains odd powers of  $B$ , as we will

discuss later. While medium field Q-slope studies have mainly been conducted for cavities operating at bath temperatures below 2.18K, in this paper we explore the effect for 141 MHz quarter wave cavities, at 4.2K. First we present an analysis of our quarter wave cavities characterization curves using the Halbritter model. Then we present a simple temperature model and compare the model results to the data. Experimentally we add two small resistive heaters on the cavity surface in contact with the 4.2K bath. This test aims to test the thermodynamic limiting magnetic field, correlate an enhanced thermal feedback to the resulting Q slope, and help us study the effect of the Nb-He interface in the nucleate pool boiling regime. The previously presented numerical model is adapted to this different geometry and results are compared to the experimental data.

## CHARACTERIZATION CURVES FITTING WITH HALBRITTER MODEL

From equation (1) we obtain the following expression for the decrease of the quality factor in terms of medium field Q slope:

$$Q(B) = \frac{G}{R_{s0}} \left[ 1 - \gamma \left( \frac{B}{B_c} \right)^2 + O(B)^4 \right] \quad (2)$$

Fig. 1 shows characterization curves for three different 141 MHz TRIUMF quarter wave cavities and the relative fitting curves based on (2). A linear fit of the Q curves is also shown, which gives a better fit than (2). This hints to a linear component of  $R_s(B)$  in addition to a quadratic.

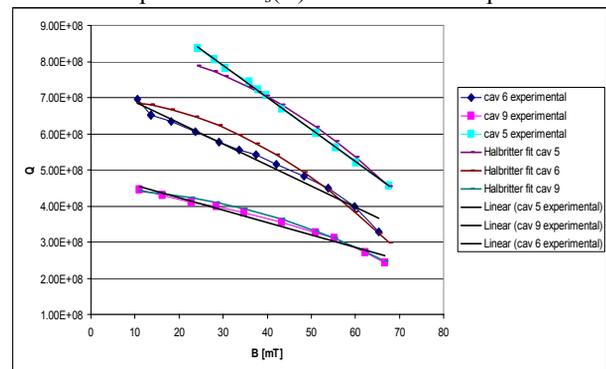


Figure 1: TRIUMF quarter wave cavities characterization curves and relative fit using Halbritter quadratic model and linear fit.

## THERMAL MODEL AND NUMERICAL SOLUTION

Consider the cavity wall as an infinite flat slab of Nb of thickness  $d$ , so that the heat transmission problem becomes one-dimensional. The heat equation is:

$$P/A = -\kappa(T) \frac{dT}{dz} \quad (3)$$

where at the Rf surface:

$$P/A = \frac{1}{2} R_s(T) H^2 \quad (4)$$

and at the Nb-He interface we assume that  $T_d = T_{\text{bath}} + \Delta T$ . where  $\Delta T$  involves the interface conductance (Kapitza or otherwise). For simplicity numerical solutions are computed for different  $\Delta T$ , constant or dependent on the RF field. The niobium slab is divided into a series of small layers producing a set of finite-difference equations. An important step is the choice of the surface resistance to use in equation (4).

### Surface Resistance and Critical Temperature

A review of models of the RF surface resistance in high gradient Nb cavities can be found in [4]. There are several field dependent contributions to  $R_s$ , but we will consider the main components:

$$R_s(T) = R_o + R_{BCS}(T) \quad (5)$$

where  $R_o$  is the residual resistance due to impurities.  $R_{BCS}$  is due to the motion of normal electrons near the RF surface and can be calculated from the BCS theory of superconductivity, but has a rather complicated form. We will use a Pippard approximation:

$$R_{BCS} = 2.78 \times 10^{-5} \frac{v^2}{t} \ln\left(\frac{148t}{v}\right) \exp\left[-\frac{1.81g(t)}{t}\right] \quad (6)$$

where:

$$t = \frac{T}{T_c}, v = \frac{f}{2.86 \text{GHz}}, g(t) = \left[ \cos\left(\frac{\pi^2}{2}\right) \right]^{1/2} \quad (7)$$

Now, another key step is including the field dependence of the critical temperature  $T_c$ :

$$T_c(H) = 9.2 \sqrt{1 - \left(\frac{H}{H_c}\right)^2} \quad (8)$$

which takes into account, from the thermodynamic model, that the critical temperature is lowered in the presence of a RF field. Substituting (8) in (7) leads to a field dependence of the BCS resistance from the applied RF field.

Results of the model are compared with the TRIUMF quarter wave cavity 9 characterization curve. The residual resistance  $R_o$  is set to achieve  $Q_0$  computed using the Pippard formula at low field.  $T_d$  is varied from  $T_{\text{bath}}$  to  $T_{\text{bath}} + \Delta T$ , to take into account a temperature drop across the metal-He interface. Simulations are run for  $\Delta T = \text{constant}$  but also for  $\Delta T$  varying with the field and the best fit comes assuming a quadratic dependence on the applied RF field. Results are shown in Fig.3. The results are not surprising, and mean that the temperature difference at the interface between Helium and metal increases with the heat flux across the niobium surface. As it can be seen from Fig. 2, the model seems to fit the experimental curves better than Halbritter's model at fields below 6MV/m. These results are consistently found in other TRIUMF quarter wave cavities.

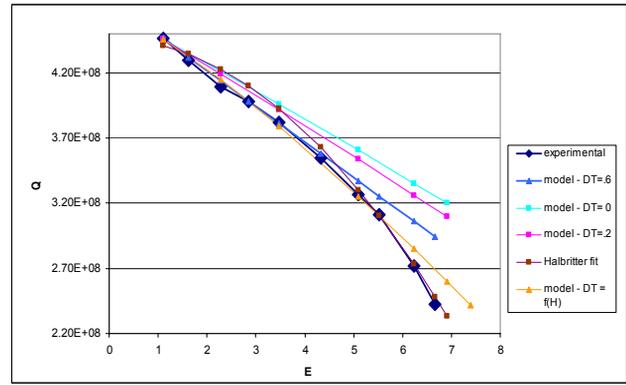


Figure 2: TRIUMF 141 MHz quarter wave cavity 9 characterization curve, compared with Halbritter's model and our thermal feedback model, which gives a better fit for fields below 6MV/m.

### HEATERS TEST

To further investigate thermal dependence of the medium field Q-slope, we place two resistive heaters on the cavity surface in contact with the He bath. Both heaters are placed in high magnetic field regions, one on the top of the inner conductor and one on the surface of the top flange. They are both Polymide resistive heaters, of values 75 and 275  $\Omega$  and radii 20 and 30 mm respectively. The Nb thicknesses are 2 mm in the inner conductor case and 12 mm on the top flange. Initial checks are done by powering the resistors and monitoring He flow through a gas meter. These calibration curves are used to estimate the percentage of power going to the bath versus the one going actually into the Nb. The response indicates  $\sim 20\%$  of the total heater power goes to the niobium. A low field ( $H_p = 20 \text{mT}$ ) is established in the cavity and the heater power is increased. For the 75 $\Omega$  heater on the inner conductor the quench point came at about 25 W. For the 275  $\Omega$  heater the quench point was found at about 110W (overall heater power plus RF). Several characterization curves at different heater power levels are taken.

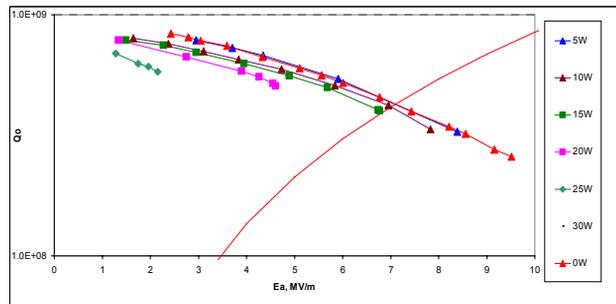


Figure 3: Q curves at different heater power levels. The heater is placed on the inner conductor surface in contact with the He bath. Cavity wall is 2 mm thick.

The results are shown in figure 3 and 4, respectively for the heater on the inner conductor and the heater on the top wall. For the inner conductor heater, an increase in the heater power reduces the quench field and as the heater

value increases to 20 W the Q-slope is increased. For the top heater results the curves seem to be 'divided' into two bunches, 0-20W and 40-60W. The 40-60W has a lower  $Q_0$  - not unexpected since the RF surface will be warmer at a higher heater power and therefore the surface resistance will be higher

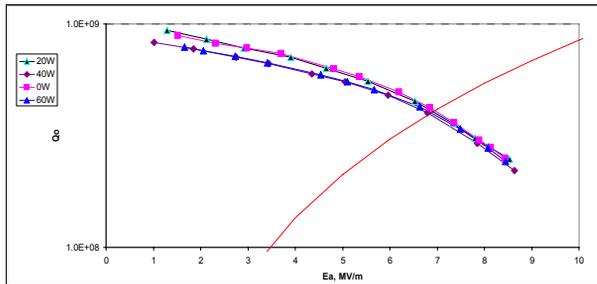


Figure 4: Q curves at different heater power levels. The heater is placed on the cavity top surface in contact with the He bath. Cavity wall is 12 mm.

### Model and Comparison with the Data

We treat the problem considering a niobium disk of height equal to the thickness of the wall and radius given by that of the heater, as shown in Fig. 5. We assume that the edge of the disk is fixed at the bath temperature. The heat flows radially outward through the lateral surface of the disk.

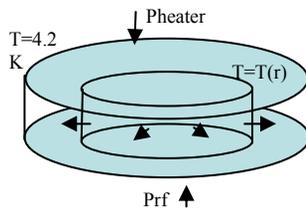


Figure 5: Schematic of the model. Heat coming from the Rf power and from the heater power flows across the lateral surface of the niobium disk. The edge of the disk is assumed at 4.2K.

We then divide the heater-sized disk into  $n$  disks and solve for the equilibrium temperature  $T(r)$ . The heat flow transport is brought into a finite difference equation, with two sources of heat: the heater power entering the metal, which is uniformly distributed across the disk and RF power, which is not uniform since disks with smaller radii will be at a higher temperature, therefore higher  $R_s$ . The temperature at every disk is then computed as:

$$T(r_i) = T(r_{i-1}) + \frac{P_{enclosed\ i}}{2\pi r_i \cdot d \cdot k(r_i)} \Delta r \quad (9)$$

$$P_{enclosed\ i} = \sum_1^i P_{RFi} + \alpha P_{heater} \cdot \frac{r_i^2}{a^2}$$

where  $d$  is the thickness of the wall,  $\Delta r$  is the radial extent of a ring,  $k$  is the thermal conductance of Nb given by  $\sim 0.075 \text{ W/mm/K}$ ,  $\alpha$  is the fraction of heater power entering the Nb and  $a$  is the radius of the heater. The quench condition corresponds to the field where the central temperature of the disk reaches  $T_c$ . Fig. 6 shows a plot of the heater power versus the quench field,

comparing the experimental data with that of the model. The percentage of heater power entering the metal is used as a fitting parameter and is chosen as 25% in close agreement with the transient analysis using the helium boil-off in the initial calibration. Several curves are shown. A result considering only rf thermal feedback does not agree. Two other curves give reasonable agreement, one considering only the thermodynamic reduction of  $T_c$  with the applied field (no rf power included) and another the thermodynamic model plus rf power in thermal feedback. The results indicate that medium field Q slope is dictated, more than by a thermal feedback, by the reduction of the critical temperature at increasing RF fields.

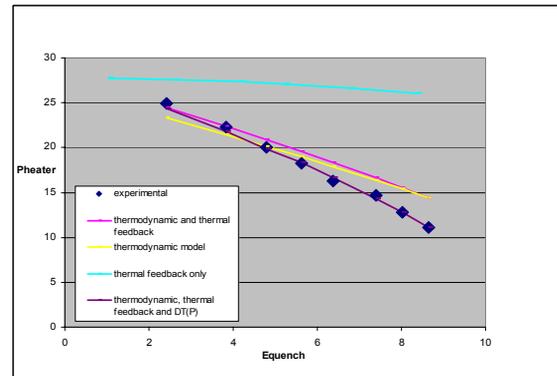


Figure 6: Comparison of experimental quench point at different heater power and the ones predicted by the model.

## CONCLUSIONS

The medium field Q-slope has been investigated for ISAC-II 141MHz quarter wave cavities. Characterization curves show both a linear and quadratic dependence on the peak magnetic field in the medium field range. Results of a heater test and comparison with a simplified heat transfer model seem to suggest a strong correlation between medium field Q slope and reduction of Nb critical  $T_c$  with the applied RF field.

## REFERENCES

- [1] J. Halbritter. RF residual losses, high electric and magnetic fields in superconducting cavities. Superconducting materials for high energy colliders, p.9, Eds Cifarelli and L. Maritato. World scientific, 1999
- [2] G. Ciovati. Proceedings of the 12th International Workshop on RF Superconductivity, p230, TuP02, 2005.
- [3] J. Vines. Systematic Trends for the Medium Field Q-Slope, Proceedings of the 13th International Workshop on RF Superconductivity, TuP27, 2007
- [4] P. Bauer. Review of Models of RF Surface Resistance in High Gradient Niobium Cavities for particle Accelerators