LINEAR-QUADRATIC-GAUSSIAN CONTROLLERS FOR SINGLE-FREQUENCY RF SYSTEMS AND SHORT BUNCHES IN NSLS-II*

N. Towne†, 1094 White Oak Lane, Farmington, NY 14425; J. Rose and H. Ma, NSLS-II Project, Brookhaven National Laboratory, Upton, NY 11973

Abstract

NSLS-II is a new ultra-bright 3-GeV 3rd-generation synchrotron-radiation light source. The performance goals require operation with a beam current of 500mA and a bunch current of at least 0.5mA with tight photon-beam position and timing specifications, which constrain rf system stability (0.15 degrees bunch jitter). This study develops computational methods for the construction of LQG controllers for discrete-time models of single-cavity rf systems coupled to short-bunch beams able to meet this tolerance. It uses Matlab's control-systems toolbox (CST) and Simulink to: 1) synthesize the LQG controller; 2) establish resolutions of state variables, ADCs, DACs, and matrix coefficients; 3) simulate closed-loop performance with floating- and fixed-point controllers controlling linear, non-linear, and Vlasov-simulation-derived linear models; and assess sensitivity to variations of the model. This machinery is applied to NSLS-II and other rings showing exceptional noise suppression and bandwidth. Thoughts are given on the validation and tuning of the rf-system model by machine measurements, and on DSP implementations.

INTRODUCTION

As with any late-generation synchrotron light source, NSLS-II users require very stable beams. The rf system [1] in particular has tight noise tolerances [2], which will be difficult to meet. The most immediate problem is noise from the high-power rf amplifiers. Although phase feedback around the amplifiers is a way to suppress phase noise, which is particularly prevalent in klystrons, before it becomes more deeply entrenched in the rest of the rf system, amplitude noise would remain. Amplifier saturation is an issue that complicates any solution to this problem [3].

This study explores the use of state-space models and linear-quadratic-Gaussian (LQG) controllers to suppress amplifier noise. Although Kalman estimators and LQG regulators [4] are widely used in other areas, I am not aware of their successful use in rf systems. Boussard and Onillon have [5], however, numerically considered their use for minimizing transients at injection.

NSLS-II is to use top-off operation, i.e., injection at full energy, which has the advantage that the operating point of the rf system is nearly constant. This greatly simplifies the problem of engineering a controller that requires a detailed model of the underlying dynamics of the system being controlled (the plant). LQG controllers minimize in a least-squares sense the deviation of critical internals of the plant plus a measure of the cost of controlling the plant via its inputs, while being driven by the noise sources. It is this formalism applied to the problem of amplifier noise in an rf system controlling a short-bunch beam that is the subject of this paper [6]. The question whether an LQG regulator synthesized from a rigid-bunch model of an rf system is actually going to work is not answered definitively here through tests in a real machine, or even with simulations using real response data. But these controllers were successfully tested numerically with linear and non-linear Simulink [7] models, and with independent linear models incorporating mode coupling derived from Vlasov simulations [8].

The rest of this paper describes the rf-system model in Simulink, its linearization into a Matlab LTI object, the steps involved in the construction of a floating-point LQG controller, construction of a fixed-point controller, model tuning and validation by machine measurements and Vlasov response functions, some elements of upper- and middle-level architecture of the fixed-point controller, the application of these ideas to NSLS-II and other light sources, and the impact of klystron saturation [3].

RF SYSTEM MODEL

Timing experiments demand the most stringent tolerances of beam phase and energy noise [2]. These experiments also require short bunches, which, having minimal mode coupling, are the simplest to model. These non-linear discrete-time models are constructed in Simulink using blocks that model physical parts of the rf system, e.g., rf mode(s), bunch(es), a klystron, etc. [9]. Using Matlab CST functions, these models are then linearized and converted to state-space linear-time-independent (LTI) Matlab objects. The most basic rf-system model requires six state variables to represent its dynamics.

Remember that the discrete-time equations implemented by a state-space model are

$$\begin{aligned} x_{n+1} &= A \cdot x_n + B \cdot u_n + G \cdot w_n \\ y_n &= C \cdot x_n + D \cdot u_n + H \cdot w_n \end{aligned} \tag{1}$$

where x, y, u, and w represent vectors of state variables, outputs, control inputs, and noise inputs, respectively. The matrices for the rf-system model are computed during linearization from the Simulink model. In this paper, the matrix A is termed the kernel.

It is important during construction of the model to incorporate delays as they are present in the machine being modeled, in the model. Without these delays, poor fits to

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machine or Vlasov response data inevitably occur. One must further include the controller processing delay in the model using the 'inputdelay' property of the LTI object.

The amplifier noise model is part of the rf-system model. It informs the model of the magnitude and frequency dependence of the amplifier noise. Amplifier delay is added to the model as a one-or-more-time-step delay within the amplifier model. All delays contribute to the model's state-variable count.

RF feedback may be part of the plant model as needed. In machines with super-conducting cavities, a small amount (open-loop gain ~ 1) is necessary for control of the reactive Robinson instability. But this feedback is better included as a function within the controller so that the controller's independent I and Q gains can properly manage amplifier gain compression [3]. This approach is feasible because this feedback requires little bandwidth.

Inputs to the model are the I and Q rf inputs. Outputs are beam energy and phase, rf cavity I and Q components, and transmission-line forward and reflected wave I and Q components. So the plant model has two inputs, eight outputs, and 16 linear response functions. Not all outputs are used for feedback.

FLOATING-POINT CONTROLLER

The steady-state Kalman estimator and LQG controller are constructed using functions of the CST in a straightforward manner. An input-noise covariance matrix is specified from the amplifier noise model, and the quadratic cost function $x^* \cdot Q \cdot x$ defined by Q comes from machine specifications [2].

An LQG controller constructed from a given subset of plant outputs is not necessarily stable. The use of an unstable controller may complicate development and operation of the system even though it may provide the best noise suppression. For this reason, of the eight plant outputs, the particular subset used for feedback is chosen for stability of the controller. Later experience with a working system may make this choice unnecessary.

The controller state variables $\hat{x}$ may be transformed by $\hat{x} \to \hat{x}' = S \cdot \hat{x}$, where S is a non-singular matrix, without changing the external behavior of the controller, implying unique transformation rules for the matrices A, B, C, and D. A transformation of this form is here termed a kernel transformation. We introduce a particular kernel transformation as a means to simplify the matrix computation $A \cdot \hat{x}$ anticipating the synthesis of fixed-point controllers in the next section. In principle, the matrix can be diagonalized, with a complex-valued diagonal. But because a complex-valued A is not directly useful in a digital controller, A is instead converted to block diagonal form with at most 2-by-2 blocks with real coefficients, this done conveniently with the Matlab's 'cdf2rdf' function. This block-diagonal form, with correspondingly transformed B, C, and D, is what is later instantiated in logic.

With the floating-point controller designed, two (Simulink) copies of the controller, one controlling the nonlinear Simulink model of the rf system and the second controller controlling the linearized model, are simulated side-by-side in another Simulink model. The closed-loop beam-noise performances are checked against each other and against noise calculated by Matlab's function 'covar'.

Performance of the closed-loop system shows that, given the noise model used and what we believe to be realistic delays, noise is reduced by nearly about an order of magnitude below the target and with bandwidths of a few tens of kilohertz in NSLS-II. Because machine response data are not yet available, a further test controlling LTI models built from response functions calculated through Vlasov simulations were performed – successfully.

Numerical experiments to assess sensitivity of closed-loop operation to variations of model parameters were also performed. First, the amplifier operating point along the saturation curve was varied with the result that the operating point could be moved down the curve over a wide range without instability or serious degradation of performance. When moving up the curve toward saturation, at some point the system becomes unstable, as is inevitable. Second, the system showed little sensitivity to I/Q rotational misalignment of the rf input.

FIXED-POINT CONTROLLER

The point of fixed-point controller synthesis is to design a controller that is efficiently implemented in logic and whose performance rivals that of the floating-point controller. Towards this end, one needs to determine bit widths for the state-variables x, inputs u, and outputs y; bit widths of the A, B, C, and D matrix coefficients; and at summation points, determine accumulator bit widths sufficient to eliminate quantization noise in the truncated sums.

We begin with the kernel. Since kernel transformations acting on the controller have no effect on the external behavior of the controller, we can use them to scale the controller state variables so that unit noise added to each state variable (while closed loop) is at the margin of significance. This is done in code by adding an additive input to the $A \cdot x$ summation point one controller state variable at a time, applying a unit noise to that input, then scaling that state variable so that the cost function $x^* \cdot Q \cdot x$ of the *plant* is one. (This process needs to be iterated to convergence.) Matlab's function 'covar' is used. One then quantizes the state variables at those levels (one). The largest signal to be represented by each state variable sets the number of bits needed for that variable, which is not determined by this procedure.

Simulations are used to determine the variation of noise levels with ADC, DAC, and matrix element resolutions. During these simulations, amplifier noise drives the closed-loop system and noise is read out at the output of the plant. Resolutions are increased a bit at a time until noise levels (or the cost function) converge to the floating-point controller levels. Results including sign bit for an NSLS-II model are as follows: ADC – 11 bits; DAC – 7 bits; A – 12 bits; B – 9 bits; and C – 6 bits. As a further condition, the finite resolution of the kernel must also not affect its eigenvalues significantly.

Where there are sums of multiple terms, there are further conditions on the resolutions of the accumulators due to the accumulation of quantization noise. Quantization noise from one term has a flat distribution with variance $\sigma_0^2 = 1/12$ in units of the resolution of the term. With n such terms added together, the noise quickly becomes normal with variance given by the central limit theorem: $\sigma^2 = n\sigma_0^2$. Additional bits added to the low end of the accumulator as shown in Fig. 1 reduce σ_0 accordingly. The result is that, depending on the number of terms, only two or three bits are needed to make the quantization noise of the result, in terms of effective number of bits, vanishingly small.

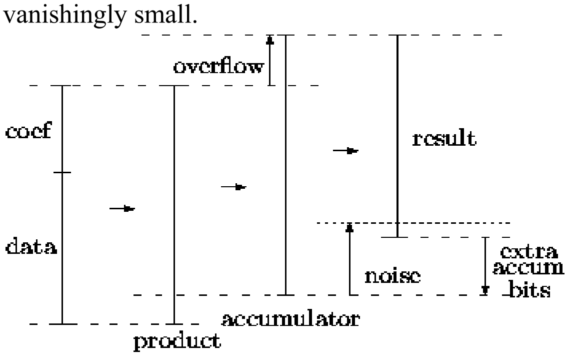

Figure 1: Bit depths along the path of a row of a matrix product. Noise refers to quantization noise.

IMPLEMENTATION IN LOGIC

Since the bit depths are modest, there is economy of the quantity of logic required to implement the LQG controller. Tests using logic synthesized by Altera Quartus II showed that these multipliers typically required 100-250 logic elements on an Excalibur demo-board target, each logic element housing a single flip-flop. Given the density of logic on modern chips, the LQG controller will easily fit on a chip (Fig. 2).

Accurate response-function measurements are an integral part of the setup and operation of LQG controllers. It is envisioned that logic for these measurements be placed on the chip and activated periodically during dedicated beam time for update of the logic. Measured response functions are required for each plant output that is used for feedback, and for the two modulation outputs to the I/Q modulator - one set of measurements for I and Q modulation types. Raw data are downloaded to a supervisory computer, processed for the machine's response functions, the measured response functions fit to the rigid-bunch model by varying two (I and Q) rf feedback parameters and a gain-compression parameter, the fixed-point controller synthesized, logic generated, and the logic uploaded to the controller board [10]. This LQG logic will coexist with other controller logic in a field-programmable gate array (FPGA).

The thought is to have the LQG controller operate in parallel with a conventional proportional-integral (PI) controller. Control is switched to the LQG controller for user data acquisition, and switched back for injection, top-off, and when any kind of fault appears.

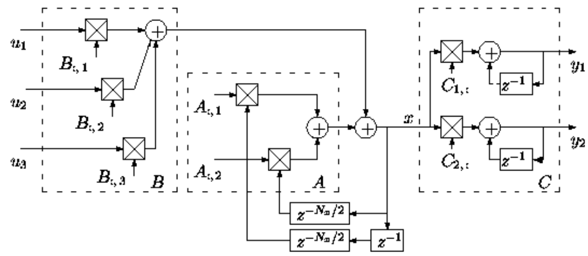

Figure 2: LQG controller numerical logic for three inputs and two outputs.

SUMMARY

Models of single-cavity rigid-bunch rf systems were developed to address the tight beam-noise requirements of timing and other experiments at NSLS-II. Numerical simulations of LQG controllers built from these models were found to realize a great deal of gain and bandwidth and show a corresponding degree of noise suppression of amplifier noise compared to simpler proportional-integral controllers. These LQG controllers do not show a prohibitive degree of sensitivity to variation of parameters of the model. These results are encouraging in that they provide evidence that robust and effective controllers for rf systems in top-off operation can be constructed. A number of tools for construction and testing of these controllers are also demonstrated. Fixed-point controllers built from these tools seem to work well and are realizable in modest FPGAs. A full response-function-measurement apparatus and logic for measurement of amplifier gain compression can coexist with controller logic and be activated as needed. Further work is needed to better characterize amplifier noise, fine-tune LTI models with real machine response data, and to fill in many details of how these controllers are to be meshed with the larger rf control system.

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