

STUDY OF DIRECT RF FEEDBACK WITH THE PEDERSEN MODEL

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Abstract

The direct RF feedback has been adopted in storage ring to reduce the beam loading effect for maximizing the stored beam current. Its performance in reducing beam loading is determined by the operational parameters, including the feedback gain, RF phase shift and the loop delay time.

This paper presents a mathematical method, based on the Pedersen model, to study the effects of the direct RF feedback on beam loading. Through an example, the influences of different operational parameters on the performance of the direct RF feedback is analyzed by examining the characteristic equation of the feedback loop. The Nyquist criterion is applied for the determination of system stability.

INTRODUCTION

Without RF feedback loops, the maximum stored beam current in the storage ring machine is limited by the beam loading effect, known as the Robinson stability limits [1]. A widely used solution to reduce the beam loading effect is the direct RF feedback, in which part of the cavity RF signal is fed back to the high power RF amplifier.

The concept of the direct RF feedback is to introduce an additional opposite beam induced voltage to the RF cavity through the feedback loop, and increase the frequency bandwidth of the RF cavity. It is equivalent to reducing the shunt impedance and quality factor of the RF cavity.

A method, based on the above explanation, was proposed to evaluate the maximum stored beam current operating with the direct RF feedback [2]. But it does not investigate the influences on system stability from the change of individual parameter in direct RF feedback operation.

In this paper, the direct RF feedback is added to the Pedersen model [3]. For focusing on the effects caused by the direct RF feedback, the model in this paper does not include other RF feedback loops like RF phase and amplitude feedback loops. Based on this model, the characteristic equation of the feedback loop is derived. Through an example, we demonstrate how to examine the system stability with the Nyquist criteria; and investigate how the delay time affects the feedback gain and the phase shift of the feedback RF signal in maximizing the stored beam current.

PEDERSEN MODEL INCLUDING DIRECT RF FEEDBACK

The model and its phasor diagram used to investigate the direct RF feedback are shown in Fig.1 and Fig.2. In the model, the cavity voltage (\vec{V}_c) is induced by the beam current (\vec{I}_B), generator current (\vec{I}_G) and the feedback RF

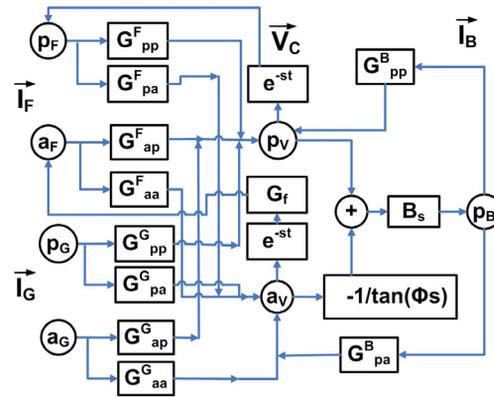


Figure 1: The Pedersen model with direct RF feedback .

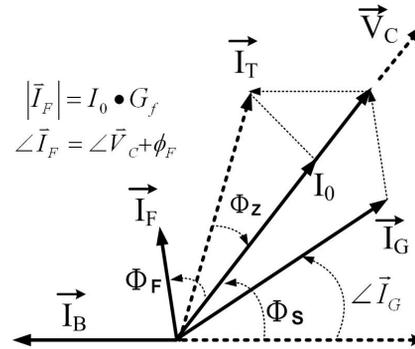


Figure 2: Phasor diagram of steady state as the cavity is operated on RF compensated condition.

current (\vec{I}_F). As the cavity is operated on the RF compensated condition, the steady state of these generating currents of the cavity voltage can be represented as in Fig.2, in which the cavity tuning angle ϕ_z is equal to

$$\tan \phi_z = \frac{I_B R_s}{V_c (1 + \beta)} \sin \phi_s \tag{1}$$

where ϕ_s is the synchrotron phase, R_s is the cavity shunt impedance, β is the cavity's RF coupling factor, I_b is the harmonic beam current which is equal to the double of the average beam current as the bunch length is negligible, compared to the RF wavelength. From Fig.2, the projection of the total generating current of the cavity voltage ($\vec{I}_T = \vec{I}_B + \vec{I}_G + \vec{I}_F$) on \vec{V}_c can be expressed as ,

$$I_0 = I_T \cos \phi_z = \frac{V_c}{R_s} \cdot (1 + \beta) \tag{2}$$

the amplitude ratio of \vec{I}_b to \vec{I}_T can be written as

$$\frac{I_B}{I_T} = \frac{I_B}{I_0} \cos \phi_z = \frac{I_B}{V_c} \frac{R_s}{1 + \beta} \cos \phi_z \tag{3}$$

If the feedback current and its phase are known, $I_F = G_F I_0$ and $\angle(\vec{I}_F) = \angle(\vec{V}_c) + \phi_F$, then the amplitude ratio of \vec{I}_G to \vec{I}_T can be obtained from Eq. (1) to (3) and the phasor diagram of Fig.2.

TRANSFER FUNCTIONS OF THE MODEL

The transfer functions between the cavity voltage and voltage generating currents can be derived from the RF cavity impedance,

$$Z(s) = \frac{2\sigma s R_s}{s^2 + 2\sigma s + \omega_R^2} \frac{1}{1 + \beta} \quad (4)$$

where ω_R the cavity resonant frequency, and σ is defined as

$$\sigma \equiv \frac{\omega_R}{2Q_L} \quad (5)$$

here Q_L is the loaded quality factor of the RF cavity. The transfer functions from the total current to the cavity voltage are given by [3]

$$G_{pp}(s) = G_{aa}(s) = \frac{1}{2} \left\{ \frac{Z(s + j\omega_{RF})}{Z(j\omega_{RF})} + \frac{Z(s - j\omega_{RF})}{Z(-j\omega_{RF})} \right\} \quad (6)$$

$$G_{pa}(s) = -G_{ap}(s) = \frac{j}{2} \left\{ \frac{Z(s + j\omega_{RF})}{Z(j\omega_{RF})} - \frac{Z(s - j\omega_{RF})}{Z(-j\omega_{RF})} \right\} \quad (7)$$

where $G_{pp}(s)$ and $G_{aa}(s)$ are phase to phase and amplitude to amplitude transfer functions, and $G_{pa}(s)$ and $G_{ap}(s)$ are phase to amplitude and amplitude to phase transfer functions. From Eq. [4] to [7] we get

$$G_{pp}(s) = \frac{\sigma^2(1 + \tan^2 \phi_z) + \sigma s}{s^2 + 2\sigma s + \sigma^2(1 + \tan^2 \phi_z)} \quad (8)$$

$$G_{pa}(s) = \frac{\sigma s \tan \phi_z}{s^2 + 2\sigma s + \sigma^2(1 + \tan^2 \phi_z)} \quad (9)$$

For the beam current component (I_B), The transfer functions to the cavity voltage are obtained from the objections of the current components on the total current I_T :

$$G_{pp}^B = G_{pp} \frac{I_B}{I_T} \cos(\angle \vec{I}_B - \angle \vec{I}_T) + G_{pa} \frac{I_B}{I_T} \sin(\angle \vec{I}_B - \angle \vec{I}_T) \quad (10)$$

$$G_{ap}^B = G_{ap} \frac{I_B}{I_T} \cos(\angle \vec{I}_B - \angle \vec{I}_T) + G_{pp} \frac{I_B}{I_T} \sin(\angle \vec{I}_B - \angle \vec{I}_T) \quad (11)$$

The similar expressions are for G_{pp}^G , G_{ap}^G , G_{pp}^F and G_{ap}^F , etc.

CRITERION FOR SYSTEM STABILITY

To determine the stability of a closed-loop system, we must investigate the characteristic equation of the system:

$$F(s) = 1 + \Delta(s) = 1 + G(s)H(s) \quad (12)$$

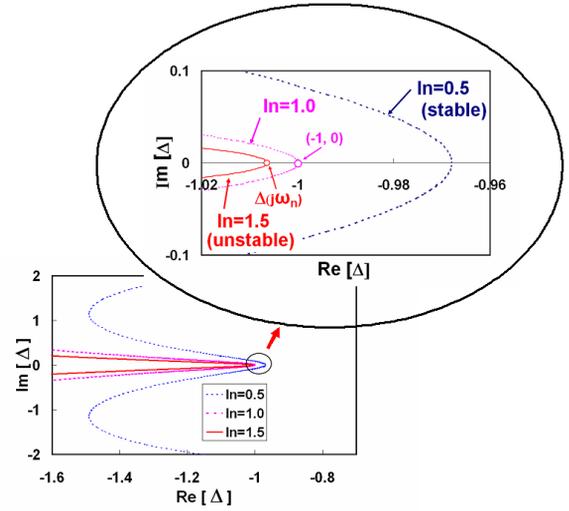


Figure 3: Nyquist plot for the model shown in Fig.1 with $G_f = 0$. The contour of Nyquist plot encircles the point $(-1,0)$ for the beam current which is higher than that limited by Robinson instability ($I_{R,max}$). ω_n is defined as the frequency (ω) of Nyquist mapping at which the Nyquist plot contour encircles the point $(-1,0)$ and intersect the real axis of the $\Delta(s)$ plane — the imagine part of $\Delta(s)$ is zero. I_n is defined as $I_n = I_B/I_{R,max}$.

$\Delta(s)$ for the system in Fig.1 can be obtained by Mason's rule:

$$\begin{aligned} -\Delta(s) = & B_s G_{pp}^B + B_s G_{pa}^B e^{(-sT_d)} G_f G_{ap}^F + e^{(-sT_d)} G_{pp}^F + \\ & e^{(-sT_d)} G_{pa}^F (-1/\tan \phi_s) B_s G_{pp}^B + e^{(-sT_d)} G_{pa}^F \\ & e^{(-sT_d)} G_f G_{pa}^F + (-1/\tan \phi_s) B_s G_{pa}^B + e^{(-sT_d)} \\ & G_f G_{aa}^F - e^{(-sT_d)} G_{pp}^F (-1/\tan \phi_s) B_s G_{pa}^B - B_s \\ & G_{pp}^B e^{(-sT_d)} G_f G_{aa}^F - e^{(-sT_d)} G_{pp}^F e^{(-sT_d)} G_f G_{aa}^F \end{aligned}$$

The system will be stable if the real part of every root of $F(s)$ is negative. There are different approaches to determine whether the system is stable. The approach used here is the Nyquist stability criterion. According to it, the system will be stable if Nyquist contour of $\Delta(s)$ does not encircle the $(-1,0)$ point as the number of poles of $\Delta(s)$ in the right-hand of the complex plane is zero. The Nyquist contours for the model in Fig.1 with the RF parameters listed in table 1 are plotted in Fig.3. G_f is set to zero for the calculation in Fig.3. It means that the system is not with the direct RF feedback. Its stored beam current is limited by Robinson instability. The harmonic current limited by Robinson instability can be expressed as

$$I_{R,max} = \frac{2 \sin \phi_s}{\sin(2\phi_s)} \cdot \frac{V_c}{R_s} \cdot (1 + \beta) \quad (13)$$

Fig.3 illustrates that the Nyquist mapping contour encircles the point $(-1,0)$ as $I_n > 1.0$, just crosses $(-1,0)$ as $I_n = 1.0$, and does not encircle $(-1,0)$ as $I_n < 1.0$.

Table 1: RF Parameters of the Example

Beam energy (GeV)	1.5
rf frequency(MHz)	500
rf voltage(kV)	1600
Radiation loss/turn (keV)	168
Momentum compaction factor	6.78×10^{-3}
harmonic number	200
Q_0	1.0×10^9
$Q_{ext} = Q_0/\beta$	2.5×10^5
R_s/Q_0	44.5

EFFECTS OF THE OPERATIONAL PARAMETERS ON THE FEEDBACK PERFORMANCE

In the direct RF feedback operation, the feedback gain (G_F) and the RF phase shift (ϕ_F) are the two parameters used to optimize the feedback performance — maximizing the stored beam current. For a given delay time, which is determined by the components and the cables of the RF system and can not be adjusted during operation, the feedback gain, as shown in Fig.4, has a threshold for the maximum stable beam current. The threshold of the feedback gain is constrained by the loop delay time. The gain threshold can be higher as the delay time is shortened.

It is interesting to find that ω_n , defined in Fig.3, is equal to zero as the feedback gain is lower than the threshold, and not equal to zero as the feedback gain is beyond the threshold. In calculation for beam current maximization, ω_n can be an indication for whether the feedback gain should be increased or decreased.

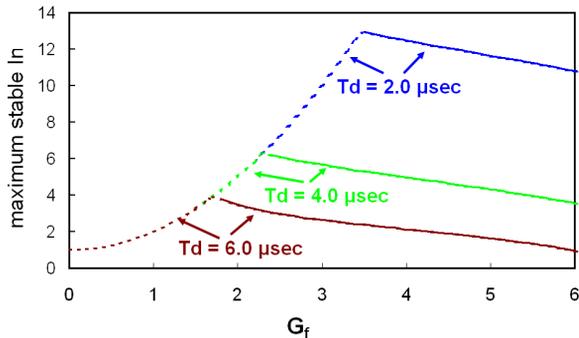


Figure 4: the maximum stable beam current versus the feedback gain, that is calculated with $\phi_F = 180^\circ$ and the RF parameters listed in table 1.

It is shown in Fig.5 that as the feedback gain is small, 0.1 in this example, the stable beam current reaches to the maximum as $\phi_F = 270^\circ$, and to the minimum as $\phi_F = 90^\circ$. And as the feedback gain becomes higher or the loop delay time is longer, the optimized ϕ_F for the maximum stable beam current is not fixed to 270° , and shifted from 270° toward 180° .

Radio Frequency Systems

T25 - Low Level RF

If the feedback RF current (I_F) is small to be negligible, compared to the generator current (I_G), the generator RF voltage should be 180° out of the beam phase while the beam current is pushed to stable limitation. Then we know from Fig.2 that the generator RF voltage will be pushed by the small feedback RF voltage to the phase stable region as $\phi_F = 270^\circ$, and to the unstable region as $\phi_F = 90^\circ$. That is just the phenomenon shown in Fig.5.

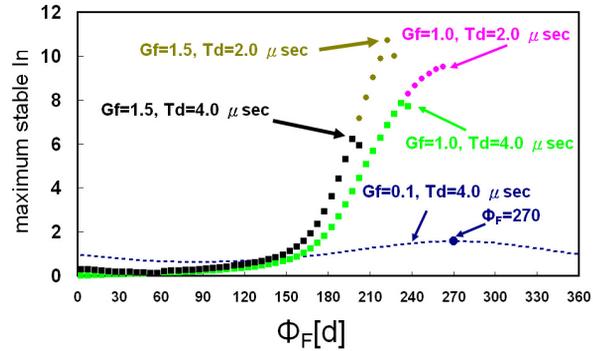


Figure 5: the maximum stable beam current versus RF phase shift of the feedback loop, that is calculated with the RF parameters listed in table 1.

CONCLUSIONS

This paper has demonstrated a mathematical method, based on the Pedersen model, to analyze the system stability in machine operation with the direct RF feedback. With the method in this paper, we can investigate the variation of the maximum beam current for different feedback gains in different loop delay time. The study confirmed that the optimized feedback gain for the maximum operational beam current is limited by the delay time. If the loop delay time can be shortened, then the performance of the direct RF feedback can be improved by increasing the feedback gain.

It is seen in the study that the frequency of the Nyquist contour encircling the unstable point (-1,0) of the Nyquist mapping plot is an indication for whether the feedback gain is beyond the limitation by the delay time. The optimized RF phase shift of the feedback loop is 270° as feedback gain is small, and shifting toward to 180° with the increase in feedback gain or delay time.

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