

ANALYSIS OF THE ORBIT RESPONSE MATRIX AND CORRECTION OF BETA FUNCTION AT THE SAGA LIGHT SOURCE

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Abstract

The method of accelerator modelling by fitting the calculated orbit response matrix to the measured is well known and widely adopted to many light sources [1], we also examined the procedure to diagnose the optics and to restore the periodicity of the twiss functions. In the analysis we use the tracking code TRACY2, because it can calculate the orbit for off-energy particle. The energy shift caused by the dipole kick was directory included in the analysis, and any free parameter of which adjust the result to experimentation was not bringing in. The multidimensional non-linear fitting was iteratively solved by SVD algorism. To resolve explicit ambiguous combinations of solution between the steering strengths and the BPM gains, we fixed strength of a couple of steering magnet for both directions to the experimentally obtained values. Before adopting the method to the real machine, we verified the accuracy of the solutions and the BPM precision required for the analysis by Monte Calro simulations. It was found by the simulations that the beam position measurement errors had to be less than 5 μm to identify the quadrupole K-value with an accuracy of 0.1 % for the condition of the SAGA Light Source electron storage ring. The parameters derived from the fitting method well reproduced not only measured orbit response matrix but also twiss functions. The root mean square error between the measured and the calculated orbit response matrix was converged in 1.1 μm . We corrected the distortion of twiss function by using the fitting result.

FORMULATION

The orbit response matrix is defined by a variation of closed orbit distortion (COD) which results from unit kick by a steering magnet. The matrix element R_{ij} represents the orbit shift at the i -th beam position monitor (BPM) due to a unit kick from the j -th steering magnet. The subject is finding best-fit parameters of which maximum likelihood reproduce the experimentally obtained data. Such parameters could be found by minimizing the quantity χ^2 . In our problems χ^2 is defined as follows,

$$\chi^2 \equiv \sum_{i,j} \frac{(R_{[\text{model}]i,j} - R_{[\text{measu}]i,j})^2}{\sigma_{ij}^2}$$

$$i = 1, 2, \dots, M, j = 1, 2, \dots, N,$$

where $R_{[\text{model}]i,j}$, $R_{[\text{measu}]i,j}$ and σ_{ij}^2 are the calculated and measured response matrix element and standard deviation of measurement errors in beam position,

respectively. M and N denote the numbers of BPMs and steering magnets, respectively. Here we introduce vectors \mathbf{a} , \mathbf{b} and a design matrix \mathbf{A} to adopt the least square method on the response matrix. The components of vector \mathbf{a} are the parameters to be fitted: BPM gains, kick angles of steering magnets, quadrupole K-values, sextupole K-values, energy offset, etc. Meanwhile the vector \mathbf{b} and design matrix \mathbf{A} are given by

$$b_k \equiv \frac{R_{[\text{model}] \text{mod}(k,M), \text{div}(k,M)} - R_{[\text{measu}] \text{mod}(k,M), \text{div}(k,M)}}{\sigma_{\text{mod}(k,M), \text{div}(k,M)}}$$

$$A_{k,l} \equiv \frac{1}{\sigma_k} \frac{\partial b_k}{\partial a_l}$$

$$k = 1, 2, \dots, M \times N,$$

$$l = 1, 2, \dots, P(\text{number of variables to be fitted})$$

Using above notations, minimization of χ^2 is expressed as

$$\text{find } \mathbf{a} \text{ that minimizes } \chi^2 = |\mathbf{A} \cdot \mathbf{a} - \mathbf{b}|^2$$

Above statement is equivalent to resolve an equation,

$$\mathbf{a} = \sum_{l=1} \left(\frac{\mathbf{U}_{(l)} \cdot \mathbf{b}}{w_l} \right) \mathbf{V}_{(l)}$$

where \mathbf{U} , \mathbf{V} and w_l are the SVD decomposition of matrix \mathbf{A} and the singular values respectively [2].

The response matrix is a linear function of the BPM gains and kicks of steering magnets but is not linear to K-value of quadrupole, K-value of sextupole and energy offset. The method of accelerator modelling by fitting response matrix is a kind of multidimensional non-linear fitting problem. Hence we perform the fitting iteratively step by step.

$$A_{k,l} \approx \frac{1}{\sigma_k} \frac{\delta b_k}{\delta a_l}$$

$$\delta \mathbf{a} = \sum_{l=1} \left(\frac{\mathbf{U}_{(l)} \cdot \mathbf{b}}{w_l} \right) \mathbf{V}_{(l)}$$

$$\mathbf{a} \Rightarrow \mathbf{a} + \delta \mathbf{a}$$

When the parameters reached to the best-fit parameters, the quantity $|\delta \mathbf{a}|$ should converge into zero. But we need to obtain fitting parameters within an accuracy of 0.1 %, the iteration has been continuing until $|\delta \mathbf{a} / \mathbf{a}| < 1\text{E} - 3$ for all parameters.

At the SVD method one of the eigenvalue w_l would be zero or be smaller than the floating-point precision in the SVD method. In order to obtain reliable results in the fitting procedure, we have introduced a threshold value κ . The threshold value κ was chosen at this modelling as

$$\kappa = 1E-5 / (\text{maximum value of } w_l).$$

When the eigenvalue is less than threshold, an infinitely large value due to the small eigenvalue is essentially rejected from the fitting by replacing $1/w_l$ to zero.

The energy shift caused by the dipole kick θ is written by [3]

$$\Delta E / E = \theta \eta \omega_{\text{rev}} / \alpha_c v$$

where η , ω_{rev} , α_c , and v are the dispersion function at the dipole, revolution frequency, momentum compaction factor, and the velocity of the electrons, respectively. Because TRACY2 can calculate the orbit of off-energy particle, we directory include this energy shift caused by the steering in the fitting procedure.

SIMULATIONS

It is not mathematically clear that the all parameters can be exactly resolved or have unique solution a priori. Moreover, the measurement errors affect the precision of the parameters to be fitted and the required accuracy of the data depends on the model. The evident ambiguous combinations of solution between the steering strengths and the BPM gains exists because multiplying a constant value to all the elements of response matrix and dividing the all element by the same constant does not make any change to the response matrix. We fixed the strength of a couple of steering magnet for both directions to the experimentally obtained values. The measurement for the kick angles was carried out using the alumina ceramic screen monitors.

There are 24 BPMs, 40 steering magnets for both directions, and 40 quadrupole magnets at the SAGA Light Source storage ring. We performed the 168 parameters fitting to the 1872 elements of orbit response matrix. The fitting variables were K-value of 40 quadrupoles, 48 BPM gains, 77 steering magnets, K-value of 2 sextupole families, and energy offset.

To confirm whether the fitting procedure could derive unique solution or not and exactly reproduce the parameters under the condition of the lattice of the storage ring, firstly we examined the ideal case in which the measurement errors are neglected. Second we changed the value of fixed steering strength to confirm the effect of the measurement errors of steering angle to the fitting. Third we investigated the required accuracy of the beam position measurement for resolving the parameters within an accuracy of 0.1%. Fourth we evaluated the fluctuation effect of power supply. At the simulation, we prepared the calculated response matrix by using randomly distributed parameter sets (original sets). Then we adopted the fitting to the matrix. After the 10 times iterative fitting, we

analysed the accuracy by comparing the fitted parameters with the original sets for each case of the above respectively. Table 1 lists the standard deviation of randomly distributed original parameter sets.

Table 1: Distribution of Original Parameter Sets

	BPM GAIN	STEERING MAGNET	QUADRUPOLE MAGNET
σ	$\pm 5\%$	$\pm 2\%$	$\pm 1\%$

Ideal Case (without any error)

Figure 1 shows the transition of the quadrupole K-value (QF1) in the iterative fitting. The other parameters were also being fitted at same time. All parameters converged into the original sets after 5 times iterations. The sextupole family and energy offset could also be converged. Figure 2 shows the ratio of fitted and the original parameters against the times of iteration.

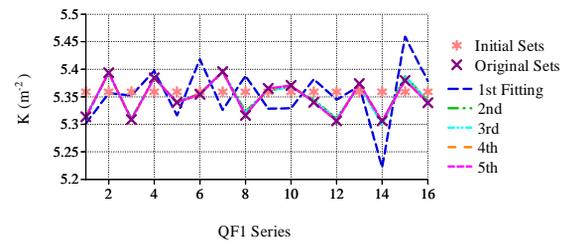


Figure 1: Transition of the QF1 K-values in the fitting.

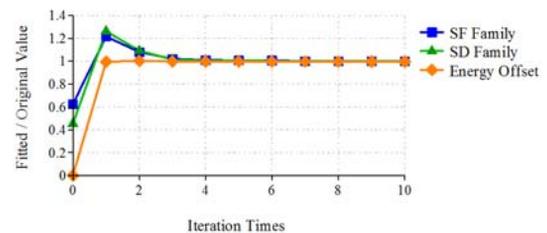


Figure 2: Fitting ratio against the iteration times.

Variation of Fixed Steering Strength

We fixed a couple of the steers to the experimentally decided values. But the kick angle measurement has about $\pm 5\%$ errors. To confirm the effect of these errors to the fitting result, we tested with the outlier fixed values. The result is illustrated in Fig. 3. The measurement error of steering angle causes the offset error of steering and BPM gains, but it affect no variation to fitted result of quadruple K-values.

BPM Noise

Figure 4 shows the reproducibility of quadruple K-Values against the BPM noise. BPM noises were included with normal distributed random errors of 2σ cut. It was

found over 5 μ m BPM noises caused unreliable solution to the fitting parameters with an accuracy of 0.1 %. In our BPM system, the standard division of the measurement of beam position is maximally 10 μ m. Therefore we took the average of 20 times beam position measurement to improve the precision of the response matrix.

Fluctuation of the Power Supply

It is clear that if the power supplies drifted with a range of 0.1 % the accurate fitting within 0.1 % could not be carried out. We investigated the effect of power supplies fluctuations to the fitting result. In worst case, the fluctuation of 0.05% caused over 0.1% fitting errors. To avoid these errors, we develop the power supply feed back system using external DCCT (STACC 2000). By this system, the drift of power supplies were decreased less than 0.05% of Peak to Peak. Finally we simulated with randomly distributed ($\sigma=2 \mu$ m) BPM noise and with the 0.05% fluctuation of the K-Values. At such actually achievable case, the parameters were reproduced within an accuracy of 0.1%.

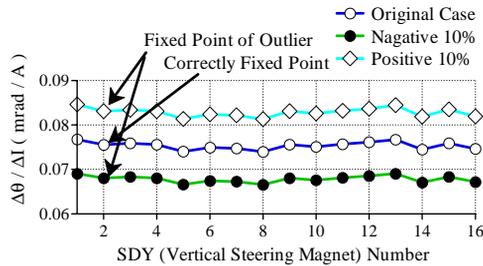


Figure 3: Mismatch of the Fixed Value.

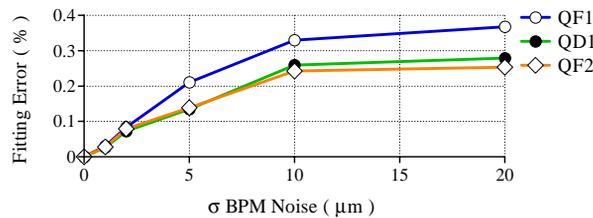


Figure 4: Reproducibility against the BPM Noise.

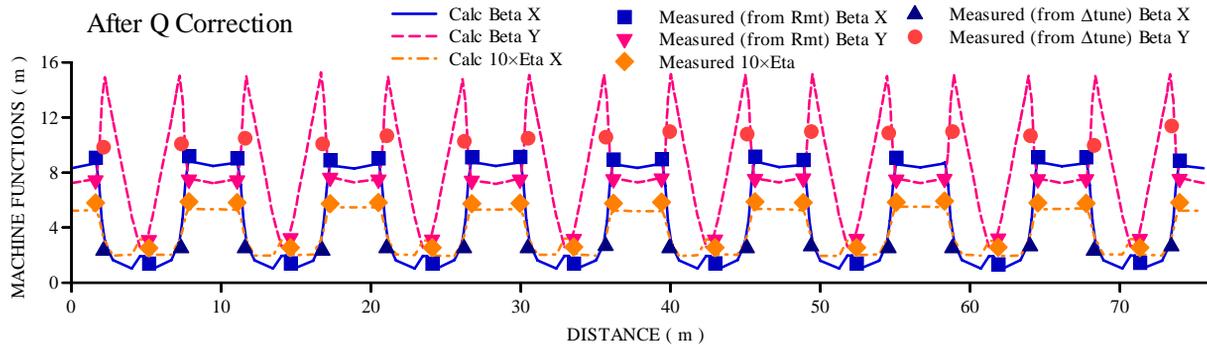


Figure 5: Calculated (using fitted parameters) and measured twiss functions after the quadrupole correction.

ADAPTING TO THE REAL MACHINE

The fitting result of the quadrupole K-values at SAGA Light Source storage ring is summarized in table 2.

Table 2: Fitting Result

	σ	Average	Designed
QF1	0.24 %	5.3129	5.3592
QD1	0.22 %	5.1798	5.1916
QF2	0.14 %	4.1385	3.9681

After the fitting, we corrected the distortion of twiss function by tuning the corrector coils of the quadrupole magnets to suppress the deviation in K-values. In Fig. 5 we compare twiss functions derived from the fitting with that obtained in the measurement. After the correction, the twiss functions show good periodicity except for the vertical beta function at QD1. Concerning the chromaticity, there was a large discrepancy between the calculation (4.3, -6.6) and the measurement (2.8, 3.0). Additionally the strength of the sextupole families was not reproduced. Such deviations may arise from quadrupole and sextupole components in bending magnets or the coupling between quadrupole and the sextupole. For more accurate modelling of the SAGA-LS storage ring, it is required to take into account the quadrupole and sextupole components in the bending magnet, and the effectiveness of the coupling between quadrupole and sextupole magnet.

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