FULL ELECTROMAGNETIC SIMULATION OF FREE-ELECTRON LASER AMPLIFIER PHYSICS VIA THE LORENTZ-BOOSTED FRAME APPROACH *

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Abstract

Numerical simulation of some systems containing charged particles with highly relativistic directed motion can by speeded up by orders of magnitude by choice of the proper Lorentz-boosted frame[1] . A particularly good example is that of short wavelength free-electron lasers (FELs) in which a high energy electron beam interacts with a static magnetic undulator. In the optimal boost frame with Lorentz factor γ_F, the red-shifted FEL radiation and blue shifted undulator have identical wavelengths and the number of required time-steps (presuming the Courant condition applies) decreases by a factor of $2\gamma_F^2$ for fully electromagnetic simulation. We have adapted the WARP code [2] to apply this method to several FEL problems involving coherent spontaneous emission (CSE) from pre-bunched e-beams, including that in a biharmonic undulator.

INTRODUCTION

It is well known that in general, explicit, fully electromagnetic simulation will have its time step Δt limited by the Courant condition corresponding to the numerical grid spacing and/or that necessary to achieve sufficient temporal resolution of the highest frequencies important to the physics of the particular situation. For problems in which a highly relativistic charged particle beam is present, the overall system time and/or length scale L_{sim} can be large and the ratio of scale lengths $L_{sim}/c\Delta t$ can become enormous. Recently, Vay [1] pointed out that for some of these problems performing the simulation in a Lorentz-boosted frame offers potentially orders of magnitude speed-up in computation time.

A natural candidate for boosted frame calculations is a short wavelength free-electron laser (FEL). Here a sample problem could have the resonant radiation wavelength $\lambda_R = 10\,\text{nm}$, an undulator wavelength $\lambda_u = 25\,\text{mm}$, and a system length $L_{sim} \geq 10\,\text{m}$. Performing this simulation in the laboratory frame requires $\sim 4\times10^9$ time steps or greater. Presuming a "moving window" type simulation centered about the electron beam of length $l_b \approx 100\,\mu\text{m}$ and radius $r_b \approx 50\,\mu\text{m}$, the number of grid points for an 2D axisymmetric (or slab) model exceeds 10^8 (and likely 3-10 times greater to model transverse diffraction effects).

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The natural boosted frame for FEL computations is the so-called "ponderomotive" frame in which the e-beam longitudinal speed (when in the undulator) is zero on average. In this frame the red-shifted FEL resonant wavelength $\lambda'_R = 2\gamma_F\lambda_R$ is equal to the blue-shifted undulator wavelength $\lambda'_u = \lambda_u/\gamma_F$. Here $\gamma_F^2 \equiv \gamma_0^2/(1+a_u^2)$ with a_u being the normalized, RMS undulator strength. The Lorentz transformation to the boosted frame shrinks the undulator by a factor γ_F and increases the radiation wavelength by the same factor times two, resulting in an overall decrease of the needed number of time steps by a factor $\approx 2\gamma_F^2$. Likewise, from the point of view of the Courant condition, the increase in Δt permits (in general) a similar increase in the spatial grid zone size so that the savings in 2- and 3-D simulations can be immense. However, in cases where the electron beam is much longer than the so-called slippage length ($l_{slip} \equiv N_u\lambda_R$) in the lab frame, this length exceeds the undulator length in the boosted frame and one factor of γ_F is lost (*i.e.*, the ratio of l_b/λ_R remains constant independently of γ_F).

To study various standard FEL problems, we used the WARP simulation code [2] with its standard full EM solver operating in slab-mode geometry (*e.g.*, $x-z$ or $y-z$). A special Python script implemented linearly-polarized undulator fields in the boosted frame. Special diagnostics measure forward- and backward-moving radiation intensity through a transverse plane fixed in the lab frame (*e.g.*, at a fixed z relative to the undulator entrance) and also on-axis values of electric and magnetic fields. In order to avoid requiring initialization of the E- and B-fields associated with a beam pulse with a net current and charge, we "added" a positron beam with the exact same charge and current distribution at $t = 0$ in the simulation (see [3] for some additional details). We now present some boosted frame results for sub-harmonically pre-bunched beams and also for prebunched beams radiating in a "biharmonic" undulator.

EMISSION BY A SUB-HARMONICALLY BUNCHED LONG PULSE

There is great current interest among many FEL groups in exploiting configurations where an e-beam is first strongly bunched in a modulator/dispersive chicane combination by an external laser at a long wavelength λ_M and then resonantly emits at a shorter wavelength in a subsequent undulator whose resonant wavelength is tuned to an integral harmonic such that $n\times\lambda_R = \lambda_M$. We studied such a configuration in the boosted frame by examining emission originating from a 180-MeV e-beam strongly pre-

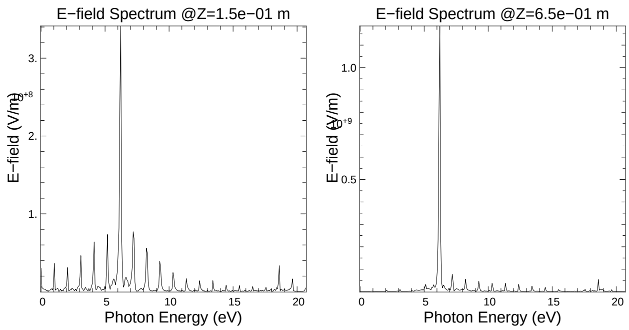

Figure 1: On-axis electric-field spectrum $|E(\omega)|$ at $z = 0.15$ m (left plot) and $z = 0.65$ m (right plot) from an electron beam sub-harmonically bunched at $\lambda = 1200$ nm in an undulator resonantly tuned to $\lambda_R = 200$ nm ($\hbar\omega = 6.1$ eV). In addition to strong sideband emission early in z whose blueward components persist later in z, there is also evident third harmonic emission at 18 eV.

bunched at $\lambda_M = 6\lambda_R = 1200$ nm propagating in a 0.5-m long, linearly polarized undulator with $\lambda_u = 25$ mm and $a_u = 1$. Here we used a 1-A current to minimize any complications due to gain and also employed negligible energy spread and transverse emittance to prevent debunching. The simulation time step was $1/24$th of the resonant radiation period in order to resolve third harmonic emission.

As expected from previous numerous studies using eikonal codes, there is strong on-axis emission centered at $\lambda = \lambda_R$ that grows (initially) quadratically with z. Unlike an eikonal code which typically can only properly model radiation in a narrow wavelength region around a central value, full EM codes (including those operating in a Lorentz-boosted frame) can study emission at wavelengths over a range $2c\Delta t \leq \lambda \leq \infty$. We find (see Fig. 1) for $z \leq 12\lambda_u$ there is significant sideband emission separated (in wavenumber) from the fundamental at λ_R by integral harmonics of λ_M. However, the sideband emission steadily weakens in a normalized sense with increasing z and is quite weak for $z \geq L_u = 0.5$ m (although still apparent on the "blue" side of λ_R in the right plot of Fig. 1).

Simulations done at a larger harmonic upshift number (*i.e.*, $\lambda_M = 12\lambda_R$) show that the sidebands persist longer in z than is true for $\lambda_M = 6\lambda_R$ case and also that the sidebands closest to λ_R are non-negligible on-axis for tens of cm beyond the undulator exit at $z = 0.5$ m. Consequently, for high gain harmonic generation (HGHG) FEL experiments, it may be possible to investigate diagnostically from the near-field radiation spectrum some details of the imposed microbunching structure.

EMISSION BY A PREBUNCHED BEAM IN A BIHARMONIC UNDULATOR

Several authors have discussed the possible utility of a biharmonic undulator configuration (see., *e.g.*, Refs. [4]-[7]) where the magnetic vector potential strength

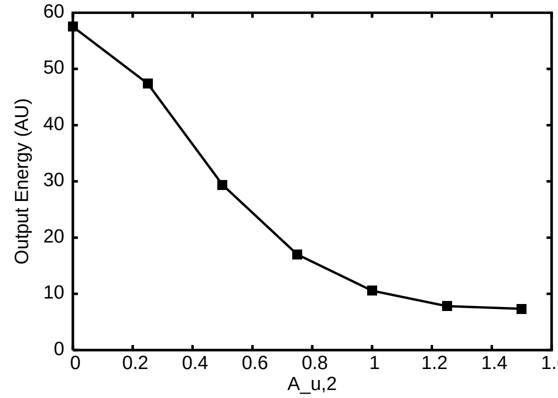

Figure 2: Outgoing radiation energy beyond the undulator exit as a function of $a_{u,2}$ for a biharmonic undulator with $\lambda_{u,2} = 3\lambda_{u,1}$ and $a_{u,1} = 1.0$.

$\vec{A} = \vec{A}_1 \cos(k_{u,1}z + \phi_1) + \vec{A}_2 \cos(k_{u,2}z + \phi_2)$ in which $k_{u,1}$ and $k_{u,2}$ are related harmonically, *e.g.*, to enhance third harmonic emission. Another possible use of a biharmonic configuration is to provide an additional "source" of A for an external laser seeded FEL amplifier where the electron beam energy must remain fixed (*e.g.*, the accelerator is feeding a multiplexed set of FEL's operating simultaneously). Then the maximum output radiation wavelength of a particular undulator depends upon the peak value of normalized undulator strength a_u available at minimum gap closure g_{min}. If the "primary" undulator has a short $\lambda_{u,1}$ and a peak on-axis value of $a_{u,1}$ limited physically to not much more than 1 because $k_{u,1}g_{min} \geq 2$, there will be a small effective tuning range in λ_R. Adding a "secondary", variable strength undulator field A_2 with a longer period can strongly increase the maximum reachable wavelength if $\max|A_2| \geq 2\max|A_1|$ because the FEL resonance relation (at the shorter resonant wavelength) obeys

$$\lambda_{R,1} = \frac{\lambda_{u,1}}{2\gamma^2} \times \left(1 + a_{u,1}^2 + a_{u,2}^2\right) \qquad (1)$$

Note that from a mathematical point of view, there is no requirement that the two undulators be related harmonically, although from a construction point of view this choice may be easiest to implement. Also, the polarity of the two undulators can be entirely different (*e.g.*, cross-polarized linear undulators).

Modeling FEL radiation emission in such a configuration poses accuracy issues for eikonal codes employing the standard wiggle-period-averaging approximations unless $\lambda_{u,2} \gg \lambda_{u,1}$. For linear undulators, the "JJ" Bessel function difference term also needs to be modified because of dephasing associated with the wiggle motion due to $\vec{A}_2$. There also can be harmonic coupling if $\lambda_{u,2}$ is an integer harmonic of $\lambda_{u,1}$. This difficulty does not arise for boosted frame EM simulation so long as the effective temporal and spatial gridding supports the shortest radiation wavelengths of interest.

We did a series of slab model, boosted frame simula-

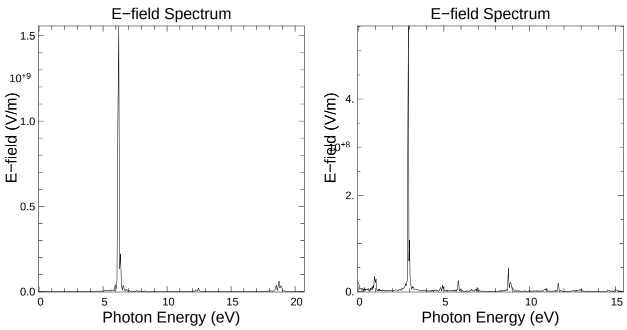

Figure 3: On-axis electric-field spectrum $|E(\omega)|$ from (left) a ”normal”, single frequency undulator with $a_{u,1} = 1.0$ and (right) a biharmonic undulator with $\lambda_{u,2} = 3\lambda_{u,1}$, $a_{u,1} = 1.0$ and $a_{u,2} = 1.5$.

tions for a 1-A, 40-fs (waterbag profile), 180-MeV e-beam propagating in a $0.75-$m length biharmonic undulator with $\lambda_{u,1} = 25$ mm, $a_{u,1}$=1.0, $\lambda_{u,2} = 75$ mm, and $a_{u,2}$ ranging from 0 to 1.5. Both undulators were linearly polarized in the same direction. As was true in the previous section, $c\Delta t = \lambda_{R,1}/24$ and there was negligible transverse emittance and energy spread to minimize kinetic debunching effects. We prebunched the beam over $\pi/4$ in phase at $\lambda_{R,1}$ which, using Eq. 1, ranges from 200 to 425 nm. Figure 2 plots the output power measured through a transverse plane 5-cm downstream of the undulator exit as a function of $a_{u,2}$. One sees an ∼eight-fold decrease in power with increasing $a_{u,2}$ although $\approx 25\%$ of this can be attributed to short-pulse slippage effects as $\lambda_{R,1}$ increases. In many situations where optimizing performance at the shortest wavelengths is most important, the power falloff with longer wavelength may be quite acceptable. In Fig. 3 we plot the on-axis electric field spectrum just outside the undulator for the separate cases of $a_{u,2} = 0$ and $a_{u,2} = 1.5$ with $a_{u,1} = 1.0$ as before. As expected, the $a_{u,2} = 1.5$ case (right plot) shows the fundamental photon energy shifts redwards by a factor of $4.25/2$ but the relative spectral width appears unchanged. The biharmonic case also shows a greater relative strength of the third harmonic and there is a somewhat greater amount of second harmonic (and sidebands to either side).

DISCUSSION

Applying the Lorentz-boosted-frame simulation method to free-electron laser problems allows study of problems where the eikonal approximation method proves insufficient, *e.g.* those where the total emission bandpass is quite large, those where wiggler-period averaging is suspect, *etc.* In our boosted-frame FEL studies to date we have been able to explore aspects of FEL emission that are essentially ”opaque” to FEL codes such as GINGER or GENESIS but, it is important to add, have not uncovered critical physics that would make one doubt the basic correctness of the eikonal approximation. Moreover, while it is true the the boosted frame method gives many orders of magnitude speedup relative to lab frame EM simulation of short wavelength FEL’s, it is still several orders of magnitude slower than use of the eikonal method.

There are at least two reasons that eikonal codes will be much faster than a boosted frame EM code. First, for most normal high-gain FEL’s where $L_{gain} \geq 50\lambda_u$, one can use $\Delta z-$steps of $\geq 5\lambda_u$ and maintain high accuracy. Moreover, the choice of Δz does not impose any limit concerning which radiation harmonics can be modeled. In contrast, the boosted frame code must use time steps where $c\Delta t \leq \lambda'_u/24$ if one needs reasonable accuracy for third harmonic emission (and less than half that to model the seventh harmonic and so on). Consequently, although the total number of such steps scales directly with the undulator length for both types of codes, the number of steps will be much greater in a boosted frame code. Second, in an eikonal code, the time resolution of slowly varying e-beam and radiation quantities can be one or two orders of magnitude larger than the resonant period λ_R/c, whereas Δz in a boosted frame code must be $\leq c\Delta t \sim \lambda'_R/24$. Consequently, the number of $t-$grid points in an eikonal code can be several orders of magnitude less than the number of $z-$grid zones in a boosted frame EM code with the total number in each case scaling with the electron beam length (presuming $l_b \gg l_{slip}$). For many long electron pulse problems, an eikonal code also has a further advantage that periodic boundary conditions in time can be readily adopted; such is not straight-forwardly possible in an EM code, whatever the frame.

All these scalings suggest that if the necessary FEL physics can be studied with an eikonal code, it will be much faster than a full EM code (in whatever frame). However, if there are optical or shorter wavelength physics that cannot be resolved properly by an eikonal code with its underlying slowly-varying envelope approximation, a boosted-frame EM code is a *very* attractive option in terms of CPU speedup relative to doing the problem in the lab frame.

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