POLARIZATION ANALYSIS OF NONLINEAR HARMONIC RADIATION IN A CROSSED-PLANAR UNDULATOR

H. Geng*, Y. Ding, Z. Huang
SLAC, Menlo Park, CA 94025, USA

Abstract

There is growing interest in producing intense, coherent x-ray radiation with an adjustable and arbitrary polarization state. The crossed-planar undulator, which was first proposed by Kim, could achieve rapid polarization control in synchrotron radiation sources and free electron lasers (FELs) through the manipulation of a phase shifter. Recently, a statistical analysis shows that a polarization degree of over 80% is obtainable for a Self-Amplified Spontaneous Emission (SASE) FEL near saturation. In such a scheme, nonlinear harmonic radiation is also generated in each undulator and the polarization of the radiation is controllable in the same manner. In this paper, we study the degree of polarization achievable at the third harmonic in a crossed-planar undulator. We also propose a method for generating second harmonic radiation with arbitrary polarization.

INTRODUCTION

Several X-ray FELs based on the SASE scheme are being built worldwide as the next generation light sources. Interest in producing radiation with polarization control in the soft X-ray regime is also growing. A SASE FEL with a planar undulator is normally linearly polarized. Circular polarization can, in principle, be provided by an APPLE-type undulator [1]. However, with this method applied to x-ray FELs: (i) the tolerances are tight and have not been demonstrated in practice, (ii) the focusing properties of the undulator are very sensitive to the polarization, and (iii) polarization adjustment involves moving the undulator and is therefore slow.

An alternative approach for polarization control is the so-called "crossed undulator" (or "crossed-planar undulator"), first proposed by K.J. Kim [2]. In this method circularly polarized light is obtained by interfering the radiation fields from two adjacent planar undulators in a crossed configuration (see Fig. 1). The first undulator is normally polarized in x and the second in y, and a phase shifter between them is used to delay the electron beam and control the final polarization state. The effectiveness of this method of controlling polarization at the fundamental wavelength in a SASE FEL was studied in [3]; it was found that over 80% circular polarization could be achieved at the end of the exponential gain regime, just before saturation. In this report, we study the use of crossed-undulators to generate circular polarization in higher harmonics, in particular, in the second and third harmonics, with the aim of achieving better than 80% polarization.

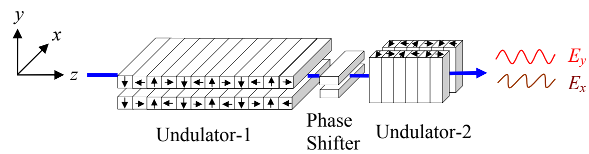

Figure 1: A sketch of crossed undulators for polarization control.

DEGREE OF POLARIZATION

The state of polarization can be described by the coherency matrix [4]

$$\mathbf{J} = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \begin{bmatrix} \langle E_x(t)E_x^*(t)\rangle & \langle E_x(t)E_y^*(t)\rangle \\ \langle E_y(t)E_x^*(t)\rangle & \langle E_y(t)E_y^*(t)\rangle \end{bmatrix} \tag{1}$$

where * means complex conjugate, and the angular bracket refers to the ensemble average. The degree of polarization can be calculated as

$$P \equiv \sqrt{1 - 4\cdot \mathrm{det}[J]/(\mathrm{tr}[J])^2}. \tag{2}$$

Stokes parameters are also generally used to describe partially polarized light. They are related to the coherency matrix by [4]:

$$\begin{aligned} S_0 &= J_{xx} + J_{yy}, \\ S_1 &= J_{xx} - J_{yy}, \\ S_2 &= J_{xy} + J_{yx} = 2\langle A_x(t)A_y(t)\cos[\theta(t)]\rangle, \\ S_3 &= i(J_{yx} - J_{xy}) = 2\langle A_x(t)A_y(t)\sin[\theta(t)]\rangle. \end{aligned} \tag{3}$$

Here $A_x(t)$ and $A_y(t)$ are the amplitudes of the radiation fields $E_x(t)$ and $E_y(t)$, and $\theta(t)$ is the phase difference between $E_x(t)$ and $E_y(t)$. Using these Stokes parameters, the degree of total polarization is defined as:

$$P = \sqrt{S_1^2 + S_2^2 + S_3^3}/S_0. \tag{4}$$

The degree of circular polarization P_c is defined as

$$P_c = |S_3|/S_0. \tag{5}$$

* Email: genghp@slac.stanford.edu. Visiting SLAC from University of Science and Technology of China

POLARIZATION AT THE THIRD HARMONIC

Method I

We begin by studying the generation of circularly polarized light at the third harmonic using the crossed undulator configuration of Fig. 1. A 1D FEL code has been modified to include third harmonic radiation, and is used here to simulate SASE. The parameters that we use are for soft x-ray FEL operation at *e.g.* the LCLS: we consider fundamental wavelength 1.5 nm, beam energy 4.3 GeV, peak current 2 kA, and undulator parameter $K = 3.5$ (other parameters are similar to those found in Ref. [3]). As was done in the previous study, the length of the second undulator L_2 is held fixed (at 1.53 m), while the first undulator length L_1 is allowed to vary for optimization. For each point the degree of circular polarization is optimized by adjusting the phase shifter. The simulation results are given in Fig. 2, where we show (i) average power radiated in both x and y, and (ii) degree of polarization in the third harmonic, giving both the degree of total polarization and of circular polarization. The results shown represent the average of 50 statistical runs.

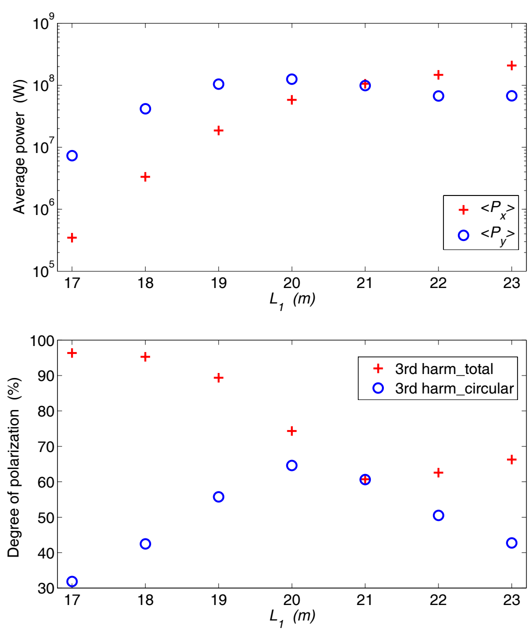

Figure 2: Power and degree of polarization of the third harmonic radiation using Method I.

In the top plot we note that up to $L_1 = 19$ m the power increases with L_1 for both polarizations. In this linear gain regime the power in y is larger than it is in x, because the beam, on entering the second undulator, is already micro-bunched. In the saturation regime ($L_1 > 19$ m), however, the micro-bunching has already saturated (as has the radiation power) within the first undulator; in the second undulator it degrades even more and the radiation power is less than in the first undulator (as seen in the figure).

In the bottom plot of Fig. 2 we see that the degree of circular polarization in the third harmonic first grows and reaches a maximum of 60% for $L_1 = 20$ m, where saturation is reached and where the power in the two polarizations is equal; beyond this point it decreases with L_1. Note that the phase shifting in the optimization for third harmonic polarization, in general, is different than it is for fundamental polarization control. Note also that to reduce the power fluctuation in the radiation one would, in principle, like to operate deep in the saturation regime; however, as we see this will reduce the circular polarization and is thus not desirable.

Method II

To increase the degree of circular polarization, we can tune the resonant wavelength in the second undulator λ_2 to the third harmonic of the first undulator λ_1, *i.e.* $\lambda_2 = \lambda_1/3$. In this way, the coherent radiation in the second undulator is stronger because $\lambda_1/3$ is now the fundamental. Thus, it takes a shorter distance for the power radiated in the second undulator to catch up with that radiated in the first undulator. The slippage (between the two fields) in the second undulator is also reduced for two reasons: (i) at optimized length the second undulator is shorter and (ii) the resonant wavelength is only 1/3$^{\rm rd}$ of what it was, so the slippage per period is also 1/3$^{\rm rd}$ of what it was.

In this method L_1 is fixed and chosen to yield a maximum in third harmonic bunching at the end, while L_2 is allowed to vary. Again, at each point the degree of circular polarization is optimized by adjusting the phase shifter. The simulation results—average power and degree of polarization as functions of L_2—are shown in Fig. 3. We see that the radiation power in the second undulator $\langle P_y \rangle$ grows and eventually reaches the level of the third harmonic power radiated in the first undulator $\langle P_x \rangle$ (after a distance of 0.8 m). When the powers are equal the degree of circular polarization reaches its maximum of 80%. Again the degree of polarization drops if the first undulator operates in deep saturation mode.

POLARIZATION AT THE SECOND HARMONIC

For polarization control at the second harmonic we need a different layout of undulators: a first (main) undulator of length L_1 that radiates, say in x; two short undulators of equal length $L_2 = L_3$ that radiate one in x, the other in y, and a phase shifter in between. The resonant wavelength of the two short undulators is chosen to be the second harmonic of the first undulator ($\lambda_2 = \lambda_3 = \lambda_1/2$), as described in [5]. After the main undulator the electron beam is sent directly into the downstream undulators, where second harmonic radiation is generated. The second harmonic content generated in the main undulator is small compared to that generated in the short undulators and can be ignored. Note that a similar scheme, but working at fundamental wavelength, has been proposed for operating a crossed-

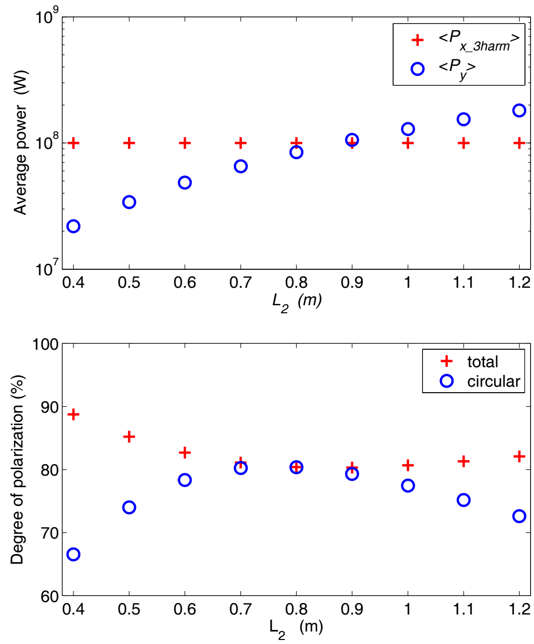

Figure 3: Power and degree of polarization of the third harmonic using Method II.

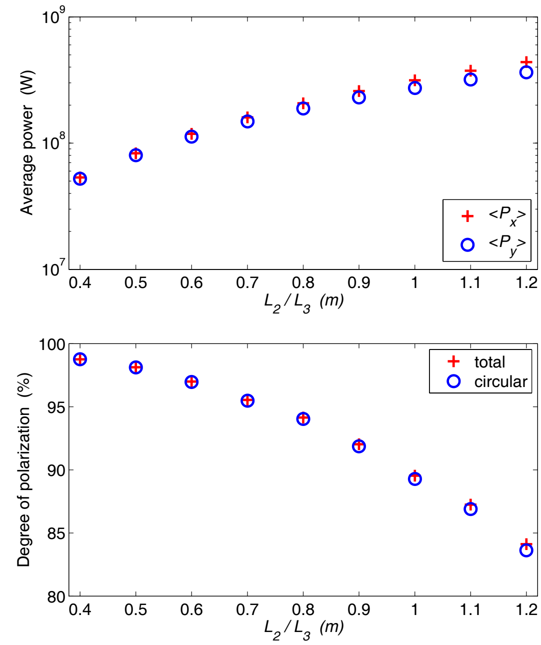

Figure 4: Power and degree of polarization at the second harmonic using our modified crossed undulator scheme.

undulator in the saturation regime by Y. Li, *et al* [6]. However, in this method, the electron beam needs to be bent in order to separate the radiation generated in the main undulator from that generated in the first short undulator—and at the same time—the electron beam micro-bunching needs to be preserved, which may be difficult to do in practice.

With our method, the radiation at the fundamental wavelength in the first undulator is not coherent with the radiation in the second and third undulators, and thus has no effect on the polarization properties of the harmonic fields. For our simulations we fix the length of the first undulator to $L_1 = 21$ m and have it operate in saturation mode. The length of the second and third undulators (with $L_2 = L_3$) is varied for optimization (and the phase shifter is again optimized for circular polarization). The simulation results are shown in Fig. 4. We see that the power radiated by the two undulators is almost always equal, and the degree of circular polarization almost equals the total polarization. A degree of circular polarization of more than 90% with an average power of $\sim$ 100 MW can be achieved when the length of the short undulators is $\lesssim 1$ m.

An important feature of this method is that the first undulator can operate in deep saturation mode and still give a high level of circular polarization, because the same power can always be radiated in the two short undulators before significant de-bunching occurs. Finally we note that this method can also be implemented with helical undulators to generate linearly polarized radiation.

CONCLUSION

In this paper we have studied the generation of higher harmonic, circularly polarized radiation using crossed-planar undulators. We have shown that one can get arbitrarily polarized radiation in both the fundamental and third harmonic by only adjusting a phase shifter. We have also shown that polarization control of the second harmonic, with a modified crossed undulator scheme, is very effective. The maximum circular degree of polarization achievable is over 90% and is insensitive to the length of the first undulator. Preliminary 3D simulations suggest these 1D results are still valid. We expect that the methods explored here, if applied to seeded FELs, will be able to produce nearly 100% polarized harmonic radiation.

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