OVERVIEW OF QUASI-PERIODIC UNDULATORS

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Abstract

The definition of quasi-periodicity and how to create an appropriate quasi-periodic undulator are presented. The one-dimensional quasi-periodic lattice can be created by projecting lattice points in a window in the two-dimensional rectangular lattice onto a irrationally inclined line. The determined quasi-periodic lattice sequence is used to modify the magnetic field of a periodic undulator.

INTRODUCTION

After the first demonstration of original quasi-periodic undulator (QPU) at the NIJI-IV [1, 2], there have been many modifications for QPU structures. One of the first most productive improvements was introducing the quasi-periodicity by modifying the magnetic field in a periodic undulator instead of modifying the period length [3]. In addition to this practical improvement, a slight modification of creation theory of one-dimensional quasi-periodicity gave another advantage for building this type of device [4]. As the result, many different types of QPUs for generating both linearly and elliptically polarized radiations have been installed in the synchrotron radiation (SR) facilities worldwide [5]. Furthermore, some more SR facilities are considering to building such devices in order to improve their performance [6].

In this paper, limitations and possible improvements of performance of QPU are discussed on the basis of synchrotron radiation physics and mathematics of quasi-periodicity.

1-D QUASIPERIODICITY

One of the way to create the one-dimensional quasi-periodicity is to project lattice points in a window in the two-dimensional periodic lattice onto an irrationally inclined line. Figure 1 shows an example of such a 2-D rectangular lattice and an inclined line. This procedure can be transformed into a simple equation as follows.

$$\tilde{z}_m = m + (r\tan\alpha - 1)\left\lfloor \frac{\tan\alpha}{r+\tan\alpha} m + 1 \right\rfloor . \quad (1)$$

In this equation, $\tilde{z}_m$ represents a normalized coordinate of m-th lattice point on the inclined axis, and the bracket $\lfloor x \rfloor$ stands for the greatest interger less than x. The letter r represents the ratio b/a [4].

This coordinate is proportional to the phase advance of emitted light in an undulator, and therefore, it can be written as:

$$\phi_m = \pi\left\{ m + (r\tan\alpha - 1)\left\lfloor \frac{\tan\alpha}{r+\tan\alpha} m + 1 \right\rfloor \right\} . \quad (2a)$$

The phase advance in each half period is written as:

$$\Delta\phi = \phi_{m+1} - \phi_m , \quad (2b)$$

and those in periodic section and in quasi-periodic section are written as: $\Delta\phi_p = \pi$ and $\Delta\phi_q = \pi r\tan\alpha$, respectively.

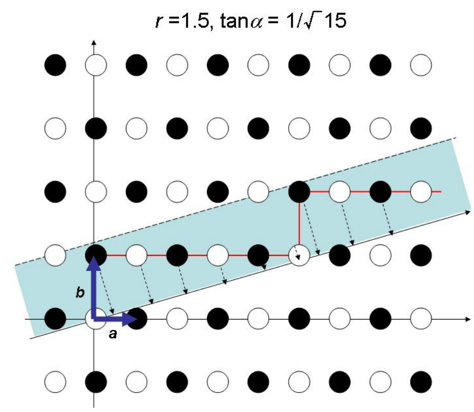

Figure 1: Relation between 2-D rectangular lattice and a 1-D quasi-periodic lattice.

As it is well known, there are two way to construct actual quasi-periodic undulators (QPU). One is to introduce a different inter pole distance at the quasi-periodic position, and another way is to introduce a different magnetic field. In Fig. 1, the position of vertical step in 2-D lattice corresponds to the quasi-periodic step at which the phase advance is smaller than that at periodic positions. For an actual QPU, the smaller phase advance can be achieved by using a shorter half-period or a smaller magnetic field.

The phase function is given in eq. (3) as follows [7]:

$$\phi = \frac{2\pi}{\lambda_{photon}}\left(\frac{z}{2\gamma^2} + \frac{\int x'^2\, dz}{2} \right), \quad (3)$$

where λ_{photon} is the wavelength of emitted photon and x' is the angle of electron trajectory in an undulator.

After some simple calculations by assuming the sinusoidal magnetic field, the phase slip ratio is found to nearly equal to the square of magnetic field amplitude ratio, that is:

$$\Delta\phi_q / \Delta\phi_p = \frac{\pi r\tan\alpha}{\pi} \cong \left(B_{0q}/B_{0p}\right)^2 . \quad (4)$$

As one can see in eq. (4), by selecting appropriate values for r and tanα, the peak field at QP position can be larger than that at the periodic position. Figure 2 shows such an example.

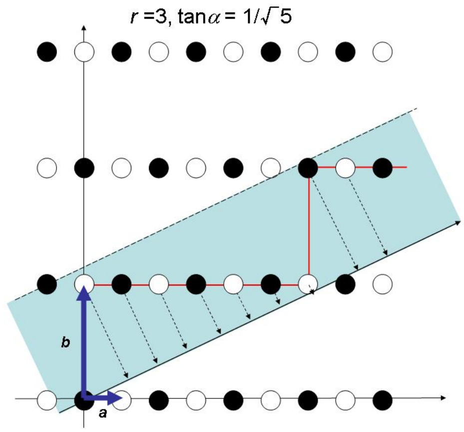

Figure 2: Another configuration of quasi-periodicity. For this case, $r\tan\alpha$=1.342, and then, B_{0q}=1.158xB_{0p}.

MAGNETIC FIELD DISTRIBUTION IN QPU

Figures 3 and 4 present the magnetic field distributions for two examples shown above. For both examples, the peak field at periodic positions were assumed to be 0.3 T, the 100 mm period length, and the 3.2 m undulator length. The deflection parameter *K* for periodic part is 2.8.

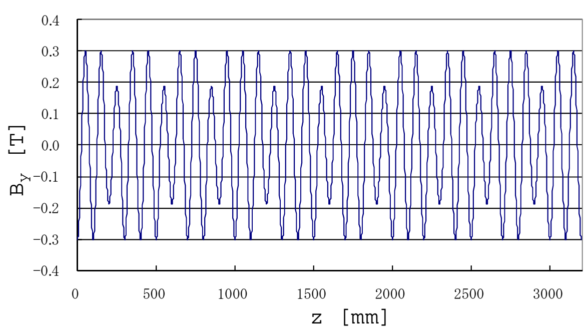

Figure 3: Magnetic field distribution of the QPU in Fig. 1.

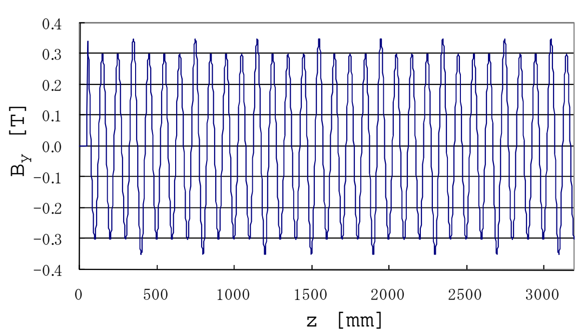

Figure 4: Magnetic field distribution of the QPU in Fig. 2.

UNDULATOR RADIATION SPECTRA

Figures 5 and 6 show expected on-axis flux density from undulators in Figs. 3 and 4, respectively. For the spectral calculation, 1 GeV electron energy, 100 mA beam current, 15 nm-rad emittance with 1 % coupling were assumed.

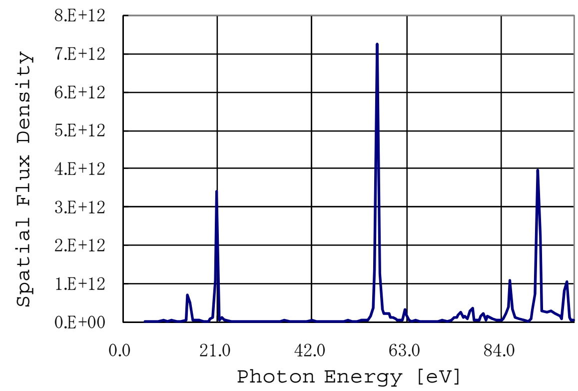

Figure 5: On-axis flux density from the QPU with the field distribution in Fig. 3.

In this configuration (reduced field strength at the QP-position), peak positions of higher harmonics shift toward lower energy from original integer positions.

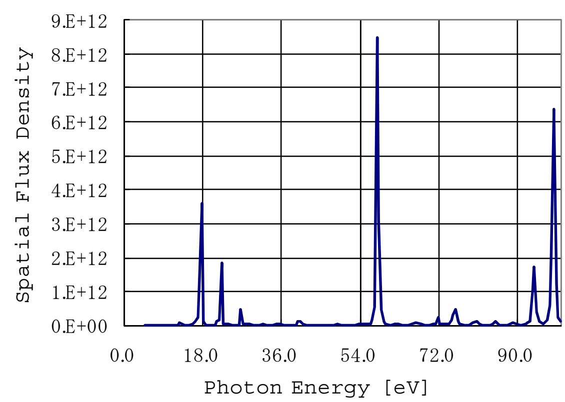

Figure 6: On-axis flux density from the QPU with the field distribution in Fig. 4.

On the other hand, in the configuration (increased field strength at the QP-positions), higher harmonic peak positions shift toward higher energy.

DISCUSSIONS AND CONCLUSION

According to the calculated results in previous section, it seems that "strong-field" QPU gives a slightly better performance with respect to the peak intensity in each (fundamental or higher) harmonic. This fact may be caused by a smaller peak field variation at the QP-position and a smaller number of QP-positions. Also, it may be worth to mention that the contamination from rational harmonics is smaller than that of "weak-field" QPU.

In conclusion, a new possibility for designing a QPU is presented in this paper. There might be further possibilities for creating QPUs with higher spectral performance by further modification of QP lattice. Also, it may be possible to introduce the phase slip for creating the quasi-periodicity by applying the orthogonal magnetic field (in the horizontal direction) in an appropriate manner.

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