BEAM DYNAMICS FOR VERY HIGH BEAM-BEAM PARAMETER IN AN e^+e^- COLLIDER

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Abstract

Beam-beam tune shift parameter characterizes the strength of the nonlinear interaction due to the beam-beam collision. The tune shift has been measured in many e+e- colliders and has been an indicator for the collider performance. The record for the tune shift is known as 0.07-0.1 depending on the parameter of the collider, especially the radiation damping rate. We discuss the fundamental limit of the tune shift can be very high (>0.2) depending on the choice of collider parameter, which concerns operating point near the half integer tune, head-on collision and travel focus.

INTRODUCTION

Various collision schemes are proposed for high luminosity B factories. In recent colliders, multi-bunch collision is crucial to get gain the multiplicity of the number of bunches. The crossing angle is introduced to avoid parasitic encounters.

An essential of crossing angle is expressed by transformations as shown in Figure 1. The electro-magnetic field is formed perpendicular to the traveling direction. The transformation which particles in the beam experience is expressed by [1,2]

$$\begin{aligned} \Delta p_x &= -F_x(x+2s\phi,y) \\ \Delta p_y &= -F_y(x+2s\phi,y) \\ \Delta\delta &= -\phi F_x(x+2s\phi,y) \end{aligned} \qquad (1)$$

where $s=(z-z_c)/2$ and f is the half crossing angle. The transformation is separated by three parts.

$$e^{\phi p_x z} \circ e^{-:H_{bb}:} \circ e^{-\phi p_x z} \qquad (2)$$

where H_{bb} is Hamiltonian for the beam-beam interaction. The first transformation is given by

$$\begin{aligned} e^{-\phi:p_x z:}x &= x-\phi[p_x z,x] = x+\phi z \\ e^{-\phi:p_x z:}\delta &= \delta-\phi[p_x z,\delta] = \delta-\phi p_x \end{aligned} \qquad (3)$$

The residual of the first and third transformations gives the transformation for δ in Eq.(1). This expression, which is called Lie operator expression, is presented in [3]. Note the operator order; $\circ$ denotes the multiplication of transformations, which is inverse order of Lie operator multiplication.

Both beams are transferred by the same transformation. The term ϕz_c appears from $2s\phi$ in Eq.(1). This transformation is actually equivalent to the appearance of z dependent dispersion (z_x) at the collision point: i.e., the revolution matrix including the crossing transformation is expressed by

$$M = e^{\phi p_x z} \circ M_0 \circ e^{-\phi p_x z} \qquad (4)$$

where M_0 is the revolution matrix of the lattice. Now the beam envelope matrix has a finite element of $\langle xz\rangle = \zeta_x\sigma_z = \phi\sigma_z$ [4], for the weak limit of the beam-beam interaction. The collision is now regarded as head-on collision with tilt beams in x-z plane as shown in Figure 2. Electro-magnetic field is the perpendicular to the moving direction now.

CRAB CROSSING AND TRAVEL FOCUSING

The crab crossing [1,6] is basically meaningful for the short bunch scheme. A transformation, which is equivalent to the crossing angle, is applied before and after the collision,

$$e^{-\phi p_x z} \circ e^{\phi p_x z} \circ e^{-:H_{bb}:} \circ e^{-\phi p_x z} \circ e^{\phi p_x z} = e^{-:H_{bb}:}, \qquad (5)$$

thus the effective transformation is the same as that for the head-on collision. To realize the transformation, crab cavities, which gives the transformation, $e^{-V':xz:/E_0}$, are placed at locations where linear transformation T_A is satisfied to,

$$e^{\phi p_x z} = T_A \circ e^{-V':xz:/E_0} \circ T_A^{-1} = e^{-(V'/E):T \circ xz:}, \qquad (6)$$

where

$$T_A \circ x = \sqrt{\frac{\beta_x^*}{\beta_{x,c}}}\,x\cos\varphi_x + \sqrt{\beta_x^*\beta_{x,c}}\,p_x\sin\varphi_x \qquad (7)$$

φ_x is the horizontal betatron phase difference between the collision point and crab cavity position, and β_x^* and $\beta_{x,c}$ are horizontal beta functions at the collision point and crab cavity position.

The well-known formula for crab angle and voltage is given by choosing the betatron phase difference of p/2.

$$\phi = \frac{\omega_{crab}V}{cE_0}\sqrt{\beta_{x,c}\beta_x^*} \qquad (8)$$

Only one crab cavity can be possible to realize the transformation

$$e^{\phi p_x z} \circ M_0 \circ e^{-\phi p_x z} = T_B \circ e^{-V':xz:/E_0} \circ T_B^{-1} \circ M_0 \qquad (9)$$

Basically this procedure is really 6x6 optics matching for the dispersion z_x.

Beam particles with z collide with the center of another beam at s=z/2 in the travel focus scheme [8]. The particles with z should have the waist position at s=z/2 to minimize the beam-beam effect. The transformation exp($-p_y^2 z/4$) realizes the travel focusing:

$$e^{zp_y^2/4} \circ e^{-:H_{bb}:} \circ e^{-zp_y^2/4} \qquad (16)$$

RF focusing is used for the transformation. However heavy development works are necessary for the RF device. We know the crab cavity exchanges x and z.

$$e^{-\phi p_x z}\,\mathrm{O}e^{xp_y^2/4\phi}\,\mathrm{O}e^{\phi p_x z}\,\mathrm{O}e^{-:H_{bb}:}\,\mathrm{O}e^{-\phi p_x z}\,\mathrm{O}e^{-xp_y^2/4\phi}\,\mathrm{O}e^{\phi p_x z}$$
$$= e^{(x-\phi z)p_y^2/4\phi}\,\mathrm{O}e^{-:H_{bb}:}\,\mathrm{O}e^{-(x-\phi z)p_y^2/4\phi} \quad (17)$$

The first and last operator exp(+-$\phi p_x z$) at the first line of Eq.(17) are actions of the crab cavities, while 3rd and 5th are the crossing angle. The 2nd and 6th operators are from two sextupole magnets located at the both sides of the collision point. Additional two sextupole magnets in both sides are added to cancel the residual nonlinear term [9].

$$e^{xp_y^2/4\phi}\,\mathrm{O}e^{-(x-\phi z)p_y^2/4\phi}\,\mathrm{O}e^{-:H_{bb}:}\,\mathrm{O}e^{-(x-\phi z)p_y^2/4\phi}\,\mathrm{O}e^{xp_y^2/4\phi}$$
$$= e^{zp_y^2/4}\,\mathrm{O}e^{-:H_{bb}:}\,\mathrm{O}e^{-zp_y^2/4} \quad (18)$$

Realistic arrangement of IR is given by chosen betatron phase so as to realize the transformation as is done in Eq.(7). Two pairs of crab cavities, which are inserted between two sextupole magnets, are located at the horizontal betatron phase difference of $(1/2+n)\pi$. The sextupole magnets are located at the vertical betatron phase difference of $(1/2+n)\pi$. The phase difference of two sextupole magnets is π or 2π depending on the sign of magnets. In this scheme two crab cavities are necessary.

In the travel focus scheme, the waist position shifts for z but does not for s: that is, particles at z have waist position for the variation of s, and the waist position is located at the centre (z=0) of the colliding beam. The hourglass effect is not avoidable even in the travel waist scheme.

The beam-beam interaction is expressed by

$$\mathbf{x}_{\pm}(+0) = S\exp\left[-:\int_{-IR}^{IR} V_0^{-1}(s_i)\varphi(\mathbf{x}_{m}, s_i)V_0(s_i)ds_i:\right]\mathbf{x}_{\pm}(-0) \quad (19)$$

where s_i, which is , is a function of the longitudinal position (z) of the particle, $s_i(z)=(z-z_i)/2$. V0 is the transformation for drift or solenoid,

$$V_0(s) = S\exp\left[-:\int_0^s \frac{p_x^2+p_y^2}{2}ds_i:\right] \quad (20)$$

Note the upper bound of the integration is a function of z now.

$$e^{-:p_y^2 z:}S\exp\left[-:\int_{-IR}^{IR} V_0^{-1}(s_i)\varphi(\mathbf{x}_{m}, s_i)V_0(s_i)ds_i:\right]e^{:p_y^2 z:}$$
$$\approx S\exp\left[-:\int_{-IR}^{IR} V_0^{-1}(z_i/2)\varphi(\mathbf{x}_{m}, z_i/2)V_0(z_i/2)ds_i:\right] \quad (21)$$

The transformation does not include z. The degree of freedom for the beam-beam interaction is reduced to two from three.

BEAM-BEAM INTERACTION FOR FLAT BEAM

For flat beam, the horizontal beam-beam force does not depend on y, but the vertical force strongly depends on x as shown in Figure 1. The horizontal motion can be regarded as one-dimensional motion. Since the horizontal force is time dependent: i.e., two degree of freedom, the horizontal motion cannot be solved. When the horizontal tune is very closed to an integer or a half integer, the horizontal motion is near solvable: that is, the transformation of the horizontal coordinates is expressed by

$$x_{n+1} = \left(1-\frac{\mu_x^2}{2}\right)x_n + \beta_x\mu_x p_{x,n}$$
$$p_{x,n+1} = -\mu_x x_n + \left(1-\frac{\mu_x^2}{2}\right)p_{x,n} - F_x(x_{n+1}, y_{n+1}) \quad (22)$$
$$F_x(x,y) \approx F_x(x,0)(1+y^2/\sigma_x\sigma_y)$$

where $\mu_x=2\pi(\nu_x-0.5)$, $x_n=(-1)^n x(s=nL)$. The horizontal motion is equivalent to that in potential

$$U(x) \approx \frac{1}{\mu_x}\int_a^x F_x(x',0)dx' \quad (23)$$

The potential increase for a large μ_x, the horizontal motion is near solvable as shown in Figure 2.

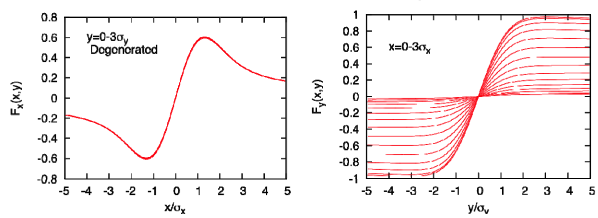

Figure 1: The beam-beam force for a flat beam with aspect ratio of 0.01.

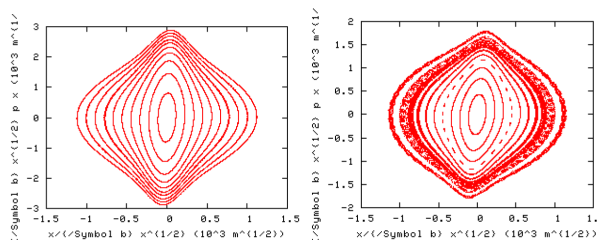

Figure 2: The Horizontal phase space plot (x-p_x) for ν_x =0.505 and 0.52, where vertical oscillation amplitude is y=2σ_x and ν_y=0.58.

The vertical transformation is expressed by

$$y_{n+1} = \cos\mu_y y_n + \beta_y \sin\mu_y p_{x,n}$$
$$p_{y,n+1} = -\frac{1}{\beta_y}\sin\mu_y y_n + \cos\mu_y p_{y,n} - F_y(x_{n+1}, y_{n+1}) \quad (24)$$

The beam particles experience vertical beam-beam force with the modulation due to the horizontal motion, which is regarded as an external function. The horizontal motion is stochastic for operating point far from half integer tune. The vertical force is

$$F_y(\bar{x}+x_r, y) = F_y(\bar{x}, y) + \frac{1}{2}\frac{\partial^2 F_y}{\partial x^2}\bigg|_{x=\bar{x}} x_r^2 + \ldots \quad (25)$$

where x_r is the stochastic term in the horizontal motion. The stochastic term gives vertical diffusion. Figure 3 shows the phase space plot in y- p_y–x space for ν_x =0.505 and 0.52. The phase space trajectories show near solvable and unsolvable ν_x =0.505 and 0.52, respectively.

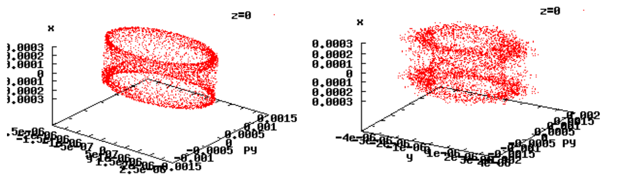

Figure 3: Horizontal amplitude (x) on y-p_y phase space for ν_x =0.505 and 0.52.

These characteristics reflect the beam-beam limit. Figure 4 shows the beam-beam parameter given by weak-strong (BBWS) and strong-strong (BBSS) simulations. The beam-beam parameter is calculated by the simulated luminosity with the formula,

$$\xi_{y,\pm} = \frac{2r_e\beta_y L_b}{N_\pm \gamma_\pm f_{rev}} . \tag{26}$$

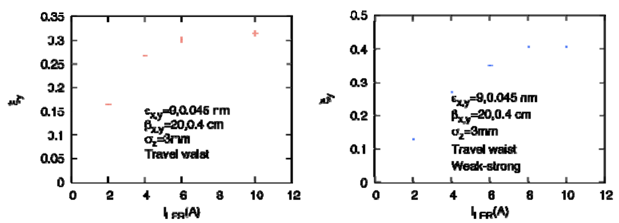

Figure 4: Beam-beam parameter given by weak-strong (BBWS) and strong-strong (BBSS) simulations.

A design candidate of Super KEKB is based on this high current and high beam-beam parameter scheme. There are many limitations for the design of Super KEKB. The coherent synchrotron radiation (CSR) limits the bunch length, especially in LER. Therefore we adopt negative momentum compaction to minimize the bunch lengthening and choose the bunch length of 5 mm and 3 mm for LER and HER, respectively.

Figure 5 shows the luminosity evolution for Super KEKB in the high current option calculated by the strong-strong simulation. The transverse damping time is 4000-6000 turns. The simulation predicted the luminosity $5.3\mathrm{x}10^{35}$ $\mathrm{cm}^{-2}\mathrm{s}^{-1}$.

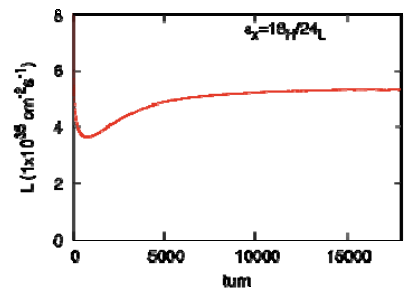

Figure 5: Luminosity evolution for Super KEKB in the high current option.

CONCLUSIONS

Table 1 shows the beam-beam parameter given by Eq.(26) in several colliders. There are no colliders to reach such high beam-beam parameter. The simulations show the beam-beam parameter is possible if machine is perfect. Do you believe it? How high is possible in our technique?

REFERENCES

[1] K. Ohmi and K. Oide, Phys. Rev. ST-AB 10 (2007) 014401.
[2] K. Ohmi, K. Oide, E. A. Perevedentsev, "Beam-beam limit and the degree of freedom", EPAC'06, Edinburgn, June 2006, MOPLS032, p. 616 (2006).
[3] K. Oide et al, in this proceedings, PAC'09, Vancouver, May 2009.

Table 1: Beam-beam parameter for several colliders in the world (except LEP).

* The tune shift enhancement due to the hourglass effect is not included.

	KEKB('06) No crab	KEKB Crab cav	PEP-II	DAFNE Crab waist	BEPC-II (Apr.09)	VEP2000 (2008)	CESR
L($\mathrm{cm}^{-2}\mathrm{s}^{-1}$)	$1.76\mathrm{x}10^{34}$	$1.93\mathrm{x}10^{34}$	$1.21\mathrm{x}10^{34}$	$4.5\mathrm{x}10^{32}$	$2.3\mathrm{x}10^{32}$	$1\mathrm{x}10^{31}$	$1.25\mathrm{x}10^{33}$
I+/- (A)	1.65/1.4	1.60/1.10	2.90/1.88	1.10/1.43	0.65/0.70	0.04	0.35
N_{bunch}	11	1584	1722	107	90	1	45
β_y (mm)	5.9	5.9	10	9.3	1.5	50	18
τ_y/T_0	4000	4000	6000	110000	32000	-	9000
Min($\xi_{y+/-}$)	0.0434	0.0598	0.033	0.0266	0.012	0.0565	0.0561