ACHROMATIC INTERACTION POINT DESIGN*

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Abstract

Designers of high-luminosity energy-frontier muon colliders must provide strong beam focusing in the interaction regions. However, the construction of a strong, aberration-free beam focus is difficult and space consuming, and long straight sections generate an off-site radiation problem due to muon decay neutrinos that interact as they leave the surface of the earth. Without some way to mitigate the neutrino radiation problem, the maximum c.m. energy of a muon collider will be limited to about 3.5 TeV. A new concept for achromatic low beta design is being developed, in which the interaction region telescope and optical correction elements, are installed in the bending arcs. The concept, formulated analytically, combines space economy, a preventative approach to compensation for aberrations, and a reduction of neutrino flux concentration. An analytical theory for the aberration-free, low beta, spatially compact insertion is being developed.

INTRODUCTION

Experiments with muon colliding beams at high energy require high luminosities of order 10^{34} cm^{-2} sec^{-1} or more, in order to obtain reasonable rates for events having parton type cross sections[1]. High luminosities require intense beams with small transverse emittance and strong, precise beam focusing at the interaction point (IP). Strong focusing at the IP requires a large space for the telescope optics as indicated in Figure 1. This leads to a significant increase of collider ring circumference, and thus a reduction of collider luminosity. More important, for very high energies, a long straight telescope section results in an intense high energy neutrino flux in the beam directions due to muon decay that produces off-site radiation [2]. This problem is significant for center of mass energies above about 3.5 TeV. The long straight sections, where the beams collide, are the most serious source of offsite radiation. In some designs, the neutrino flux from this section is spread out by steering the muon beam using slowly time-varying dipole fields introduced in the telescope section. This makes the IP design even more cumbersome and adds to the cost.

Another source of lengthening the interaction region is the chromatic compensation. Small beta is effective for short bunches, implying large momentum spread. This large momentum spread will introduce extra beam size at the focal point due to chromaticity. So, to focus the beam with finite energy spread at a small spot, the chromaticity at the collision point must be corrected. This problem drastically complicates high luminosity muon collider design. A solution of this problem was proposed earlier [3,4,5,6], by installing a chromatic compensating block (CCB) before the final focusing block (FFB). This method allows compensation for IP chromaticity while preserving the focusing strength of the FFB. This advance, however, has still not been implemented in muon collider designs as a standard element of the state of the art. A reason for this "inertia" seems to be that the CCB causes significant lengthening of the IR region, since the length of the CCB is as long as or somewhat longer than that of the FFB.

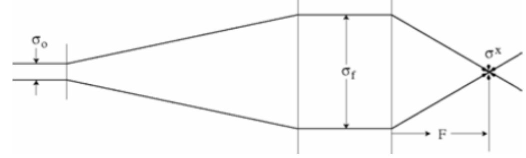

Figure 1: General schematic of the interaction region of a collider

Here, a new concept for achromatic low beta designs is being developed, in which the interaction region telescope and the optical correction elements are installed in the bending arcs. The concept combines space economy, compensation for aberrations, and a reduction of neutrino flux concentration. It includes the following advances.

1) The Beam Extension Sections (BES) and the bending arcs are designed together. This design can be performed by modifying a conventional focusing lattice interleaved with combined bending magnets. The extension method can be described as a strong parametric resonance in betatron motion, implemented along a number of periods in the arc. Such a 'zooming' lattice will be combined with periodic dispersion in a normal level assumed for arcs.
2) The CCB immediately follows the BES and uses non-alternating dipoles to drive dispersion in cooperation with quadrupoles. Being a part of the arc before the FFB, this block includes sextupoles to develop a momentum-angle correlation in the beam which will neutralize the chromatic aberration of the FFB to prevent chromatic spread at the collider focal point. With lattice symmetry features, the CCB will not generate second order effects.
3) A compact, short quadrupole focal block will be used to reach the minimum transverse beam size at the IP.

THE FINAL FOCUSING BLOCK

Table 1 shows representative design parameters of a low-emittance muon collider with a luminosity of 7.1034 cm^{-2} sec^{-1} at a center-of-mass energy of 5 TeV [7]. The tabulated beta function of 5 mm at the interaction point drives the choice of bunch length down to 3 mm to avoid the "hour glass" effect, resulting in a large beam energy spread of 0.3%.

Figure 2 illustrates the linear optics for the FFB with the parameters described above. It uses anti-symmetric triplet focusing, 8 meters away from the Interaction Point (IP), which leaves a reasonable space for the detector. That distance combines with very tight spot at the IP of

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$\beta^* = 5mm$. With the relation $\beta^* \beta_f = F^2$, where, β_f is the beta function at the entrance of FFB, it results in rather large values of $\beta_{\max} \sim 100$ km and consequently large beam sizes at the FF quads. Based on Figure 2, the dispersion [8] at the collider central point is designed to be exactly 0, while the dispersion prime is not necessary 0, and the FFB is an anti-symmetric structure. A combined function magnet with weak dipole field is used where the beta function is large.

Table 1: Low Emittance Muon Collider Parameters

Circumference	L	km	6
Beam Energy	E	TeV	2.5
Energy spread	$\Delta E / E$	%	0.3
Bunch length	σ_z	mm	3
Transverse emittance, norm.	$\varepsilon_{\perp n}$	µm	2
Longitudinal emittance, norm.	ε_{zn}	cm	40
Large beta	β_f	km	100
Focal parameter of IR	F	m	25
Beta-star	β^*	mm	5
Beam-beam tune shift	$\Delta\nu$		0.1
Average circulating current	<N$_c$>	s^{-1}	2.5x10^{16}
Average injected muon current	<N$_{in}$>	s^{-1}	2 x 10^{13}
Luminosity	L	cm^{-2}s^{-1}	7x 10^{34}

For a β^* of 5 mm as shown in the Table, the tolerance to focal point errors can be estimated as:

$$\frac{\Delta F}{F} << \frac{\beta^*}{F} \approx 2 \cdot 10^{-4} \text{ @ 2.5 TeV}$$

which seems not to present a serious challenge to designers. One may consider even smaller beta-star, if the effects of imperfections could be corrected. On the other hand, the achievable bunch length of about 3 mm limits the β^* to no less than 5 mm.

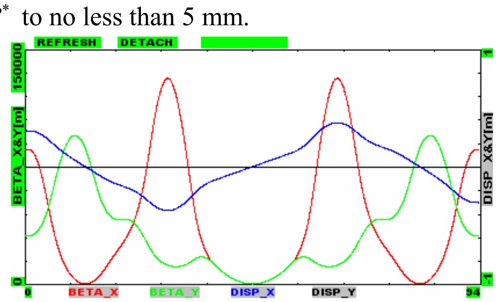

Figure 2: Optics of a final focusing block

The chromatic spread at the IP can be estimated as

$$\Delta s^* = \Delta F = \frac{\partial F}{\partial p}\Delta p = p\frac{\partial F}{\partial p} q; \qquad p\frac{\partial F}{\partial p} \geq F$$

The impact on beam transverse size at the IP is: $\delta x^* = -x'^* \Delta F$, and a tolerance criterion is then

$$\delta x^* << \sigma^* = \theta^* \beta^* \rightarrow \Delta F << \beta^*$$

An estimate of chromatic spread of the focus point gives:

$$\Delta F > F\frac{\Delta p}{p} = 25m \cdot 3.10^{-3} = 7.5cm$$

So the chromatic spread at the focal point presents a problem. Though designers have counted on the possibility to compensate this spread, no systematic approach to IP design has been yet developed.

CHROMATIC COMPENSATING BLOCK

To compensate the chromatic spread at the IP point, the design will be based on the following methodology: design a special compensating block of magnets, which is located before the FFB and after the beam extension section. This section has to be composed of a set of dipoles, quadrupoles, and sextupoles with certain symmetry, the function of which is to create in the entering parallel beam an angle spread negatively correlated with the chromatic kick of the FFB, so the chromatic effects at the collider point will be cancelled.

Specifically and briefly, the design requirements up to second order can be expressed analytically in terms of five rather nonlinear equations as follows [9]:

1) $2\int_0^* Dn_s x_0^2 ds = \int_0^* n x_0^2 ds$ 2) $2\int_0^* Dn_s y_0^2 ds = \int_0^* n y_0^2 ds$

3) $\int n_s x_0 y_0^2 ds = 0$ 4) $\int n_s x_0^3 ds = 0$ 5) $\int (n_s D - n) D x_0 ds = 0$

Here, $x_0(s)$, … are the horizontal and vertical betatron part of particle trajectory that becomes parallel before it enters the CCB. These equations are analytic expressions of requirements that designers of interaction regions face regularly. However, by imposing simple symmetry requirements relative to the middle of the CCB, namely,

$$n(s) = n(-s), \; y^2(s) = y^2(-s), \; D(s) = \pm D(-s),$$

$$x_0(s) = \mp x_0(-s), \text{ and } n_s(s) = \pm n_s(-s),$$

the five equations can be reduced to three, 1), 2) and 5). Condition 5) is still a constraint due to contributions of second order dispersion outside the CCB. But, there is a simple way to compensate for it, introducing 180 degree phase space rotation in the horizontal plane before the CCB. Then, five conditions are finally reduced to only two to be satisfied. This can be achieved by using the difference in behaviour of the beta-function between the horizontal and vertical planes.

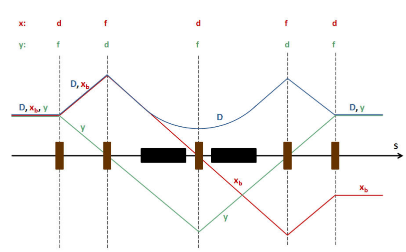

Figure 3: A CCB schematics with an even symmetry dispersion.

While creating the chromatic kick, the CCB does not produce non-linear aberrations due to second order dispersion effects of transverse emittances in sextupole fields. All other terms are automatically cancelled due to the symmetry of the lattice and the focusing and dispersion design inside the CCB. A preliminary conceptual drawing of a CCB with even symmetric type of dispersion and sextupoles but odd symmetry in horizontal betatron trajectory $x_0(s)$ is shown in Figure 3.

Figure 4 illustrates a prototype of the CCB lattice's beta function, dispersion and beam trajectory. It includes a continuous bending magnet and combined function magnet, with symmetric dispersion. This CCB is part of the arc. Sextupoles in pairs are located in high dispersion regions to compensate the chromaticity effect in the collider point. The FFB follows this CCB.

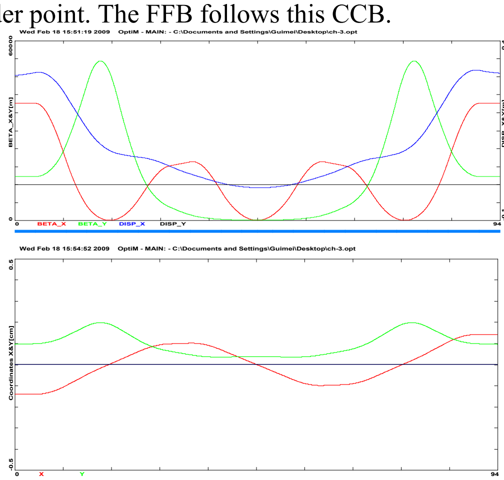

Figure 4: CCB with symmetric dispersion pattern. Upper: dispersion and beta functions. Lower: betatron part of beam trajectory.

BEAM EXTENSION SECTION

The BES design proposed here will be performed along the arcs. The large momentum spread of the muon beam constrains such a design by demanding that one has to eliminate the dispersion function growth along with beam size. Here, the special condition of a periodic zooming focusing lattice can prevent the dispersion function enlargement, making it periodic similar to the case of stable focusing. Apparently, one can design a single cell of a periodic lattice including bend in a way that the betatron part grows in each cell while the dispersion part repeats periodically. It remains to match the dispersion function between the end of a regular lattice of the arc and beginning of the extension section of the arc. In this way, the dispersion and related beam size due to momentum spread will be maintained at a "normal" level as designed for the arcs. The zooming rate should be modest in order to avoid too large dispersion beat in a single cell.

The periodic cell in x/ y plane can be described as

$$\begin{pmatrix} x_f \\ x_f' \end{pmatrix} = \begin{pmatrix} \lambda_i & M_{12} \\ 0 & 1/\lambda_i \end{pmatrix} \begin{pmatrix} x_i \\ x_i' \end{pmatrix}$$

Where $|\lambda_i|$ is larger than 1, but moderates to avoid large dispersion beating in one cell.

Figure 5 illustrates the BES optics. Each period is composed of four combined function magnets, with continuous bend. The total length of BES is 288m. One can see that the beta function in the x and y planes is gradually amplified from several ten meters to about 100 km, while the dispersion is stable along the BES.

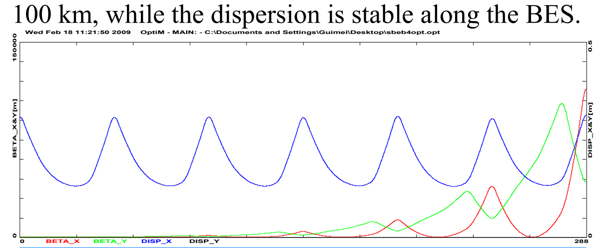

Figure 5: Beam Extension Optics

CONCLUSION

In this work, we have demonstrated the possibility of a bent achromatic IP design for a muon collider. A preventive approach to compensation for IP chromaticity allows one to preserve the focusing strength of the final quadrupole triplet despite no-aberration IP design constraints. We also have shown that beam extension and compensation sections can be combined with the continuous bend of the arcs, such that the neutrino flux is spread out and the collider circumference is reduced.

The most attractive version of an IR for a muon collider would be a continuous-bend IR, including the final focusing section. One possible approach would be to use combined function magnets everywhere (all magnets with dipole component) in order to maximally reduce the brilliance of the neutrino flux from muon decay.

The methodology based on an explicit analytic method is being developed. Studies of high order optics, dynamic aperture, momentum acceptance, and error sensitivity are being made to compare to previous designs.

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