

## REVERSE EMITTANCE EXCHANGE FOR MUON COLLIDERS\*

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### Abstract

Muon collider luminosity depends on the number of muons in the storage ring and on the transverse size of the beams in collision. Ionization cooling as it is currently envisioned will not cool the beam sizes sufficiently well to provide adequate luminosity without large muon intensities. Six-dimensional cooling schemes will reduce the longitudinal emittance of a muon beam so that smaller high frequency RF cavities can be used for later stages of cooling and for acceleration. However, the bunch length at collision energy is then shorter than needed to match the interaction region beta function. New ideas to shrink transverse beam dimensions by lengthening each bunch will help achieve high luminosity in muon colliders. Analytic expressions for the reverse emittance exchange mechanism were derived, including a new resonant method of beam focusing.

### INTRODUCTION

A Conceptual Picture of a Muon Collider (MC) Complex is presented in Figure 1. It includes an intense H<sup>-</sup> source, 8 GeV proton accumulator, target, muon cooling systems, 30 GeV coalescing ring, ILC-type linac with recirculating arcs, and a 1.5 TeV low-emittance muon collider ring.

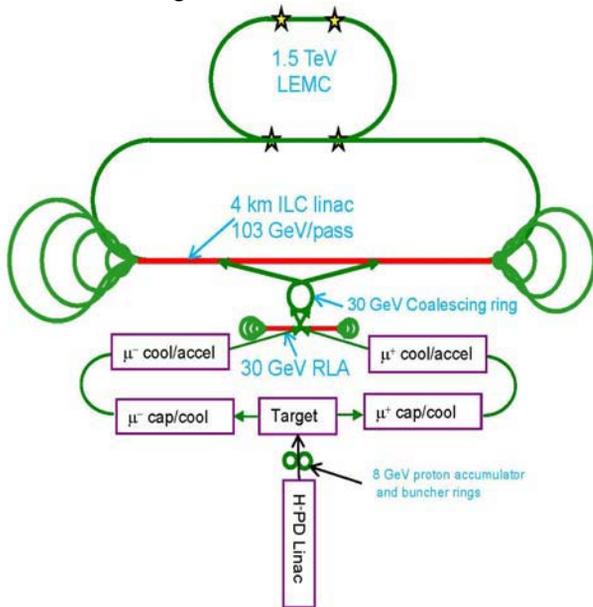


Figure 1: MC with high energy bunch coalescing. After initial 6D cooling in helical cooling channels, PIC and REMEX reduce transverse emittances before coalescing.

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The muon beam cooling scheme shown in Figure 2 is one of the most difficult part of the entire project. The use of a continuous absorber as provided by a gas-filled RF system implies an idea to provide a natural, very effective means of achieving emittance exchange and true six-dimensional (6D) cooling. Namely, if the superimposed magnetic field provides dispersion down the beam channel such that higher momentum corresponds to longer path length and larger ionization energy loss, the momentum spread can be reduced. Recent simulations of cooling channels using a Helical Cooling Channel (HCC) of superimposed helical dipole, helical quadrupole, and solenoid fields show a 6D emittance reduction of factor of 50,000 in a channel only 150 meters long. That cooling factor is very much larger than other cooling channels of comparable length.

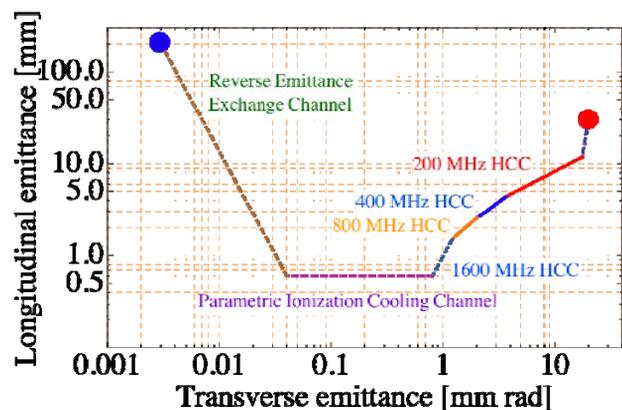


Figure 2: Fernow-Neuffer Plot of emittance evolution needed for a Low Emittance Muon Collider. The solid lines represent G4beamline simulations of Helical ionization Cooling Channels (HCC) using realistic magnetic fields based on helical solenoid (HS) magnets, but hydrogen-pressurized RF cavities that are yet to be engineered.

### PARAMETRIC-RESONANCE IONIZATION COOLING

The PIC concept shown in Figure 3 is to excite a half-integer parametric resonance in a beam line or ring to cause the usual elliptical motion on a phase-space diagram to become hyperbolic, much as is used in half-integer extraction from a synchrotron. This causes the beam to stream outward to large  $x'$  and/or  $y'$  while the spatial dimensions  $x$  and/or  $y$  shrink.

Ionization cooling is then employed to constrain or shrink the  $x'$  dimension. Thus the beam size is reduced by the action of the resonance and the angular divergence is reduced by ionization cooling. An essential aspect of this method of cooling, which we anticipate can reduce each transverse dimension another factor of 10, is the control of the aberrations that cause detuning and loss of the resonance condition.

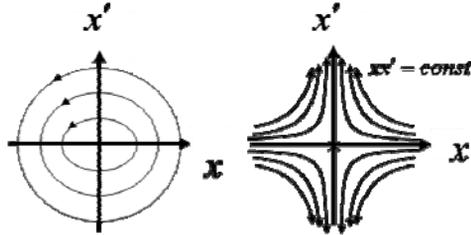


Figure 3: Comparison of particle motion at periodic locations along the beam trajectory in transverse phase space for: LEFT ordinary oscillations and RIGHT hyperbolic motion induced by perturbations at a harmonic of the betatron frequency.

We assume initially that the tune spread for a beam in a focusing channel is zero, leaving tuning demands and compensation for aberrations to be discussed later. Figure 4 illustrates an arrangement for resonance beam focusing along a cooling channel. Weak lenses installed every half oscillation period drive a half-integer parametric resonance that creates a hyperbolic beam evolution at the absorber plates.

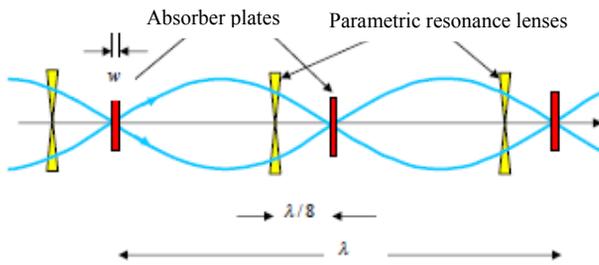


Figure 4: Conceptual diagram of a beam cooling channel in which hyperbolic trajectories are generated in transverse phase space by perturbing the beam at the betatron frequency, a parameter of the beam oscillatory behavior. Neither the focusing magnets that generate the betatron oscillations nor the RF cavities that replace the energy lost in the absorbers are shown in the diagram.

As the resonance instability evolves, the amplitudes grow while the phase distribution shrinks, so the beam size  $\sigma$  and angle spread  $\theta$  alternate with 90 degrees relative phase shift. At points where the size is minimal (focused at the absorber plates), the angle spread is maximal, and vice versa—in accordance with Liouville's theorem. There are no correlations in the  $(x, x')$  distribution at these points, so that the normalized emittance can be defined as the product  $\beta\gamma\sigma\theta = \epsilon_{\perp}$  at the plates, where  $\beta\gamma = p/mc$  are Lorentz factors. By using ionization cooling to impose a damping force on transverse velocities in the absorber with decrement

$\Lambda_c = 2\Lambda_d$ , we obtain damping for both the angle spread and the beam size at the plate.

Due to scattering in the absorber plates, the angle spread at the plates and oscillation amplitudes evolve to a conventional equilibrium regardless of the thickness of the plates. However, the diffusion of phases and particle transverse positions at the plates is suppressed drastically due to the small width of the plates with respect to the focal parameter  $\lambda/2\pi$ :  $\delta x = -s\delta x'$ , where  $-w \leq 2s \leq w$ .

### REVERSE EMITTANCE EXCHANGE

The key feature of 6D phase space cooling is emittance exchange. This process takes place by manipulating the path of a particle as a function of its momentum. With the proper dispersion, a higher (lower) momentum particle traverses a longer (shorter) path length in a cooling absorber with respect to the reference particle.

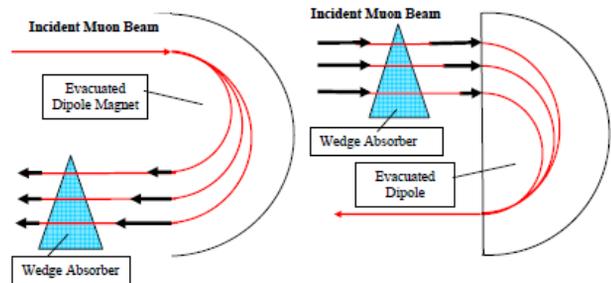


Figure 5: Use of a wedge absorber for direct (left) and reverse (right) emittance exchange.

The left side of Figure 5 shows the usual mechanism for reducing the energy spread in a muon beam by emittance exchange. An incident beam with small transverse emittance but large momentum spread (indicated by black arrows) enters a dipole magnetic field. The dispersion of the beam generated by the dipole magnet creates a momentum-position correlation at a wedge-shaped absorber. Higher momentum particles pass through the thicker part of the wedge and suffer greater ionization energy loss. Thus the beam becomes more monoenergetic. The transverse emittance has increased while the longitudinal emittance has diminished. For the new mechanism of the transverse emittance reducing the incident beam with large transverse emittance but small momentum spread passes through a wedge absorber creating a momentum-position correlation at the entrance to a dipole field. The trajectories of the particles through the field can then be brought to a parallel focus at the exit of the magnet. Thus the transverse emittance has decreased while the longitudinal emittance has increased.

### COMPENSATION FOR ABERRATIONS

A principal challenge of the PIC and REMEX designs is to have all particles reach the minimum radial position at the absorber positions along the beam path. This synchronism is violated by the spread of betatron oscillation tunes, which is still quite large even after basic 6D cooling. In a linear focusing field, there are two fundamental mechanisms of optical aberration:

1) chromatic aberration due to tune dependence on particle energy and 2) spherical aberration due to particle path dependence on the square of transverse momentum. Compensation for both destructive factors is necessary in order to realize the maximum cooling effect. Compensation for chromaticity requires introduction of a large dispersion together with sextupole magnetic fields. Large dispersion, however, is detrimental to PIC and REMEX because of the energy straggling impact on transverse emittance discussed above. A resolution of this difficulty is to design the dispersion function with a period  $\frac{1}{2}$  of the betatron wavelength by applying alternating bends as shown in Figure 6.

Compensation for spherical aberrations also seems achievable in this type of beam transport using an appropriate modulation of sextupole field along the beam path. An optimized design for resonance cooling requires comprehensive analytical and simulation studies of these possibilities which are under way.

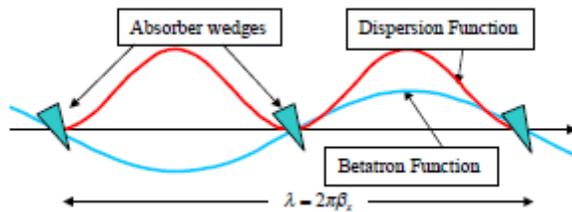


Figure 6: Schematic of an achromatic wiggler, where wedge absorbers are placed at symmetric locations relative to the dispersion function, which has a period half that of the betatron function.

### MUON BUNCH COALESCING

Recombining muon bunches at high energy has three important advantages:

- 1) The charge/bunch can be low for cooling and low energy acceleration for a collider. This is needed for credible Linac operation and for the reduction of space charge effects for deep cooling (as in PIC).
- 2) Time dilation means fewer muons will be lost from decay since accumulation is done at high energy, and
- 3) The muon collider R & D can be a compatible continuation of neutrino factory efforts.

Once you know how to recombine bunches, the problem is to use that ability in a sensible choreography for collider operation. One interesting point is that you must cool transversely before the beam will fit into high frequency RF cavities. In the scheme described below, we cool transversely to the point that the beam fits into 1.3 GHz RF cavities, and then capture the beam into a short train of 16 bunches. In this scheme we use a special ring to recombine or coalesce short bunch trains into single bunches, where the transverse emittance of the cooled muons is preserved and the beam is stacked in momentum space. That there is momentum space to use comes from scaling the  $2\% \Delta p / p$  from the  $\sim 200$  MeV/c cooling energy to the multi TeV/c of the collider.

### NEXT STEPS

A new concept for beam transport is being developed for PIC, called Epicyclic Parametric Ionization Cooling (EPIC) [2], which is being examined with the G4beamline code [3]. We believe that the control of aberrations that is possible with this channel will be very useful for the design of REMEX beam lines.

Recent progress with the EPIC concept, both analytical and in numerical simulations is presented elsewhere in this conference.

Numerical simulations for REMEX using the EPIC beam concept are just beginning.

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