

# CONDITIONS ON THE GRAZING FUNCTION $g$ FOR EFFICIENT COLLIMATION

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## Abstract

The grazing function  $g$  is introduced – a synchrotron optical quantity that parametrizes the rate of change of total angle with respect to synchrotron amplitude for particles grazing a collimator or aperture. The grazing function is particularly important for crystal collimators, which have limited acceptance angles. The implications for RHIC, SPS, Tevatron and LHC crystal implementations are discussed. An analytic approximation is derived for the maximum value of  $g$  in a matched FODO cell, and is shown to be in good agreement with a realistic numerical example. The grazing function scales linearly with FODO cell bend angle, but is independent of FODO cell length.

## INTRODUCTION

The total horizontal displacement  $x_T$  of a particle as it passes a collimator is the sum of its betatron ( $x_T$ ) and synchrotron ( $x_s$ ) displacements, where the betatron displacement and angle oscillate according to

$$x_\beta = a_x \sin(\phi_x) \quad (1)$$

$$x'_\beta = \frac{a_x}{\beta} (\cos(\phi_x) - \alpha \sin(\phi_x)) \quad (2)$$

Here  $\beta$  and  $\alpha$  are horizontal Twiss functions at the collimator,  $a_x$  is the betatron amplitude, and the betatron phase advances with turn number  $t$  according to

$$\phi_x(t) = 2\pi Q_x t + \phi_{x0}, \quad (3)$$

Similarly, the synchrotron displacement and angle are

$$x_s = \eta \delta = a_s \sin(2\pi Q_s t + \phi_{s0}) \quad (4)$$

$$x'_s = \eta' \delta = a_s \sin(2\pi Q_s t + \phi_{s0}) \quad (5)$$

where  $\delta = \Delta p/p$  is the relative momentum offset, which performs synchrotron oscillations according to the synchrotron tune  $Q_s$ . The variables  $\eta$  and  $\eta'$  (dispersion and dispersion-prime) are optical quantities at the collimator, complementing  $\beta$  and  $\alpha$ . Only one of these four,  $\beta$ , is positive-definite. The total angle  $x'_T$  of a particle is thus written in general as

$$x'_T = \frac{a_x}{\beta} [\cos(\phi_x) - \alpha \sin(\phi_x)] + \eta' a_s \sin(\phi_s) \quad (6)$$

A *grazing particle* is one that only just touches the edge of a collimator displaced by  $x_c$  when its betatron and synchrotron displacements are simultaneously in time at their extrema – either maxima or minima – such that

$$a_x + |\eta| a_s = |x_c| \quad (7)$$

Simultaneous betatron and synchrotron oscillation extrema are achieved on turn number  $t$  when the phases are

$$\phi_x(t) = \text{sgn}(x_c) \pi/2 \quad (8)$$

$$\phi_s(t) = \text{sgn}(x_c) \text{sgn}(\eta) \pi/2 \quad (9)$$

where the possibilities of negative displacement  $x_c$  and negative dispersion  $\eta$  are explicitly taken into account.

The *grazing angle* – the total angle of a grazing particle – is found by substituting these phases into Eqn. 6 and by using Eqn. 7 to eliminate  $a_x$ . It is

$$x'_G = -\frac{\alpha}{\beta} x_c + \text{sgn}(x_c) \text{sgn}(\eta) \left( \frac{\alpha}{\beta} \eta + \eta' \right) a_s \quad (10)$$

Thus the grazing angle depends linearly on the synchrotron amplitude  $a_s$  and the linear slope of grazing angle with respect to synchrotron amplitude is

$$\frac{dx'_G}{da_s} = \text{sgn}(x_c) \text{sgn}(\eta) g \quad (11)$$

The dimensionless optical *grazing function*  $g$  is

$$g \equiv \left( \frac{\alpha}{\beta} \eta + \eta' \right) \quad (12)$$

Inspection confirms  $g$  to be the slope of the normalized dispersion  $\eta_N = \eta/\sqrt{\beta}$  scaled by the square root of  $\beta$

$$g = \sqrt{\beta} \eta'_N \quad (13)$$

Any linear dependence of the grazing angle on the synchrotron amplitude may cause particles with some synchrotron amplitudes to fall outside the angular acceptance of the collimator. The rigorous synchrotron condition for constant grazing angle is

$$g = \sqrt{\beta} \eta'_N = \frac{\alpha}{\beta} \eta + \eta' = 0 \quad (14)$$

This is a condition on the optics, independent of the emittance and the energy spread of the beam. Since  $\beta$  is positive-definite, a collimator is ideally placed at a location where normalized dispersion is at a local maximum or minimum ( $\eta'_N = 0$ ). This condition has already been noted in the literature [1, 2, 3, 4]. Two particular trivial solutions are immediately obvious:

1.  $\eta = \eta' = 0$ : anywhere in a dispersion-free straight.
2.  $\alpha = \eta' = 0$ : e.g. in the middle of a quadrupole at the boundary of a matched half-cell.

The following sections of this paper go beyond previous work by recognizing that the behavior of  $g$  is worth studying in its own right.

## GRAZING FUNCTION IN A FODO CELL

Consider a half-cell of length  $L$  with a quadrupole at each end, enclosing one or more dipoles. If this half-cell is matched ( $b' = \eta' = 0$  at both ends) then  $g$  is zero at both ends, and it is close to zero within the quadrupoles, as illustrated in Fig. 1. However,  $\eta'_N$  and (hence)  $g$  are non-zero within the half-cell, since the normalized dispersion  $\eta_N$  is not exactly the same at both ends. A reasonable approximation is that  $\eta'_N$  evolves quadratically with the azimuthal co-ordinate  $s$  according to

$$\eta'_N(s) \approx \frac{6 \Delta\eta_N}{L^3} s(s-L) \quad (15)$$

where  $\Delta\eta_N$  is the total change from end to end. The extreme values of  $\eta'_N$  and hence  $g$  are therefore expected near the middle of the half-cell, at  $s = L/2$ , so that

$$|g|_{max} \approx \sqrt{\beta_{mid}} \frac{3|\Delta\eta_N|}{2L} \quad (16)$$

The accuracy of this approximation depends on the detailed layout of the dipoles within the half-cell.

A case of particular interest is a matched FODO half-cell containing one dipole with a total bend angle of  $\theta$ , filling the half-cell. Substituting the standard thin-lens approximation for the optical functions in a matched FODO cell [5] into Eqn. 16, we get:

$$|g|_{max} \approx \theta \frac{3}{4\sqrt{2}} \frac{\sqrt{1+C^2}}{S^2C} \cdot \left( (2-S)\sqrt{1+S} - (2+S)\sqrt{1-S} \right)$$

where  $S = \sin(\phi/2)$ ,  $C = \cos(\phi/2)$ , and  $\phi/2$  is the phase advance per half cell. In the case at hand when  $\phi = 90$  degrees  $|g|_{max}$  is predicted to be

$$|g|_{max} \approx 0.39 \theta \quad (17)$$

with no dependence on the half-cell length  $L$ .

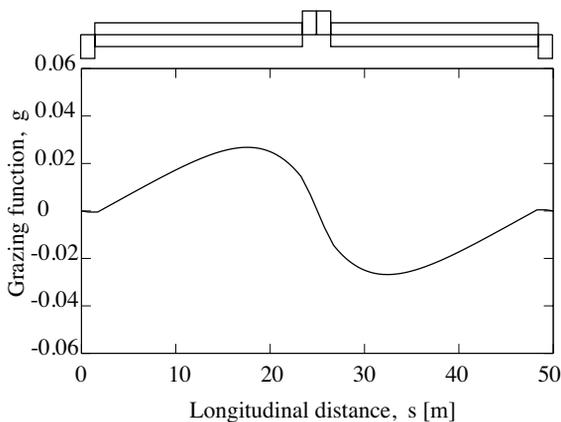


Figure 1: The grazing function in a matched FODO cell with thick quads and filled with dipoles.

Figure 1 shows that when  $L = 25$  m and  $\theta = \pi/50$ , then the maximum value occurs close to the mid half-cell, with a value of  $|g|_{max} = 0.0268$  that is reasonably close to the value of 0.0248 that is predicted by Eqn. 17. Numerical testing confirms the prediction of Eqn. 17 that the maximum value of the grazing function scales like

$$g_{max} \approx 0.427 L^0 \theta^1 \quad (18)$$

when the phase advance per full-cell is 90 degrees. These results apply *only* to a matched FODO cell. The grazing function can become much larger in absolute magnitude when an unmatched dispersion or betatron wave is present, and in non-FODO locations.

## THE GRAZING FUNCTION IN RHIC, SPS, TEVATRON AND LHC

Table 1 shows that primary collimators in four hadron colliders – RHIC, SPS, Tevatron and LHC – have grazing functions with an order of magnitude of 0.003, either positive or negative [6, 7, 8, 9, 10, 11]. The rigorous condition  $g = 0$  has not been attained in these implementations of amorphous and crystal primary collimators. Inspection of the pairs of  $\eta'$  and  $g$  values in Tab. 1 shows that there is a systematically strong cancellation between the two terms  $(\alpha/\beta)\eta$  and  $\eta'$  that comprise  $g$  in Eqn. 12, reflecting the tendency for the normalized dispersion  $\eta_N$  to remain approximately constant in well matched optics, so that  $\eta'_N$  and hence  $g$  are small. How small is small enough for the absolute value of the grazing function? How significant are the non-zero  $g$ -values in Tab. 1?

### A Relaxed Condition for Crystal Collimators

A discussion of implementation-specific details of multiple collimation systems is beyond the scope of this paper. Nonetheless, a general discussion of the “acceptance angle”  $\sigma'_A$  for protons incident on a crystal primary collimator is possible. Even though  $\sigma'_A$  depends strongly on crystal material, geometry and beam energy, a rule of thumb the crystal acceptance angle in channeling mode for Si crystals is

$$\sigma'_A [\mu\text{rad}] \sim 4 E^{-1/2} \quad (19)$$

The crystal acceptance angle can be compared with the grazing angle spread from the center to the edge of the RF bucket (from synchrotron amplitude  $a_s = 0$  to  $a_s = a_{Bucket}$ ). The grazing angle spread across the bucket

$$\Delta x'_{TB} = |g| a_{Bucket} \quad (20)$$

is a natural scale that is especially relevant if a collimator is being used to intercept beam escaping from the RF bucket. Uncaptured beam is a major concern for the Tevatron and the LHC, because such beam migrates into the abort gap and can quench superconducting magnets – or even do irreversible damage – during an emergency abort [12]. The grazing angle spread across the bucket is recorded in the

Table 1: Nominal optics, grazing functions, and other values at primary collimators in four accelerators. The collimator type is “A” for amorphous or “C” for crystal. The last column records the grazing angle spread across the RF bucket.

	Type	$\alpha$	$\beta$ [m]	$\eta$ [m]	$\eta'$ [ $10^{-3}$ ]	$g$ [ $10^{-3}$ ]	$E$ [TeV]	$a_{Bucket}$ [ $10^{-3}$ ]	$\sigma_p/p$ [ $10^{-3}$ ]	$\sigma_x$ [mm]	$\Delta x'_{TB}$ [ $\mu\text{rad}$ ]
RHIC	C	-26.5	1155.0	-0.864	-16.2	3.6	0.10	1.50	0.50	7.36	5.40
SPS (UA9)	C	-2.21	96.1	-0.880	-19.0	1.2	0.12	1.10	0.40	1.06	1.32
Tevatron (T-980)	C	-0.425	67.5	1.925	15.0	2.9	0.98	0.45	0.14	0.55	1.31
LHC (IR3)	A	1.72	131.2	2.100	-30.1	-2.5	0.45	0.97	0.31	1.01	2.43
LHC (IR7)	A	2.06	152.0	0.36	-5.6	-0.7	7.0	0.35	0.11	0.26	0.88
LHC (IR7)	A	2.06	152.0	0.36	-5.6	-0.7	0.45	0.97	0.31	1.09	0.68
LHC (IR7)	A	2.06	152.0	0.36	-5.6	-0.7	7.0	0.35	0.11	0.28	0.25
LHC (crystal)	C	1.93	136.1	0.341	-5.6	-0.8	0.45	0.97	0.31	1.03	0.78
LHC (crystal)	C	1.93	136.1	0.341	-5.6	-0.8	7.0	0.35	0.11	0.26	0.28

last column of Tab. 1. In general (avoiding details) it is desirable for this spread to be much less than the collimator acceptance angle  $\sigma'_A$ . Thus the relaxed condition on the grazing function for efficient collimation is

$$|g| \ll \frac{\sigma'_A}{a_{Bucket}} \quad (21)$$

### How Significant are Actual $g$ -Values?

Figure 2 shows how the grazing angle spread across the RF bucket  $\Delta x'_{TB}$  compares with the (approximate) channeling acceptance angle  $\sigma'_A$  given in Eqn. 19, across two orders of magnitude in beam energy  $E$ . Both amorphous and

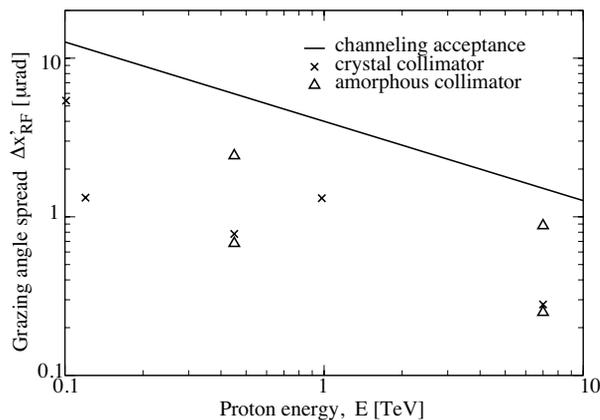


Figure 2: Variation of the grazing angle spread across the RF bucket as a function of energy for the collimators listed in Tab. 1. The solid line is the channeling acceptance.

crystal primary collimator locations are shown. In all cases the grazing function values  $g$  lead to total angular spreads that are “safe” by about half an order of magnitude. As expected, the grazing angle spread across the RF bucket decreases (roughly) with the square root of energy  $E$ .

## SUMMARY

The grazing function  $g$  parametrizes the rate of change of total angle with synchrotron amplitude for grazing particles. The grazing function is a pure optics function, closely

related to the slope of the normalized dispersion function. It has an ideal value of  $g = 0$  at the collimator.

Design values for past, present and future crystal implementations in RHIC, SPS, Tevatron and LHC suggest that the natural realistic values of  $g$  are acceptably small, although they are not negligible. Planning for future crystal implementations should always include a grazing function analysis, both in design (making  $g$  zero, or small enough) and in error analysis (ensuring that  $g$  cannot become anomalously large).

We would like to thank R. Assmann, A. Drees, R. Fliller, J-B. Jeanneret, E. Laface, N. Mokhov, G. Robert-Demolaize, W. Scandale, D. Still, and A. Valishev, for their support and encouragement.

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