MULTI-MODE ACCELERATING STRUCTURE WITH HIGH FILLING FACTOR

S.V. Kuzikov#, M.E. Plotkin, Institute of Applied Physics, Nizhny Novogorod, 603950, Russia

Abstract

A new two-beam accelerating structure based on periodic multi-mode cavities was suggested recently [1-3]. The structure is aimed to increase threshold surface fields and thus to provide a high gradient.

In order to increase the filling factor of the structure it is suggested to operate with several TM-modes with non-zero longitudinal indices. These modes are able to provide the long effective interaction of a moving bunch with RF fields along the cavity. Such regime requires for the longitudinal index to be strictly proportional the mode frequency.

INTRODUCTION

In order to achieve multi-TeV energies of the colliding particles in e^-e^+ colliders, the accelerating gradient should be as high as possible. Increasing of the gradient allows reducing of the total length of multi-kilometer machine while maintaining the same energy of accelerated particles, thus reducing accelerator cost.

In "warm" e^+e^- colliders accelerating gradient is limited, first of all, by thresholds of RF breakdown and pulse heating [4]. In order to avoid these limitations a new type of accelerating structure was suggested in [1-3]. Such the structure is built of the oversized cavities operated with a superposition of modes with equidistant frequencies. In multi-frequency structure surface fields at any point appear as a sequence of narrow peaks, separated by intervals of close to zero fields. Since RF breakdown and thermal fatigue thresholds depend on exposition time (see [4]), we expect significant increase of the accelerating gradient.

ACCELERATION PRINCIPLE IN A MULTI-FREQUENCY STRUCTURE

The accelerating structure suggested in [1-3] consists of periodic, metallic, multi-mode cavities with equidistant spectrum of eigenmodes. The accelerated bunches of particles are transported through small holes at the centres of the cavities (Fig. 1). The coupling of the cavities due to these holes is neglected. Periodic bunches move relative to the cavities with velocity close to speed of light c and being spaced in time with period T_b.

The idea of multi-frequency structure implies that strong accelerating field must appear at resonator axis only for a short interval of time when a bunch is resident in the given cavity. All the rest time, when the cavity is empty, it is desirable to reduce these fields in order to minimize exposition of cavity walls. The field, produced by superposition of many modes, is localized in space and bounce between the structure axis and the wall at the bunch period.

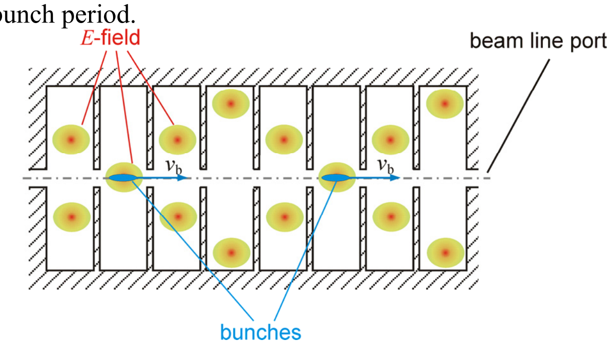


Figure 1: Acceleration of moving periodic bunches by cavities operated with a superposition of synchronized eigenmodes.

The proposed solution means that fields are periodic functions of time:

$$\vec{E}(\vec{r},t+T_b)=\vec{E}(\vec{r},t). \tag{1}$$

This requires the RF field in each cavity to be represented as superposition of equidistantly spaced eigenmodes:

$$\begin{gathered}\vec{E}(\vec{r},t)=\sum_n a_n\cdot\vec{F}_n(\vec{r})\cdot\exp(i\omega_n t),\\ \omega_n=\omega_0+n\cdot\Delta\omega,\quad T_b=q2\pi/\Delta\omega,\end{gathered} \tag{2}$$

where $\omega_0/\Delta\omega=p/q$; n, p, and q are positive integers, $\vec{F}_n(\vec{r})$ and a_n - spatial field distribution and magnitude of n-th mode.

The duration of each field peak is determined by a condition that phase difference between the lowest and highest modes reaches π:

$$\Delta t_{cor}\approx\pi/(\omega_N-\omega_0). \tag{3}$$

We assume that the ratio of peak's width to time interval between peaks $\Delta t_{cor}/T_b$ is a small parameter. Hence, the field at an arbitrary point is pulsed in time with significant intervals between peaks. The desired reduction of the exposition time is inversely proportional to this parameter.

Any multi-mode structure is assumed to be fed by means of a high-current drive beam. This beam, bunched with the same period as accelerated beam, excites fields which accelerate a low-current main beam.

In particular, the structure which is based on the described idea, can be assembled of the rectangular cross-section cavities with sizes $a\times 2a\times l_r$ [3]. Each cavity in

#kuzikov@appl.sci-nnov.ru

this case works with the eigenmodes $TM_{n,2n,0}$ (n = 1,3,5,…) with pure equidistant spectrum.

CAVITIES WITH EQUIDISTANT SPECTRUM OF MODES HAVING NON-ZERO LONGITUDINAL INDICES

According to (3), a bunch is allowed to stay in cavity during the time interval which is inversely proportional to the difference of the highest and the lowest frequencies. This condition comes from the requirement that a bunch must always be in the accelerating phase of each partial mode. This leads to maximum of allowed cavity length:

$$l_r^{\max} = c\Delta t_{cor} = \pi \cdot c / (\omega_N - \omega_0). \quad (4)$$

This cavity length brings a limit to the filling factor of the structure because acceleration in the sections between the cavities is absent (length of these sections is conditioned by technological reasons). On the other hand, the effect of reduction of surface exposition time and gradient increase the stronger the larger number of the involved modes. A good way out of this situation is to use modes with non-zero longitudinal indices. Frequencies of such modes must be, of course, equidistant ones. In addition its must provide location of a bunch in accelerating phase for each mode during the whole travel through the cavity. This is reached, for example, if the mode with longitudinal index n=1 has twice as big frequency compared to the mode with zero number of longitudinal variations. Such mode will change field sign when a bunch comes to the centre of the cavity.

A square cavity is a simple system which allows involving a mode with one longitudinal variation to the set of equidistant modes without longitudinal variations. This is achieved by means of the proper choice of the cavity length. If we require $f_{1,1,1} = 2f_{1,1,0}$ for a resonator with sidewall a, we immediately get $l_r = a/\sqrt{6}$.

Let us investigate also a possibility to involve a big number of modes with nonzero longitudinal indices. The analysis can be performed by means of a one-dimensional model problem (Fig. 2). Here the cavity, consisted of two infinite metallic planes, is excited by a sequence of plane bunches traveling with the space period l_B.

The current inside the cavity appears as an infinite sequence of delta functions with time spacing $T_b = l_b/c$:

$$j_x(x,t) = j_0 \sum_k \delta(x - c(t - kT_b)). \quad (5)$$

This expression can be expanded in series:

$$j_x(x,t) = \sum_n j_x^n(x) \cdot e^{-i\omega_n t}, \quad \omega_n = \frac{\pi n c}{l_r}. \quad (6)$$

We will assume that each harmonic of the current j_x^n excites the corresponding cavity mode of the same frequency: $E_n = E_0 \cdot \cos(\pi n x/l_r) \cdot \exp(-i\omega_n t)$.

The total field in the cavity is given as a superposition of all eigenmodes:

$$E(x,t) = \sum_n C_n E_n(x,t) \quad (7)$$

with corresponding excitation coefficients C_n. In the case of infinite number of modes with equal Q-factors we get, using (6-7), the analytical solution:

$$E(x,t) = A\sum_k \left\{ \delta\left(\frac{x}{c} - (t + kT_b)\right) + \delta\left(\frac{x}{c} + (t - kT_b)\right) \right\} \quad (8)$$

where A - is a constant. The formula (8) represents a sum of two periodical sequences of delta functions moving in opposite directions with velocity c. This solution demonstrates also that peak value of the accelerating field automatically follows for the bunch along its path in cavity.

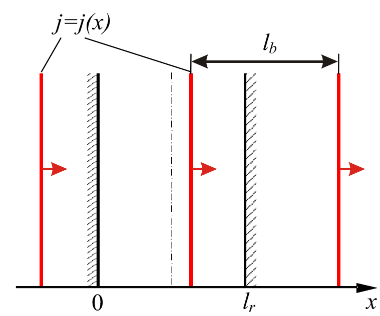


Figure 2: One-dimensional model of resonator with passing sequence of bunches.

In case of the finite number of modes the fields, given by (7), have been calculated numerically. However, the numerical solution shape well fits analytical solution (8) while the number of modes, taken into account, is big enough (N>>1).

A square cavity with length $l_r = a/2\sqrt{7}$ gives a realistic example of the cavity with equidistant spectrum of "longitudinal" modes. In particular, the frequencies of three modes $TM_{3,3,0}$, $TM_{5,5,2}$, $TM_{1,1,4}$ in such cavity are strictly equidistant.

ACCELERATING STRUCTURE BASED ON TWO-MIRROR CAVITIES

A two-mirror cavity is a natural development of the one-dimensional cavity considered in the previous section. The accelerating structure composed of such cavities (Fig. 3) potentially has unique mode selection properties.

In case of the two-mirror cavity with square mirrors, the operating modes $TM_{1,1,n}$ have highest Q-factors amongst all others. Its frequencies can be written using a quasi-optical approach:

$$\omega_n \approx \frac{\pi n c}{l_r} + \frac{\pi c l_r}{(na)^2}. \tag{9}$$

According to (9) in order to keep the equidistance of the spectrum, the length of the cavities l_r should be taken smaller than its cross-section size a.

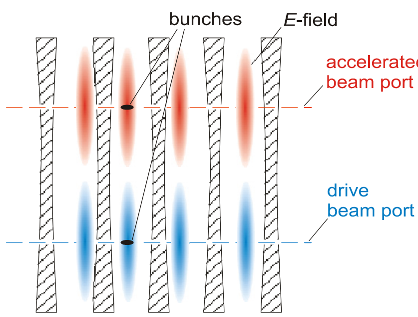
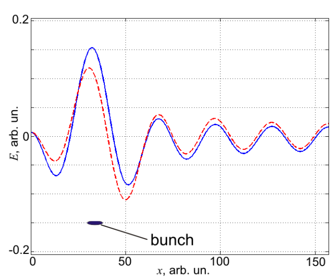


Figure 3: Two-beam accelerating structure based on two-mirror cavities.

The accelerating field distribution has been calculated numerically for the realistic copper cavity using the approach described for the ideal cavity. The frequency of the lowest mode ($TM_{1,1,1}$) has been chosen 6 GHz and the solution has been compared with the ideal one-dimensional case. Results of that comparison are shown in Fig. 4 where longitudinal field distributions in the cavity are shown for the ratio $l_r/a = 30$ and number of modes $N = 10$. The larger number of modes the better coincidence of the resulting field distribution with the delta-function and, therefore, the higher moving field maximum.



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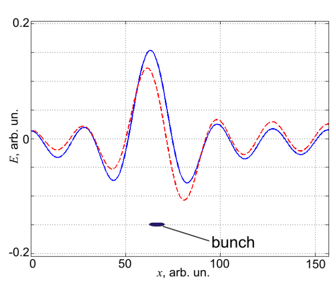

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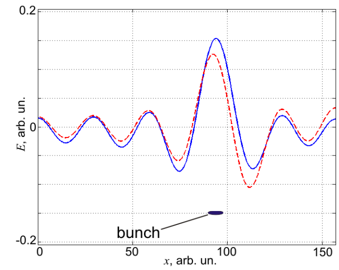

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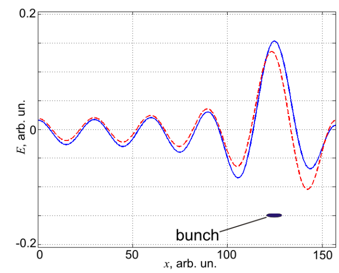

d

Figure 4: Longitudinal distributions of accelerating field in one-dimensional resonator (blue curve) and two-mirror resonator (dashed red curve) at four sequent times: a - $t = 0.1T_b$, b - $t = 0.2T_b$, c - $t = 0.3T_b$, d - $t = 0.4T_b$.

CONCLUSION

A new accelerating structure which uses a big number of modes with equidistantly spaced frequencies and non-zero longitudinal indices is suggested. It is shown that maximum of accelerating field strictly follows the accelerated bunch along the cavity. This allows increasing the filling factor and average on length accelerating gradient of the structure. The prospects to use a structure composed of quasi-optical two-mirror cavities are shown.

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