Dynamic Beta/Emittance Effects in the Measurement of Horizontal Beam Sizes

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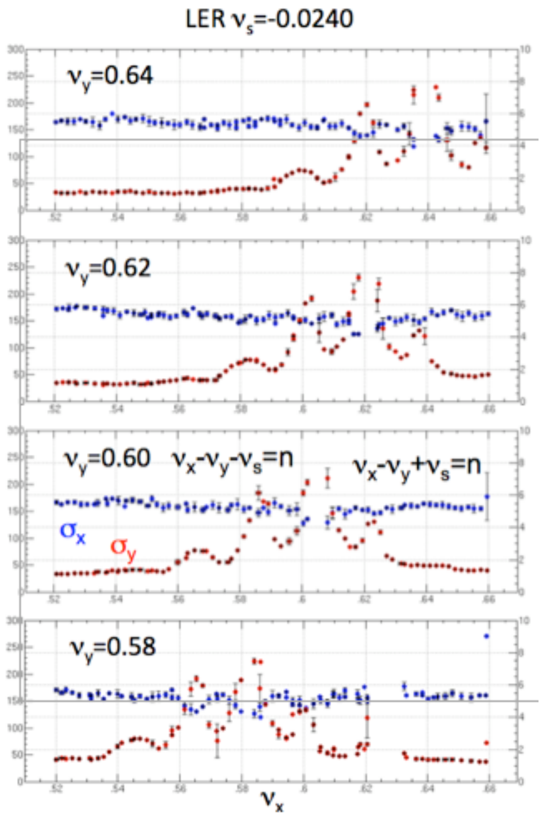
KEKB luminosity is now 19.33×10^{33} $cm^{-2}s^{-1}$

KEKB Operation Summary

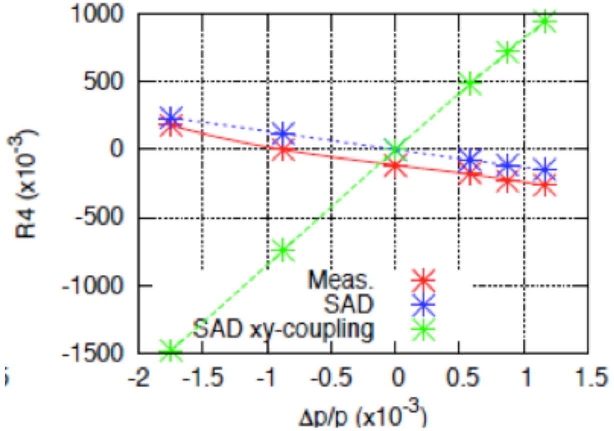
Records: 17.6 w/o crab, 16.8 w crab

- R chromaticity correction works very well in the crab crossing.
- Study of R chromaticity since last year bears fruit.

Beam size measurement in tune space



R chromaticity measurement and comparison with simulation



Effect of chromaticity on the luminosity

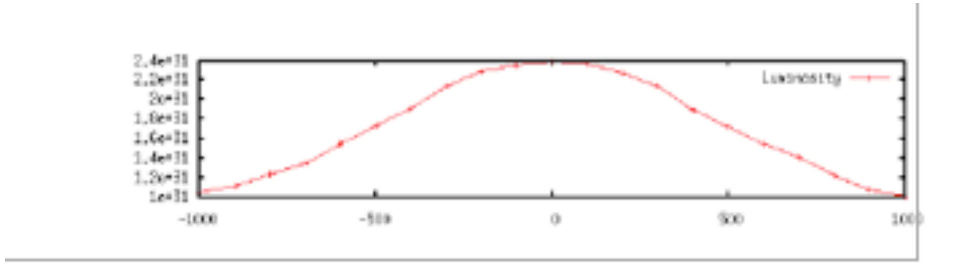


Fig.8 Horizontal beam size, Vertical beam size and Luminosity vs. $dR4/d\delta$

Dynamic beta

- Distortion of optics due to the beam-beam force
- The beam-beam force is determined by the beam size of the colliding beam

$$\Delta p_{x,\pm} = -q_{\pm} x = -\frac{2N_{\mp} r_e}{\gamma} \frac{1}{\sigma_{x,\mp}^2} x_{\pm} = -\frac{4\pi\xi}{\beta^*} x_{\pm}$$

- Revolution matrix including the beam-beam force

$$M_{\pm} = K_{\pm} M_{0,\pm} K_{\pm} \qquad K_{\pm} = \begin{pmatrix} 1 & 0 \\ -q_{\pm}/2 & 1 \end{pmatrix}$$

- Beta function for M, ($\overline{\beta}^*$)

$$\left(\frac{\beta^*}{\overline{\beta}^*}\right)^2 = 1 + q\beta^* \cot\mu - \frac{q^2\beta^{*2}}{4}$$

Beam Envelope Formalism

- The beta modulation due to the beam-beam force changes the emittance.
- Beam envelope formalism is based on the transfer for the radiation effect, thus is not necessary to care for the optics distortion.

$$\Sigma_0 = (1 - D^t) M_0^t \Sigma_0 M_0 (1 - D) + B$$
$$D = \oint M_0(s^*, s)^{-1} D_0(s) M_0(s^*, s) ds$$
$$B = \oint M_0(s^*, s)^t B_0(s) M_0(s^*, s) ds$$

D ~ diagonal matrix with the element of the radiation damping time, $T_0/\tau_{x,y,z}$

- Integrate $B_0(s)_{ij} = \frac{55}{24\sqrt{3}} \frac{r_e \hbar}{mc} \frac{\gamma^5}{|\rho|^3} \delta_{i6} \delta_{j6}$ along the ring.

Dynamic Emittance

- Including the beam-beam force is only to replace M_0 with M in the bam envelope formalism.

$$\Sigma = (1 - D^t) M^t \Sigma M (1 - D) + B$$

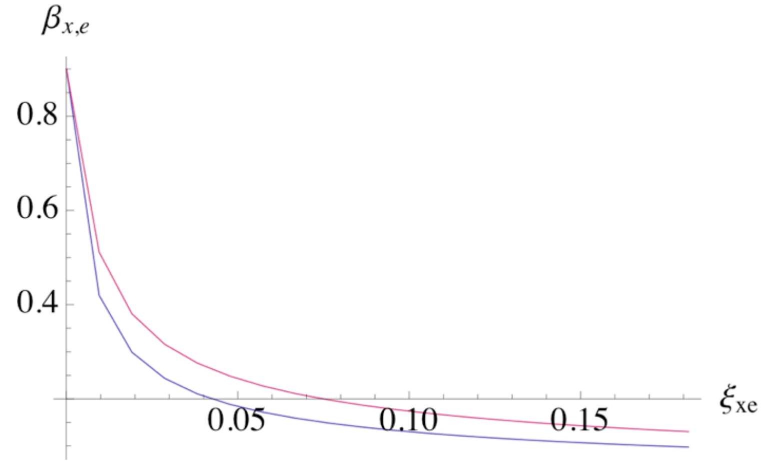
- One typical choice of B, diagonal matrix with 2 $\beta_x \varepsilon_x T_0 / \tau_x$,…, radiation excitation to the normal mode.
- Actually it depends on the transfer matrix between radiation positions and the collision point.

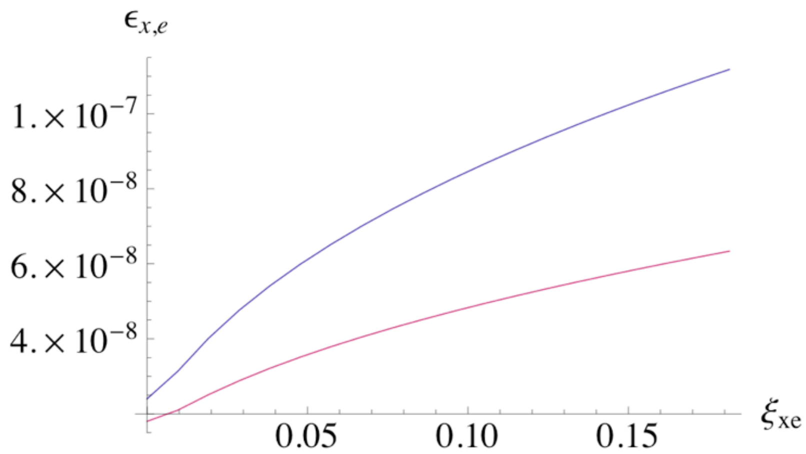
- The dynamic beta and emittance have to be solved for both beams with the consistency.

- Alternative approach: find normal coordinate for M, calculate radiation integral with beta beat.

Dynamic beta/emittance for the radiation excitation B to the normal mode

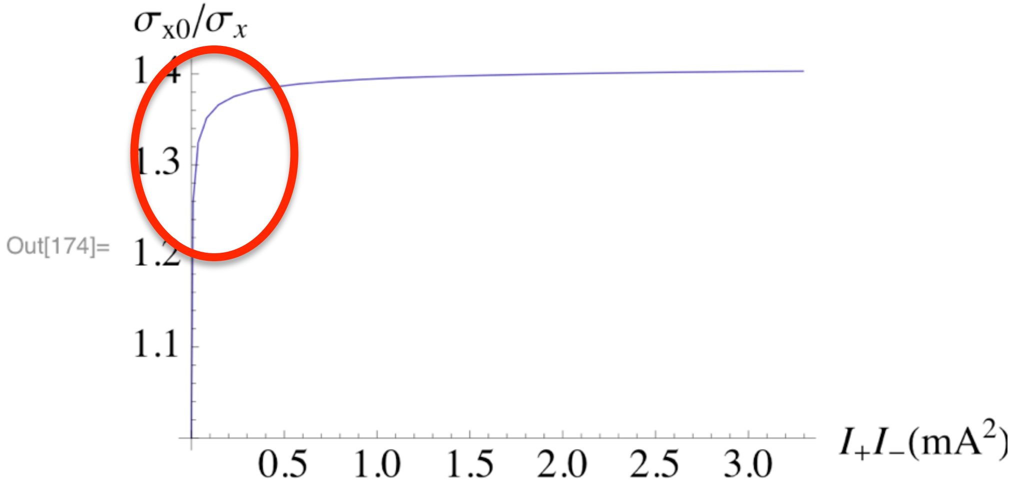
- Two envelope equations are solved with iteration
- β_x* decreases for increasing current, but ε_x increases.



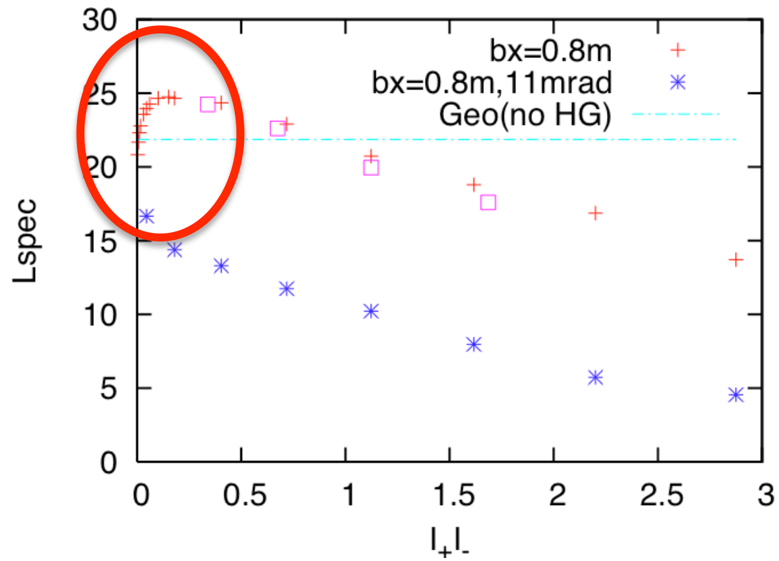


Luminosity increase due to the dynamic beta/emittance of colliding two beams

- σ_x including dynamic beta/emittance



- Luminosity calculated by a strong-strong simulation (BBSS)



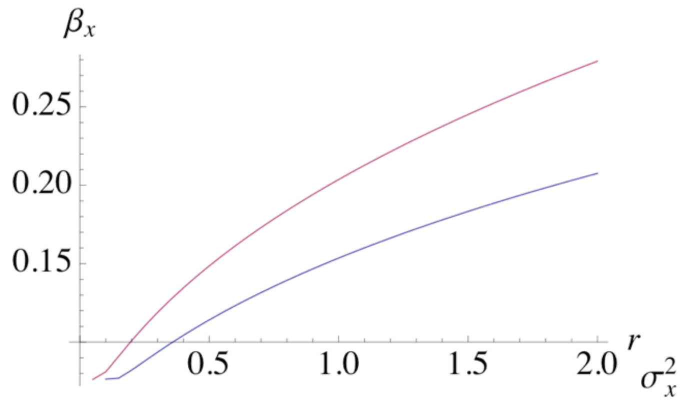
Radiation excitation is not always along normal axis

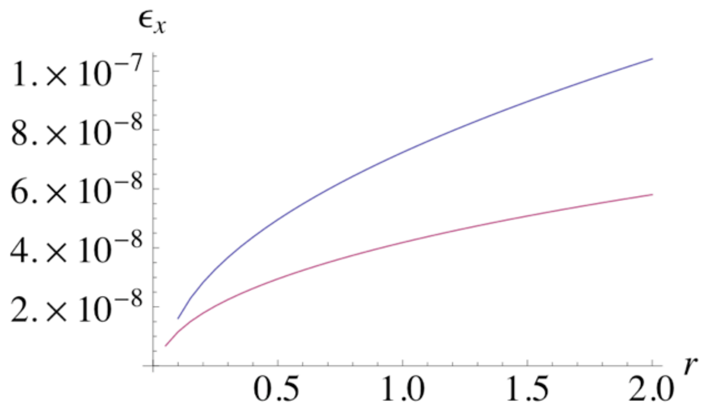
- For general excitation, diffusion assignment, r, is introduced.
- B matrix is modeled as

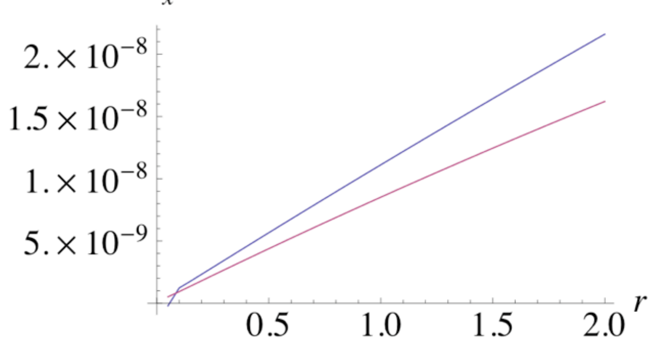
  $B_{11}=2\, r\, \beta_x \varepsilon_x T_0/\tau_x$, $B_{22}=2\,(2-r)\, \beta_x \varepsilon_x T_0/\tau_x$

The same emittance for M_0

- The diffusion assignment strongly affects the dynamic beta and emittance.







Evaluation of dynamic beta/emittance and the diffusion assignment, based on measurements

- Transfer matrix from IP to monitor

$$M_0(s_m, s^*) = \begin{pmatrix} \sqrt{\frac{\beta_m}{\beta^*}} \cos\Delta\mu & \sqrt{\beta_m \beta^*} \sin\Delta\mu \\ -\sqrt{\frac{1}{\beta^* \beta_m}}(\sin\Delta\mu + \alpha_m \cos\Delta\mu) & \sqrt{\frac{\beta^*}{\beta_m}}(\cos\Delta\mu - \alpha_m \sin\Delta\mu) \end{pmatrix}$$

- The relation of β and σ_x without and with beam-beam

$$M(s_m, s^*) = M_0(s_m, s^*)K$$

$$\begin{pmatrix} \bar{\alpha}^* & \bar{\beta}^* \\ -\bar{\gamma}^* & -\bar{\alpha}^* \end{pmatrix} = M(s_m, s^*) \begin{pmatrix} \bar{\alpha}_m & \bar{\beta}_m \\ -\bar{\gamma}_m & -\bar{\alpha}_m \end{pmatrix} M(s_m, s^*)^{-1}$$

$$\frac{\bar{\beta}_m}{\bar{\beta}^*} = \frac{\beta_m}{\beta^*}\left[\left(\cos\Delta\mu - \frac{q\beta^*}{2}\sin\Delta\mu\right)^2 + \left(\frac{\beta^*}{\bar{\beta}^*}\right)^2 \sin^2\Delta\mu\right]$$

$$\frac{\bar{\sigma}^2_{m\pm}}{\bar{\sigma}^{*2}_{\pm}} = \frac{\beta_{m\pm}}{\beta^*_{\pm}} \times \left[1 + \frac{2N_{\mp} r_e \beta^*_{\pm}}{\gamma} \frac{1}{\bar{\sigma}^{*2}_{\mp}} (\cot\mu_{\pm} \sin^2\Delta\mu_{\pm} - \sin\Delta\mu_{\pm} \cos\Delta\mu_{\pm}\right]$$

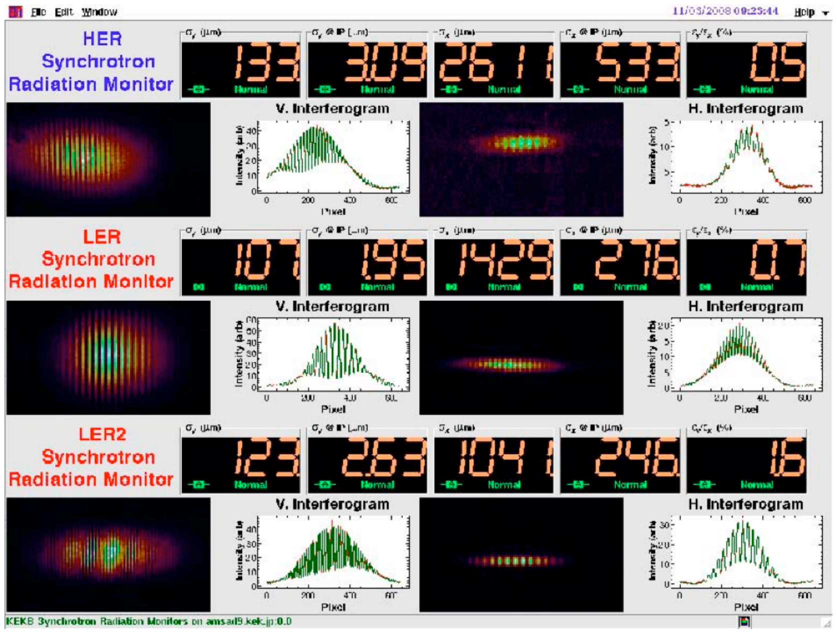
Measurement

- Beam size at IP is estimated from that at monitor with considering dynamic beta/emittance of colliding two beams.

$$\bar{\sigma}_{\pm}^{*2} = \frac{-A_{\pm}A_{\mp} + B_{\pm}B_{\mp}}{A_{\pm} + B_{\mp}}$$

$$A_{\pm} = \frac{2N_{\mp} r_e \beta_{\pm}^{*}}{\gamma_{\pm}} (\cot\mu_{\pm} \sin^2\Delta\mu_{\pm} - \sin\Delta\mu_{\pm} \cos\Delta\mu_{\pm})$$

$$B_{\pm} = \frac{\bar{\sigma}_{m\pm}^{2} \beta_{\pm}^{*}}{\beta_{m\pm}}$$



Measured beam size

$$\bar{\sigma}_{m+} = 1041\ \mu\text{m},\quad \bar{\sigma}_{m-} = 2611\ \mu\text{m}$$

$$\bar{\beta}_{m+/-} = 14.23/21.89\ \text{m},\quad \Delta\nu_{+/-} = 20.951/35.358$$

$$\nu_{x,+/-} = 0.5054/0.5108,\quad N_{+/-} = 6.20/3.66\times 10^{10}$$

$$\beta_{x\pm}^{*} = 0.9\ \text{m}$$

Estimated beam size at IP

$$\bar{\sigma}_{x,+}^{*} = 100\ \mu\text{m},\quad \bar{\sigma}_{x,-}^{*} = 132\ \mu\text{m}$$

Diffusion assignment r~1.2-1.5

Conclusions

- The dynamic beta, emittance and diffusion assignment are estimated by the horizontal beam size measurement.
- $\bar{\sigma}^*_{x,+} = 100\ \mu\text{m},\ \bar{\sigma}^*_{x,-} = 132\ \mu\text{m},\ \bar{\beta}^*_{x,+} = 0.2 - 0.25\ \text{m},\ \bar{\beta}^*_{x,-} = 0.15 - 0.2\ \text{m}$
  $\bar{\varepsilon}_{x,+} = 40\ \text{nm},\ \bar{\varepsilon}_{x,-} = 80\ \text{nm},\ r = 1.2 - 1.5\ \text{m}$
- The horizontal size measurement has not perfectly calibrated yet, thus these results are preliminary.
- Beam-beam simulation (BBSS) gives a larger luminosity for smaller r<1.
- B integral in SAD gave r=0.9, but measurement gives r=1.2-1.5.
- This result r>1 indicates lower emittance, though it does not seem to explain the measured luminosity in KEKB.