



Simulating Electron-Ion Dynamics in Relativistic Electron Coolers

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Acknowledgments

G. I. Bell, R. Busby, J.R. Cary, P. Messmer, A. Sobol



I. Ben-Zvi, V. Litvinenko, A. Fedotov, E. Pozdeyev



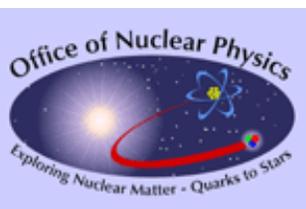
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Outline

- Motivation and Overview
- Simulation of “conventional” high-energy e- cooling
- Simulating the modulator of a coherent e- cooling system

Themes

- Simulating important subtleties in basic plasma physics
 - exposes underlying assumptions (which are now violated)
 - physical intuition was frequently incorrect and misleading
- Hard to find/modify/invent the right numerical algorithm
 - implementation for parallel computing
 - noise reduction
- Simulating the modulator of a coherent e- cooling system



Long-term motivation for electron cooling of relativistic hadron beams: the **Electron-Ion Collider (EIC)** concept

Unanimous recommendation of the Quantum Chromodynamics Town Meeting, at Rutgers University, New Jersey, January, 2007

A **high luminosity** Electron Ion Collider (EIC) is the highest priority of the QCD community for new construction after the Jlab12 GeV and RHIC II upgrades. EIC will address compelling physics questions essential for understanding the fundamental structure of matter:

- Precision imaging of the sea-quark and gluons to determine the spin, flavor and spatial structure of the nucleon
- Definitive study of the universal nature of strong gluon fields in nuclei

The collider and the detector designs must be developed expeditiously.

C. Aidala *et al.* (The EIC Working Group), "A High Luminosity, High Energy Electron-Ion-Collider; A New Experimental Quest to Study the Glue that Binds Us All," White Paper prepared for the NSAC LRP (2007).

http://www.phenix.bnl.gov/WWW/publish/abhay/Home_of_EIC/NSAC2007/070424_EIC.pdf



High luminosity relativistic ion beams require electron cooling

- EIC requires orders-of-magnitude higher ion luminosity
 - can only be achieved by reducing phase space volume of beams
 - ions have no natural mechanism for phase space damping
 - hence, external cooling techniques are required
 - in some cases, stochastic cooling can be used; however...
 - 8.9 GeV antiprotons in Fermilab accumulator ring require e- cooling
 - 250 GeV polarized protons will require e- cooling
- Two EIC concepts are being considered in the US
 - eRHIC (add energy recovery linac to the RHIC complex)
 - ELIC (add e- and ion rings to Jefferson Lab complex)
 - electron cooling is included in all present designs

ERL-based Layout for eRHIC

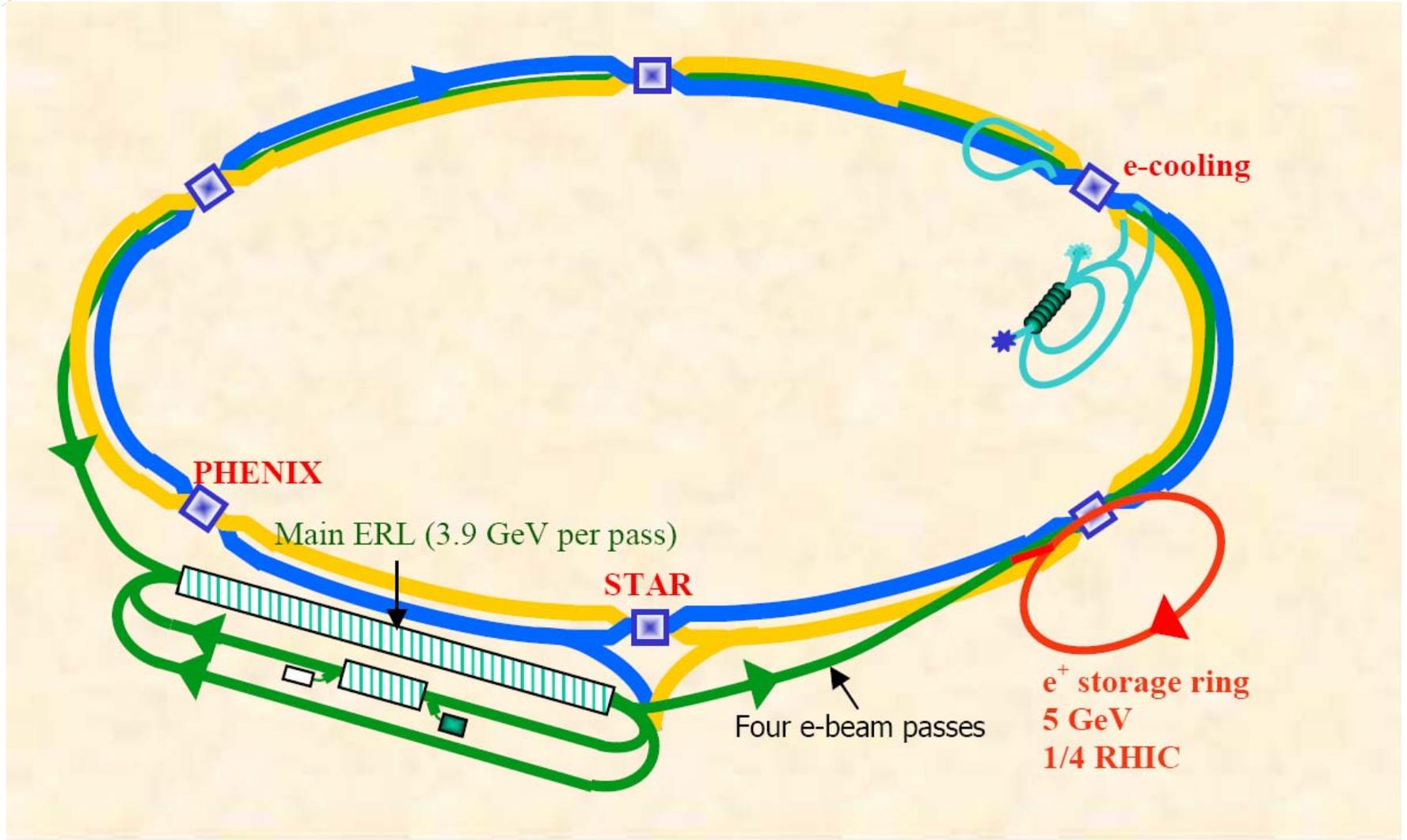
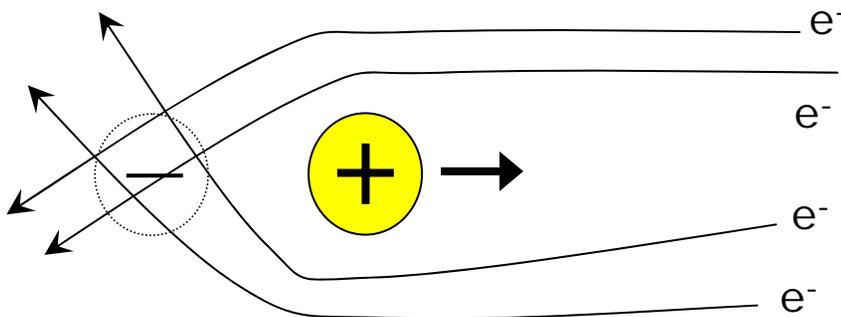


Image taken from 2007 eRHIC position paper



Dynamical friction is the key physical process for e- cooling

- Case of isotropic plasma, with no external fields, was first explained 65 years ago
 - S. Chandrasekhar, *Principles of Stellar Dynamics* (U. Chicago Press, 1942).
 - B.A. Trubnikov, *Rev. Plasma Physics* **1** (1965), p. 105.



$$\mathbf{F} = -\frac{4\pi n_e k^2}{m_e} \int \log\left(\frac{\rho_{max}}{\rho_{min}}\right) \frac{\mathbf{v}}{|\mathbf{v}|^3} f(\mathbf{v}_e) d^3 v_e$$

$$\mathbf{v} = \mathbf{v}_i - \mathbf{v}_e$$

$$k = Ze^2/(4\pi\epsilon_0) \quad \rho_{min} = \frac{k}{m_e|\mathbf{v}|^2}$$

- Physics can be understood in two different ways
 - Binary collisions (integrate over ensemble of e-/ion collisions)
 - Dielectric plasma response (ion scatters off of plasma waves)



Electron cooling is not yet well-established for relativistic ion beams

- Budker developed the concept in 1967
 - G.I. Budker, *At. Energ.* **22** (1967), p. 346.
- Many low-energy electron cooling systems in operation

- Electron cooling occurs in the beam frame
 - implies (holding many factors fixed) that the cooling rate $\sim 1/\gamma^2$
 - e- density & interaction time both decrease by factor γ in beam frame

- Fermilab has shown cooling of relativistic p-bar's
 - S. Nagaitsev *et al.*, *PRL* **96**, 044801 (2006).
 - 4.3 MeV DC electrons; $\gamma \approx 9.4$
- RHIC II, eRHIC need “high-energy” cooler
 - >100 GeV/n $\rightarrow \gamma > 100 \rightarrow >50$ MeV bunched electrons

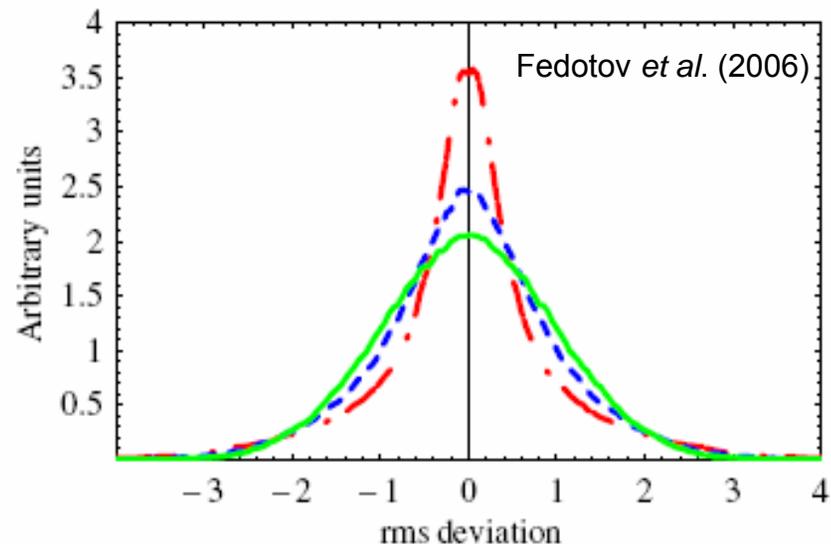
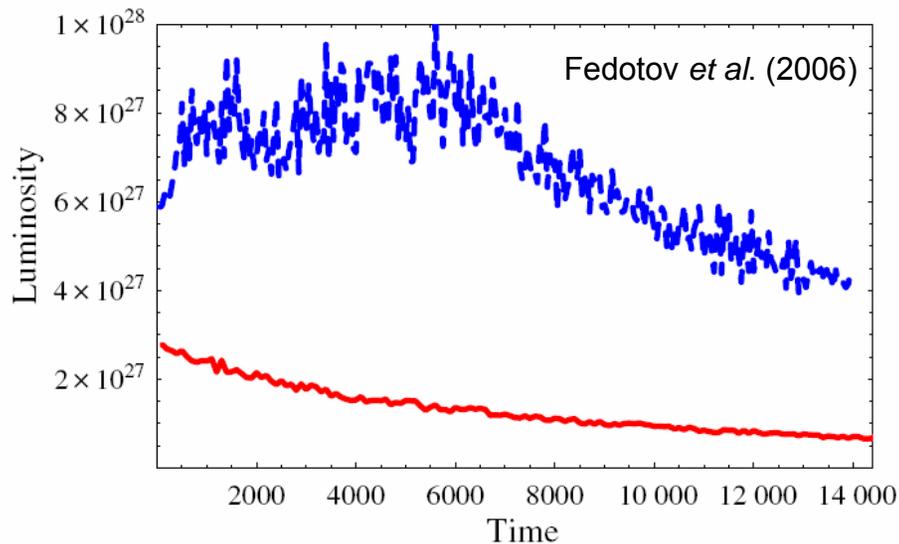


Relativistic e- cooling requires detailed simulations to reduce risk

- Early BETACOOOL simulations of RHIC II cooling raised serious concerns
 - alternate friction force models differed greatly
 - friction must be understood to within a factor of two
 - no theory for arbitrary external magnetic fields
- Clear mandate for this computational effort
 - **simulate dynamical friction of ions moving through e-'s**
 - work in beam frame of co-moving beams
 - particle motion is nonrelativistic; self-fields are electrostatic
 - lab frame fields (solenoid, wiggler, errors) are Lorentz boosted
 - for a single interaction, diffusion exceeds friction
 - short interaction + low e- density → no Debye shielding
 - electron velocity distribution can be highly anisotropic
 - **provide actionable information for BETACOOOL models**

Full e- cooling sim's are distinct from simulating micro-physics of a single pass

- BETACOOOL code is used to model many turns
 - A.O. Sidorin *et al.*, *Nucl. Instrum. Methods A* **558**, 325 (2006).
 - A.V. Fedotov, I Ben-Zvi, D.L. Bruhwiler, V.N. Litvinenko, A.O. Sidorin, *New J. Physics* **8**, 283 (2006).
- a variety of electron cooling algorithms are available
 - i.e. simple models for dynamical friction and diffusion
- various models for “heating” are included
 - intra-beam scattering (IBS). beam-beam collisions. etc.



Many useful algorithms are implemented in VORPAL framework for e- cooling simulations

- Fast multipole method (FMM) and tree-based algorithms
 - requires constant time step; inefficient for beams with close collisions
 - **this algorithm was abandoned**
- 4th-order predictor-corrector “Hermite” algorithm
 - taken from astrophysical dynamics community
 - generalized to include solenoid field
 - used successfully for e-/ion interactions only, with only a few ions
 - didn’t parallelize well, so we used a task farming approach
 - astrophysicists use special “Grape” hardware to parallelize
- Semi-analytic binary collision model
 - accurately models arbitrarily strong Coulomb collisions
 - arbitrary external fields included via 2nd-order operator splitting
- Electrostatic particle-in-cell (PIC)
 - cannot capture close Coulomb collisions
 - combined effectively with “binary collision” model for e-/e- interactions
 - appropriate for CeC modulator simulations, but too noisy
- δf PIC
 - macro-particles simulate deviations from analytic distribution function
 - lower noise ideal for CeC modulator simulations

4th-order predictor-corrector “Hermite” algorithm was used successfully

Algorithm developed and used extensively by the astrophysical dynamics community to simulate globular clusters

–J. Makino, The Astrophysical Journal **369**, 200 (1991)

–J. Makino & S. Aarseth, Publ. Astron. Soc. Japan **44**, 141 (1992)

Predictor step:

$$\mathbf{v}_{p,j} = \frac{1}{2} (t - t_j)^2 \dot{\mathbf{a}}_j + (t - t_j) \mathbf{a}_j + \mathbf{v}_j$$

$$\mathbf{x}_{p,j} = \frac{1}{6} (t - t_j)^3 \dot{\mathbf{a}}_j + \frac{1}{2} (t - t_j)^2 \mathbf{a}_j + (t - t_j) \mathbf{v}_j + \mathbf{x}_j$$

where

$$\mathbf{a}_i = \frac{q_i}{m_i} \mathbf{v}_i \times \mathbf{B} + \frac{q_i}{4\pi\epsilon_0 m_i} \sum_j \frac{q_j \mathbf{r}_{ij}}{(r_{ij}^2 + r_c^2)^{3/2}}$$

$$\mathbf{r}_{ij} = \mathbf{x}_{p,j} - \mathbf{x}_{p,i}$$

$$\dot{\mathbf{a}}_i = \frac{q_i}{m_i} \mathbf{a}_i \times \mathbf{B} + \frac{q_i}{4\pi\epsilon_0 m_i} \sum_j q_j \left[\frac{\mathbf{v}_{ij}}{(r_{ij}^2 + r_c^2)^{3/2}} + \frac{3(\mathbf{v}_{ij} \cdot \mathbf{r}_{ij}) \mathbf{r}_{ij}}{(r_{ij}^2 + r_c^2)^{5/2}} \right]$$

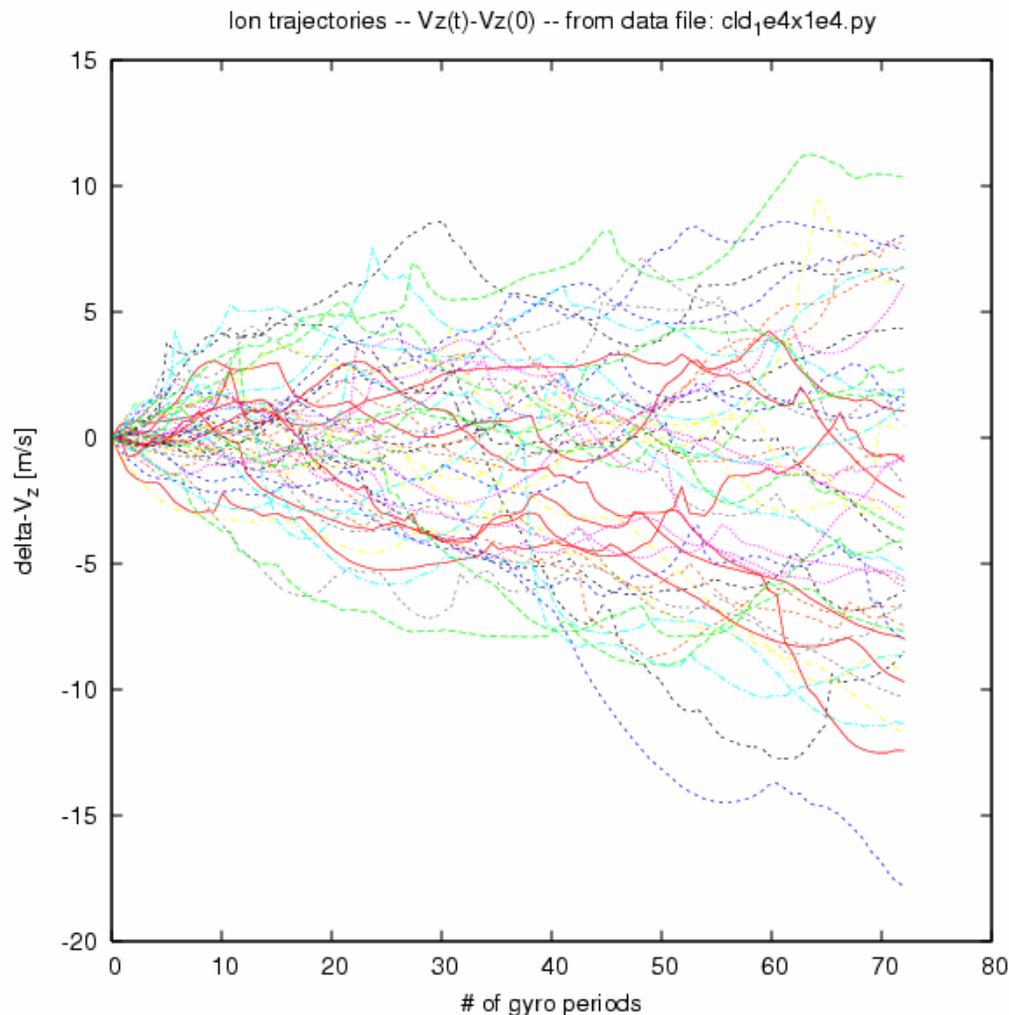
$$\mathbf{v}_{ij} = \mathbf{v}_{p,j} - \mathbf{v}_{p,i}$$

“cloud” radius

$$r_c \rightarrow 0$$

Diffusive dynamics obscures dynamical friction in a single pass

- Diffusive spreading of ion trajectories obscures any velocity drag due to dynamical friction.
- For many millions of turns (e.g. in RHIC), friction forces will dominate diffusion.



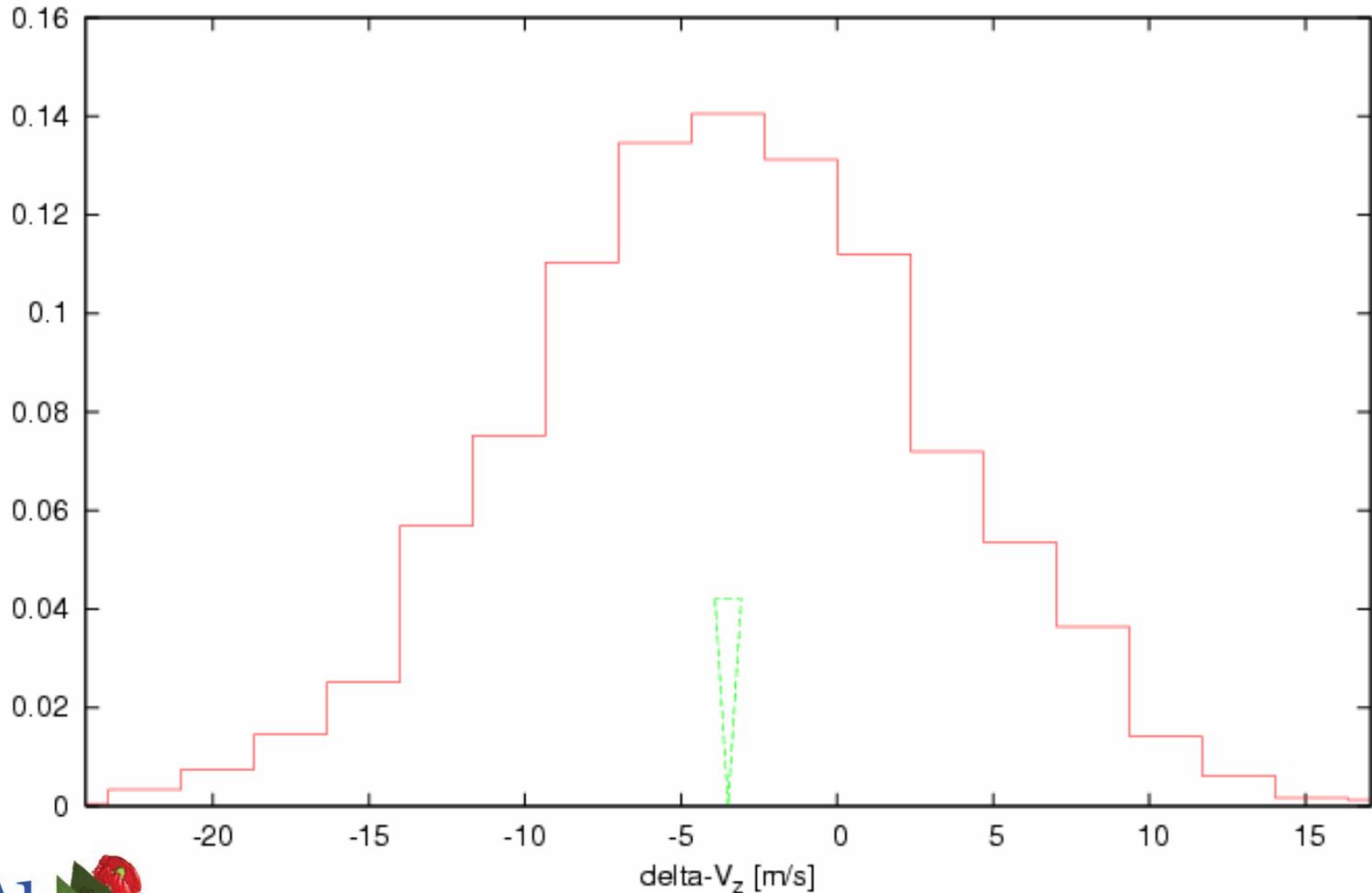
Noise reduction and statistics are required to extract the dynamical friction

- Numerical trick of e-/e+ pairs can *suppress diffusion*
 - simulate with e-/e+ pairs that have identical initial conditions
 - sign of external fields must be flipped for the positrons
 - friction force, independent of sign of charge, is unchanged
 - diffusive kicks are approximately cancelled
- Central Limit Theorem is our only other friend
 - mean friction force is drawn from a Gaussian distribution
 - this assumption is **not correct**; see poster by Sobol *et al.* (Friday morning)
 - RMS is reduced by $N_{\text{traj}}^{1/2}$ from that of the original distribution
 - hence, rough error bars given by $\pm 1 \text{ rms} / N_{\text{traj}}^{1/2}$
- how to routinely generate 1000's of trajectories?
 - “task farming” automates runs and uses processors efficiently
 - used for Hermite algorithm, which did not parallelize well
 - replace physical electrons with many “micro” particles
 - OK if e-'s see each other via PIC, or not at all
 - used with binary collision algorithm, which does parallelize well



The Central Limit Theorem is used to extract $\langle F \rangle$ and error bars from binned data

Histogram of 2392 ion trajectories -- from data file 'cld₁e4x1e4.py'



Differing analytical models for dynamical friction of magnetized electrons are found in the literature

Ya. S. Derbenev and A.N. Skrinsky, “The Effect of an Accompanying Magnetic Field on Electron Cooling,” Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, “Magnetization effects in electron cooling,” Fiz. Plazmy **4** (1978), p. 492; Sov. J. Plasma Phys. **4** (1978), 273.

I. Meshkov, “Electron Cooling; Status and Perspectives,” Phys. Part. Nucl. **25** (1994), 631.

$$F_{\parallel}^A = -\frac{3}{2} \omega_{pe}^2 \frac{(Ze)^2}{4\pi\epsilon_0} \left[\ln\left(\frac{\rho_{\max}^A}{\rho_{\min}^A}\right) \left(\frac{V_{\perp}}{V_{ion}}\right)^2 + \frac{2}{3} \right] \frac{V_{\parallel}}{V_{ion}^3}$$

$$F_{\perp}^A = -\omega_{pe}^2 \frac{(Ze)^2}{4\pi\epsilon_0} \ln\left(\frac{\rho_{\max}^A}{\rho_{\min}^A}\right) \frac{(0.5V_{\perp}^2 - V_{\parallel}^2)}{V_{ion}^2} \frac{V_{\perp}}{V_{ion}^3}$$

$$r_L = V_{rms,e,\perp} / \Omega_L(B_{\parallel})$$

$$\rho_{\min}^A = \max(r_L, \rho_{\min})$$

$$\rho_{\max}^A = \min(r_{beam}, \rho_{\max})$$

$$\rho_{\max} = V_{rel} / \max(\omega_{pe}, 1/\tau)$$

$$V_{rel} = \max(V_{ion}, V_{e,rms,\parallel})$$

$$V_{ion}^2 = V_{\parallel}^2 + V_{\perp}^2$$

V.V. Parkhomchuk, “New insights in the theory of electron cooling,”

Nucl. Instr. Meth. in Phys. Res. **A 441** (2000), p. 9.

$$\mathbf{F} = -\frac{1}{\pi} \omega_{pe}^2 \frac{(Ze)^2}{4\pi\epsilon_0} \ln\left(\frac{\rho_{\max} + \rho_{\min} + r_L}{\rho_{\min} + r_L}\right) \frac{V_{ion}}{(V_{ion}^2 + V_{eff}^2)^{3/2}}$$

$$r_L = V_{rms,e,\perp} / \Omega_L(B_{\parallel})$$

$$\rho_{\min} = (Ze^2 / 4\pi\epsilon_0) / m_e V_{ion}^2 \quad \rho_{\max} = V_{ion} / \max(\omega_{pe}, 1/\tau)$$

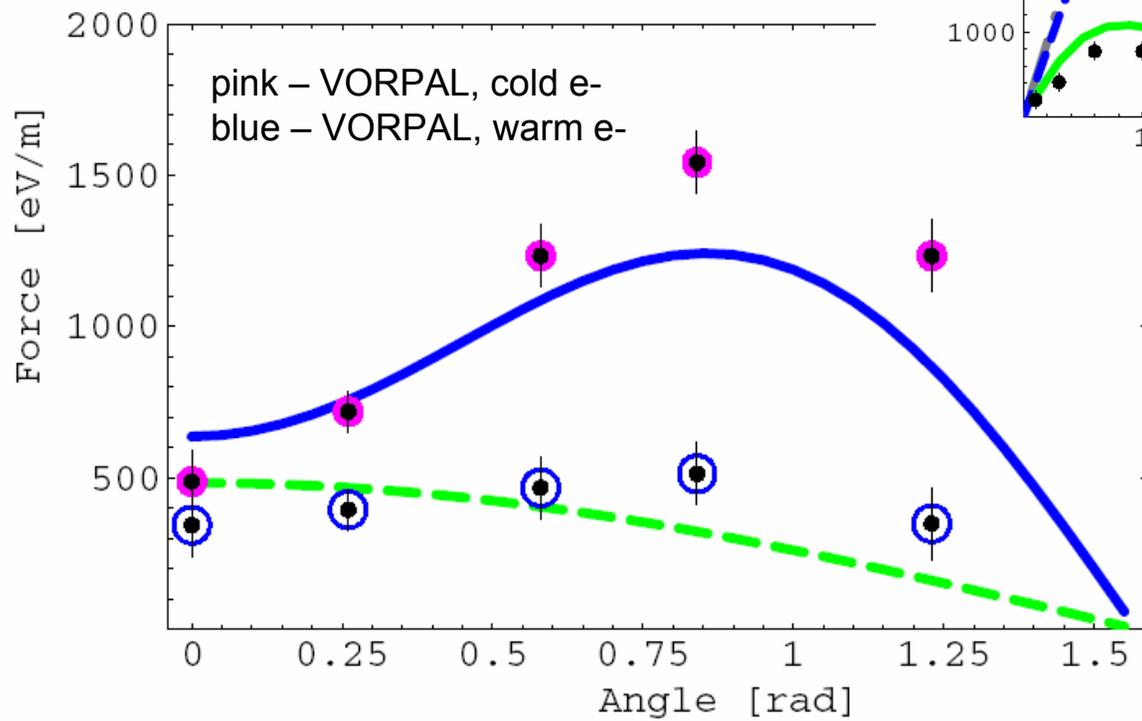
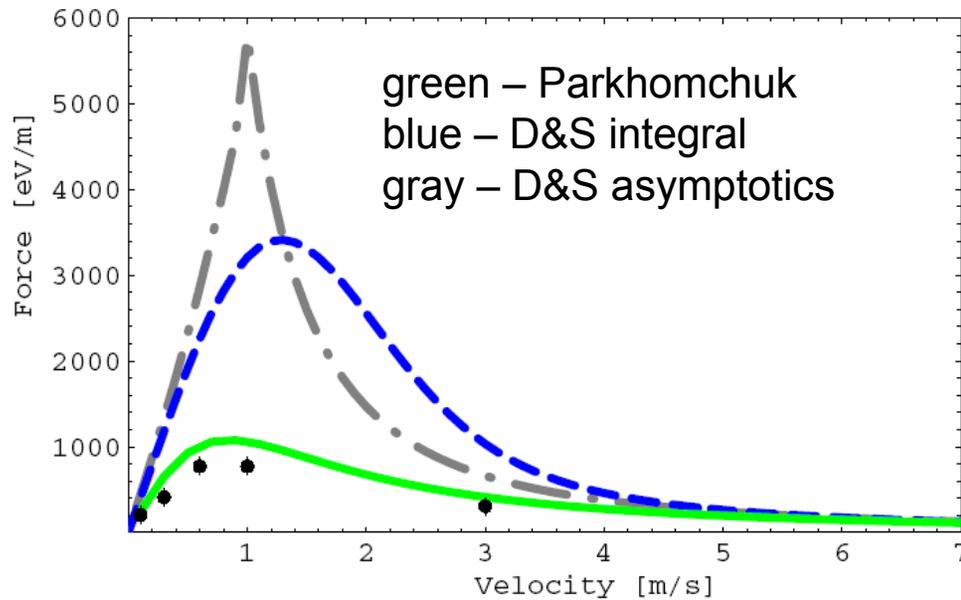
$$V_{eff}^2 = V_{e,rms,\parallel}^2 + \Delta V_{\perp e}^2$$

VORPAL simulations clarify differences between formulae for magnetized friction

VORPAL



A.V. Fedotov, D.L. Bruhwiler, A.O. Sidorin, D.T. Abell, I. Ben-Zvi, R. Busby, J. R. Cary and V.N. Litvinenko, "Numerical study of the magnetized friction force," Phys. Rev. ST/AB **9**, 074401 (2006)



- D&S algorithm is accurate for cold electrons – **not for warm**
- Parkhomchuk formula is approximately correct for typical parameters, **but not always**
- 3D quadrature of standard formula with modified ρ_{\min} is better for **idealized solenoidal** field
- In general, direct simulation is **required**

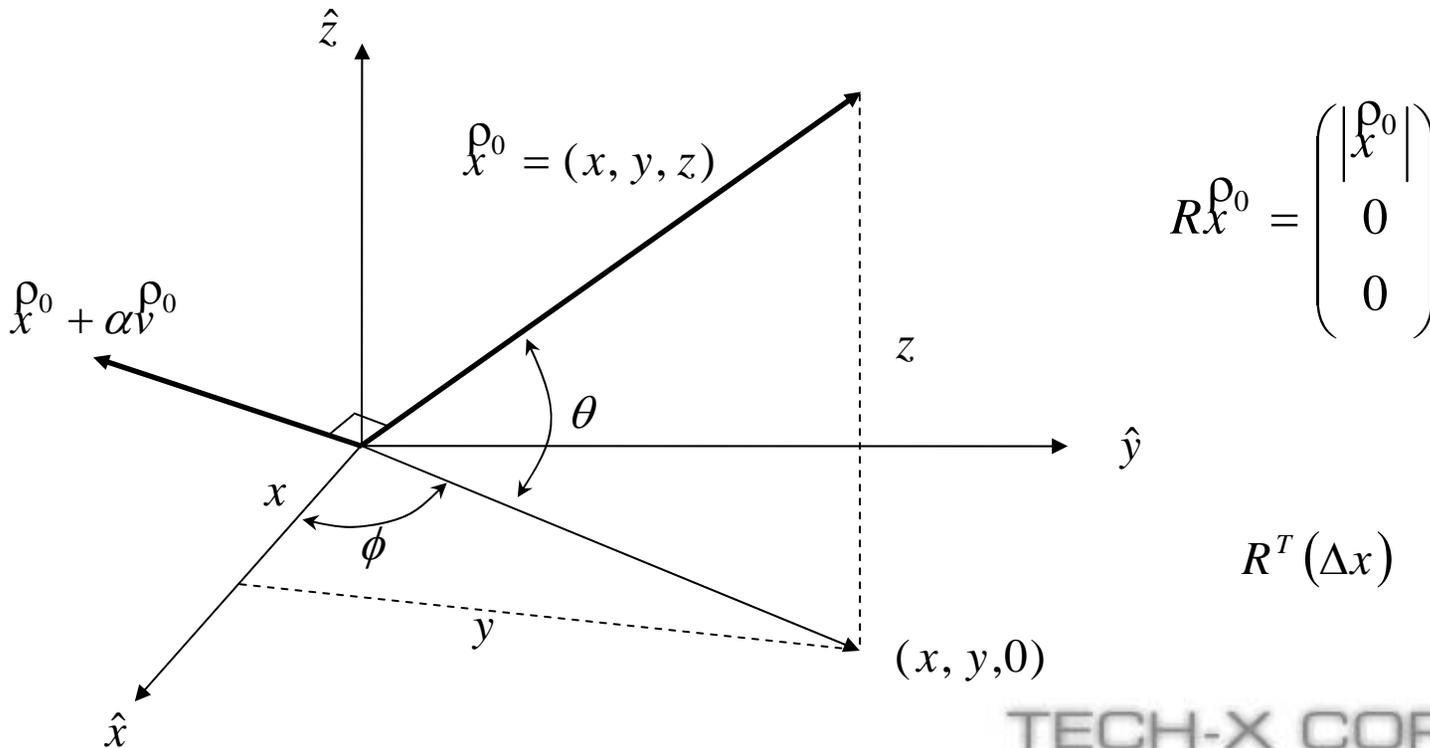
Replacing SC solenoid with a conventional wiggler offers lower cost & technical risk

- Why look for alternatives to solenoid design?
 - solenoid design & beam requirements for RHIC are challenging
 - 80 m, 5 T, superconducting, field errors $<10^{-5}$
- Advantages of a wiggler
 - like a solenoid, it provides focusing & suppresses recombination
 - modest fields (~ 10 Gauss) effectively reduce recombination via ‘wiggle’ motion of electrons:

$$\rho_w = \frac{\Omega_{gyro}}{k_w^2 v_{beam}} \sim 1.4 \times 10^{-3} \lambda_w^2 [m] B_w [G] / \gamma$$
 - e- bunch is easier: less charge and un-magnetized
 - lower construction costs; less technical risk
- What’s the effect of ‘wiggle’ motion on cooling?
 - independent suggestion of V. Litvinenko & Ya. Derbenev
 - increases ρ_{min} of Coulomb logarithm: $\rho_{min} \rightarrow \rho_w$
 - strong need for simulations

Semi-analytic “binary collision” algorithm handles arbitrary external fields

- Binary collision approach
 - neglects possibility of 3-body, 4-body etc. collisions
 - captures close binary collisions with great accuracy
 - find rotations (yaw, pitch, roll) to plane in which motion lies
 - “initial” positions & velocities obtained in this plane
 - then standard orbital parameters are calculated





VORPAL simulations support decision to use conventional wiggler for e- cooling of 100 GeV/n Au⁺⁷⁹

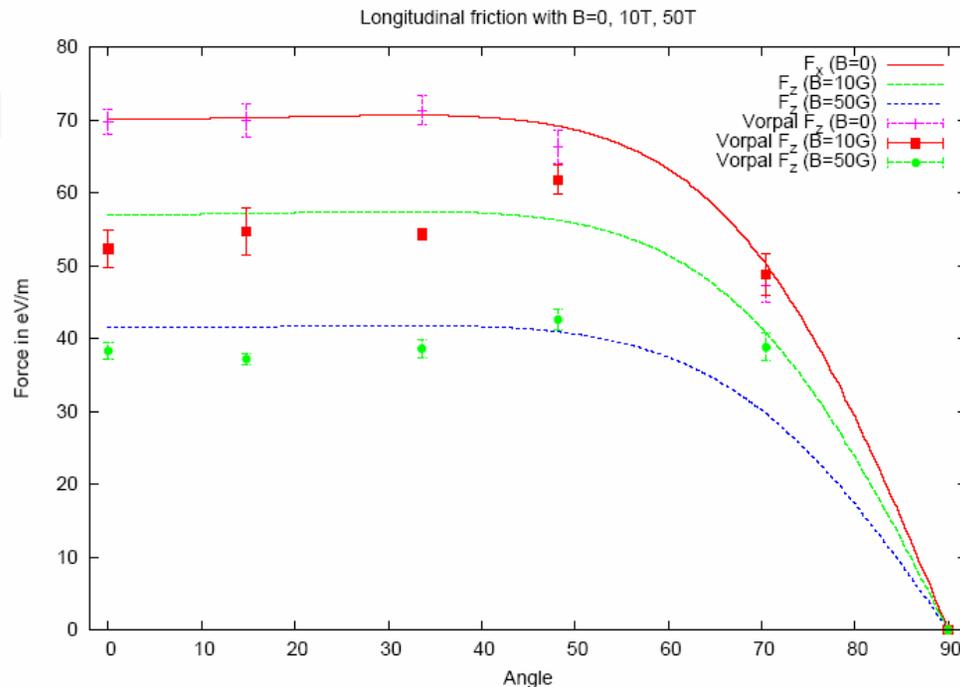
• Culmination of years of work, beginning in 2002

A.V. Fedotov, D.L. Bruhwiler, A. Sidorin, D. Abell, I. Ben-Zvi, R. Busby, J.R. Cary, and V.N. Litvinenko, "Numerical study of the magnetized friction force," Phys. Rev. ST Accel. Beams **9**, 074401 (2006).

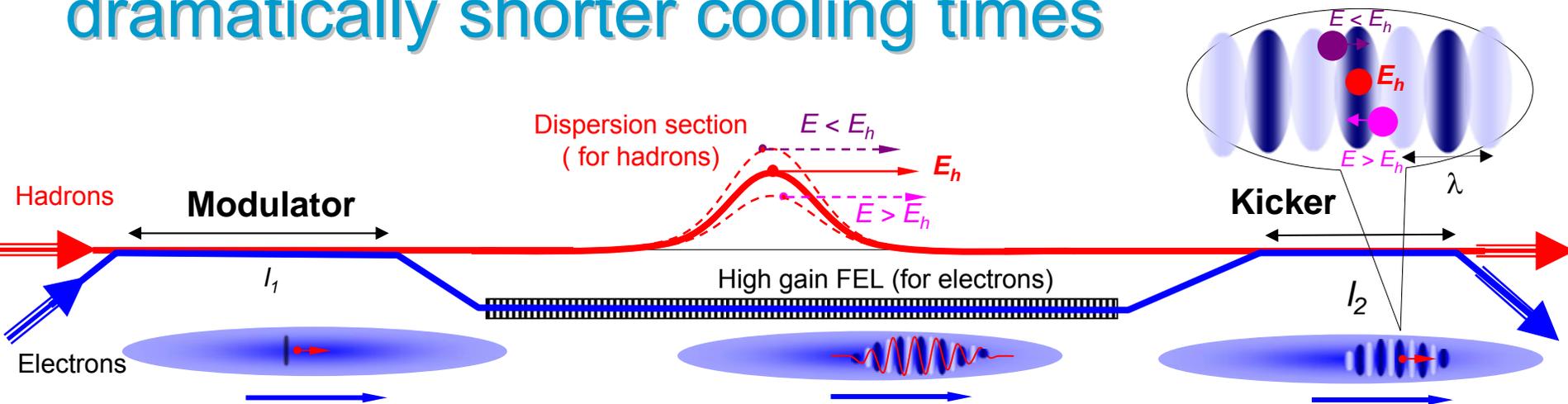
A.V. Fedotov, I. Ben-Zvi, D.L. Bruhwiler, V.N. Litvinenko and A.O. Sidorin, "High-energy electron cooling in a collider," New J. Phys. **8** (2006), p. 283.

G.I. Bell, D.L. Bruhwiler, A. Fedotov, A.V. Sobol, R. Busby, P. Stoltz, D.T. Abell, P. Messmer, I. Ben-Zvi and V.N. Litvinenko, "Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam", J. Comp. Phys. **227** (2008), p. 8714.

- Conventional wiggler could replace expensive solenoid
 - friction force is reduced only logarithmically



Coherent e- Cooling (CeC) offers dramatically shorter cooling times



Litvinenko & Derbenev, "Coherent Electron Cooling," Phys. Rev. Lett. 102, 114801 (2009).

- Coherent Electron Cooling concept
 - uses FEL to combine electron & stochastic cooling concepts
 - a CEC system has three major subsystems
 - **modulator:** the ions imprint a "density bump" on e- distribution
 - **amplifier:** FEL interaction amplifies density bump by orders of magnitude
 - **kicker:** the amplified & phase-shifted e- charge distribution is used to correct the velocity offset of the ions



CeC modulator looks like standard e- cooling section; key physics & sim.'s are very different

- **Very different from previous e- cooling simulations**

- previously, dynamical friction force was the key metric
 - details of close binary collisions were of essential importance
- now, the electron density and velocity wake is what matters
 - close binary collisions are not important

- **New algorithms are being used**

- close binary collisions required special algorithms
- electrostatic PIC, with noise reduction, can be used
 - however, noise levels are unacceptable
- most recently, δf PIC for higher fidelity
 - algorithm taken from the plasma fusion community

$$f \Rightarrow f_o + \delta f$$

Hu and Krommes, “Generalized weighting scheme for δf particle-simulation method,” Phys. Plasmas **1**, p. 863 (1994).
Xiang, Cary and Barnes, “Low-noise electromagnetic δf particle-in-cell simulation,” Phys. Plasma **13**, 062111 (2006).

For semi-infinite e- distributions, modulator has 4 dimensionless param's

- Infinite e- beam size
 - only 4 dimensionless parameters
 - courtesy, V. Litvinenko
 - finite beam size will be simulated in future

Parameter	Definition	Description
R	$R \equiv \sigma_{vx} / \sigma_{vz} = 3$	Ratio of transverse to longitudinal RMS velocity spread.
T	$T \equiv v_{ix} / \sigma_{vz}$	Ratio of transverse ion velocity to RMS velocity spread.
Z	$Z \equiv v_{iz} / \sigma_{vz}$	Ratio of longitudinal ion velocity to RMS velocity spread.
ζ	$\zeta \equiv Z_{ion} / (4 \pi n_e R^2 \lambda_D^3)$ $\zeta = 0.1$ in the following simulations	Plasma nonlinearity parameter.

- VORPAL uses MKS
 - use param's relevant to Au⁺⁷⁹ at RHIC

Parameter	Value	Definition
n_e	$1.60 \times 10^{16} \text{ e-}/\text{m}^3$	Electron Density
$\omega_p = (2\pi)8.98 n_e^{1/2}$	$7.14 \times 10^9 \text{ radians/second}$	Plasma frequency in radians per second
$f_p = 8.98 n_e^{1/2}$	$1.14 \times 10^9 \text{ cycles/second}$	Plasma frequency in cycles per second
$1/f_p$	0.88 nanoseconds	Plasma frequency time scale
$\lambda_D = \sigma_{vz} / \omega_p$	1.26 microns	Nominal longitudinal Debye radius
$(\sigma_{vx}, \sigma_{vy}, \sigma_{vz})$	$(27, 27, 9) \times 10^3 \text{ m/sec}$	RMS electron velocity spread



Comparison of simulations with theory is underway, with good results

- Recent analytical results for e- density wake

G. Wang and M. Blaskiewicz, Phys Rev E 78, 026413 (2008).

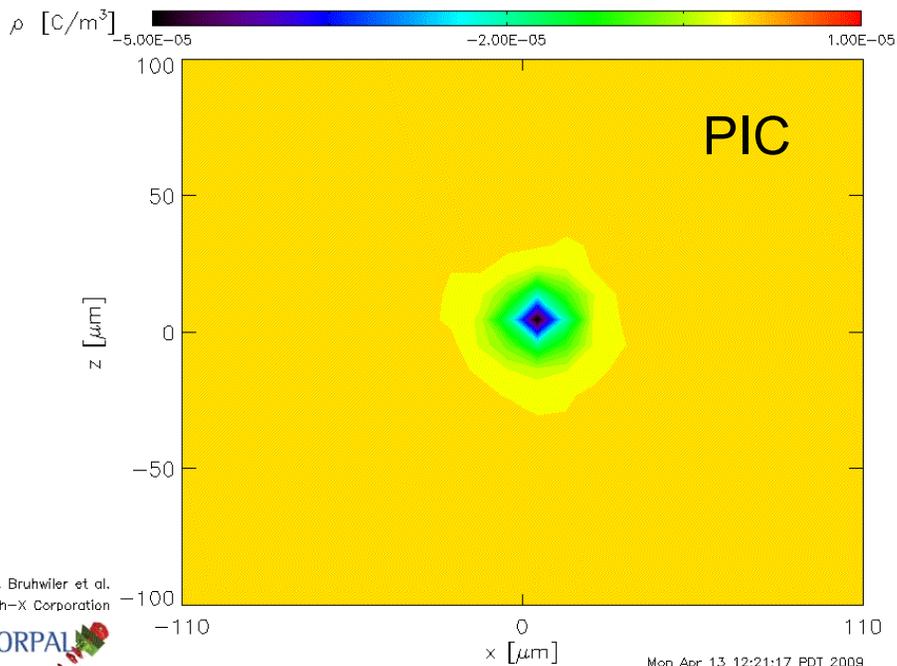
$$\tilde{n}(\mathbf{r}, t) = \frac{Zn_0\omega_p^3}{\pi^2\sigma_{vx}\sigma_{vy}\sigma_{vz}} \int_0^{\omega_p t} \tau \sin \tau \left(\tau^2 + \left(\frac{x - v_{hx}\tau/\omega_p}{r_{Dx}} \right)^2 + \left(\frac{y - v_{hy}\tau/\omega_p}{r_{Dy}} \right)^2 + \left(\frac{z - v_{hz}\tau/\omega_p}{r_{Dz}} \right)^2 \right)^{-2} d\tau$$

- theory makes certain assumptions:
 - single ion; arbitrary velocities
 - uniform e- density; *anisotropic* temperature
 - Lorentzian velocity distribution
 - now implemented in VORPAL
 - linear plasma response; *fully 3D*
- Dynamic response extends over many λ_D and $1/\omega_{pe}$
 - thermal ptcl boundary conditions are important

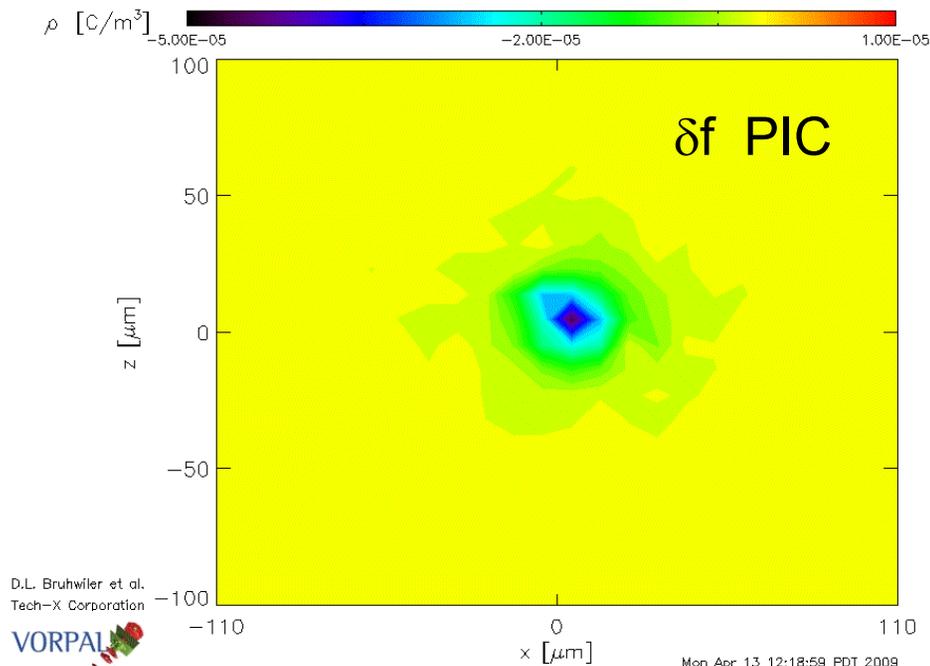


Electrostatic PIC can be used to simulate the electron response in an idealized modulator, but δf PIC is faster, quieter & more accurate

Data file: /scratch/scratchdirs/bruhwile/m327/k-es-1c/k-es-1c_SumRhoJ_1.h5



Data file: /scratch/scratchdirs/bruhwile/m327/k-df-1c/k-df-1c_SumRhoJ_1.h5



- Movies above show 2D slice through e- density of 3D simulations
 - R=1 (isotropic e- temperatures); T=Z=0 (stationary ion)



PIC shows ~30% deviations from theory

- Movie shows 1D integral of e- density perturbation

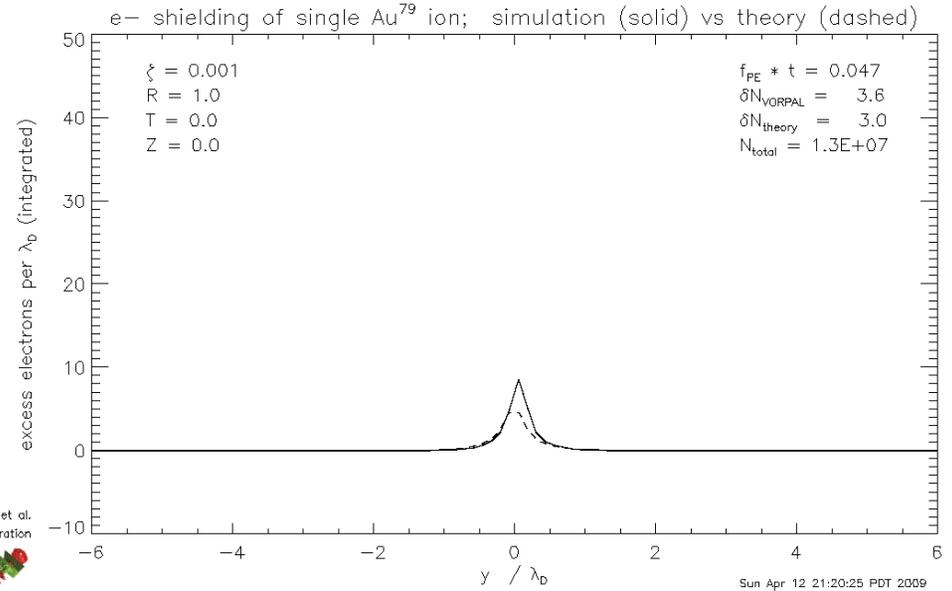
$$F(z) = \int f_e(\bar{\rho} - \hat{z} \cdot Z\tau, \bar{v}, \tau) d\bar{v}^3 dx dy$$

- R=1 (isotropic e- temperatures)
- T=Z=0 (stationary ion)
- local deviations < 50%

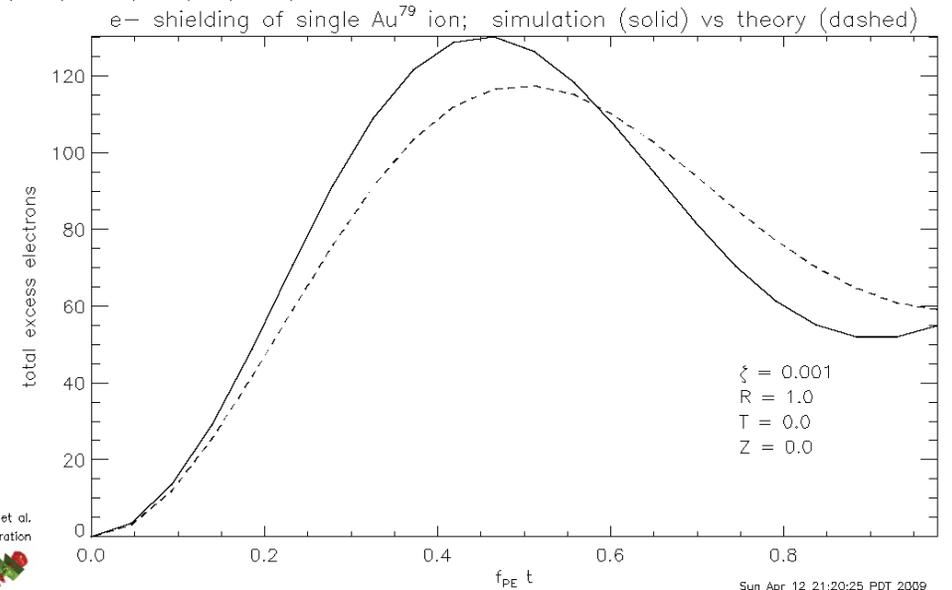
- Total e- shielding is shown in figure below

- peak response is seen after 1/2 of a plasma period
- integrated deviations ~30%

Data file: /scratch/scratchdirs/bruhwile/m327/k-es-1c/k-es-1c_SumRho.L1.h5



File sequence: /scratch/scratchdirs/bruhwile/m327/k-es-1c/k-es-1c_SumRho.L1.h5 ; N = 1-21



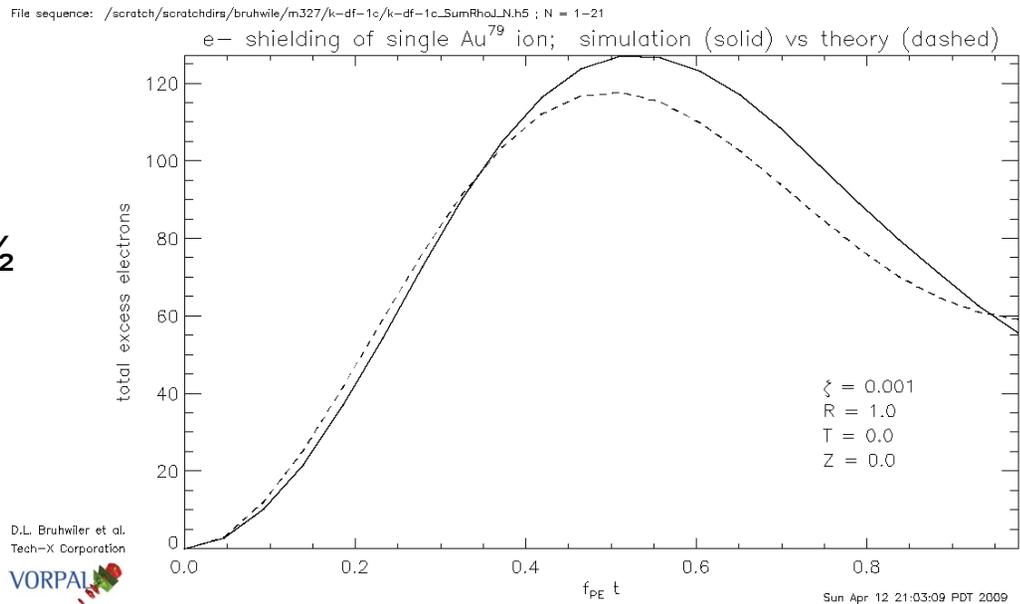
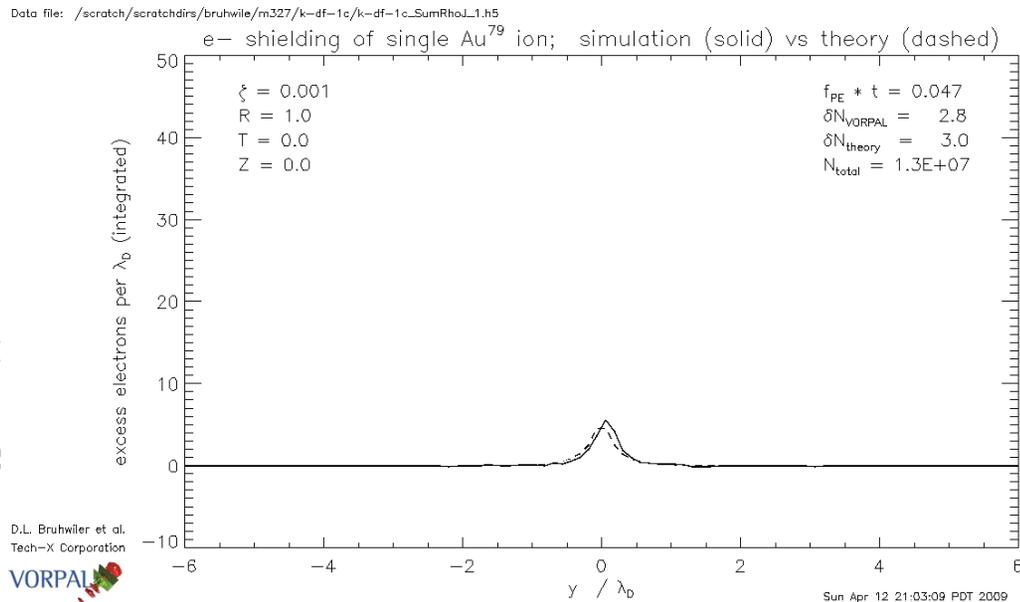
δf PIC shows $\sim 10\%$ deviations from theory

- Movie shows 1D integral of e- density perturbation

$$F(z) = \int f_e(\bar{\rho} - \hat{z} \cdot Z\tau, \bar{v}, \tau) d\bar{v}^3 dx$$

- R=1 (isotropic e- temperature)
- T=Z=0 (stationary ion)

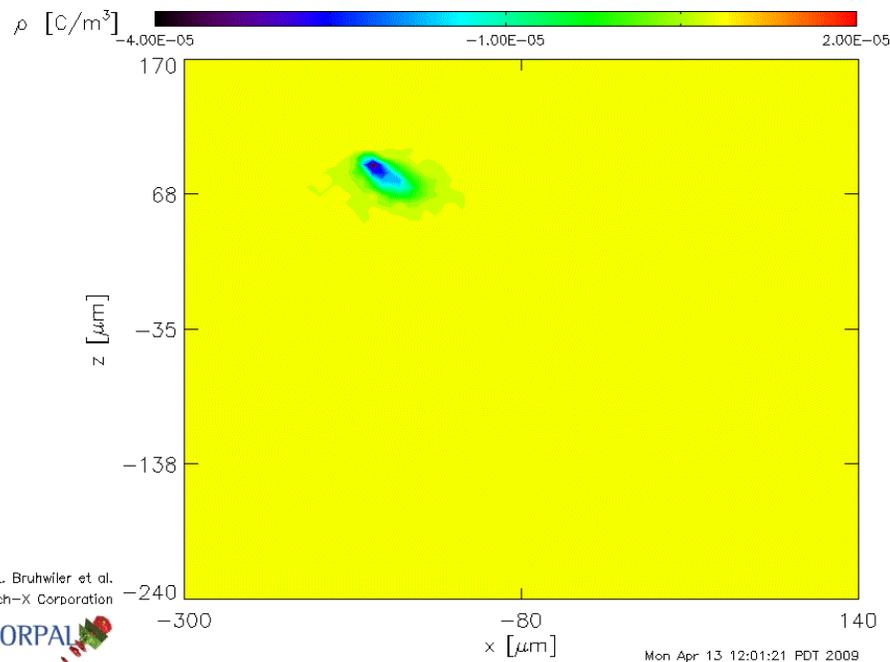
- Total e- shielding is shown in figure below
 - peak response is seen after $\frac{1}{2}$ of a plasma period
- $\sim 5x$ faster than PIC, due to fewer particles per cell



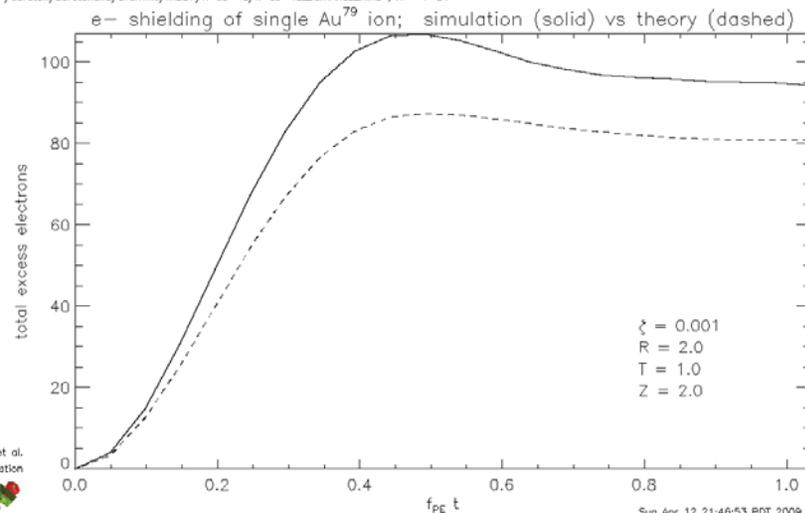
Simulated e- response to ion moving in x & z; anisotropic e- temp.

- Au⁺⁷⁹ ion is moving in both x & z
 - R=2 (transverse e- temp. 4x larger)
 - T=1 ($v_x = -1 * v_{e,x,rms}$)
 - Z=2 ($v_z = 2 * v_{e,z,rms}$)
- Total e- shielding is shown in figure below
 - peak response is seen after 1/2 of a plasma period
 - subsequent oscillation (for stationary ion) is not seen
- Deviations from theory ~20%
 - δf algorithm is not yet working in VORPAL for moving ion

Data file: /scratch/scratchdirs/bruhwiler/m327/k-es-4d/k-es-4d_SumRhoJ_1.h5

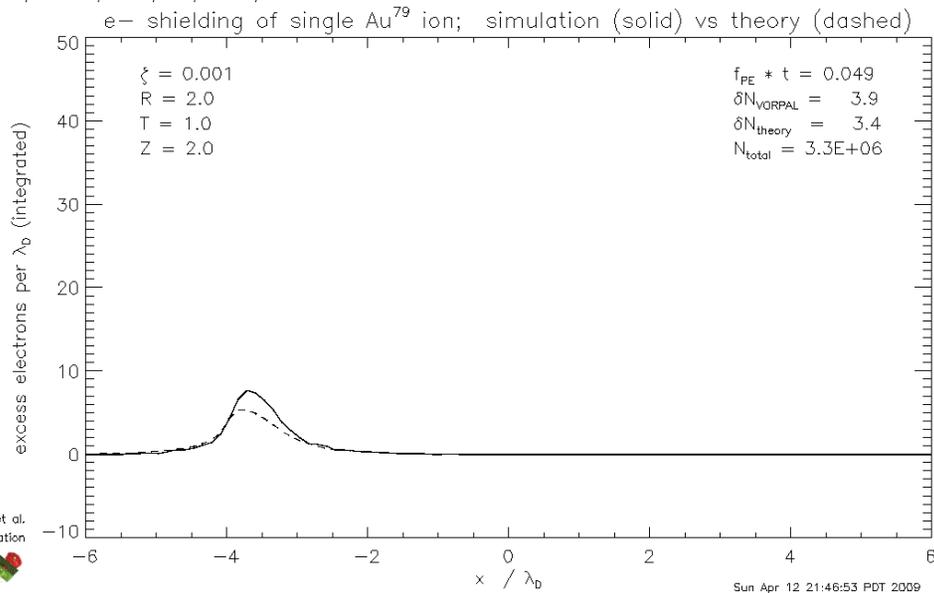


File sequence: /scratch/scratchdirs/bruhwiler/m327/k-es-4d/k-es-4d_SumRhoJ_N.H5 ; N = 1-21



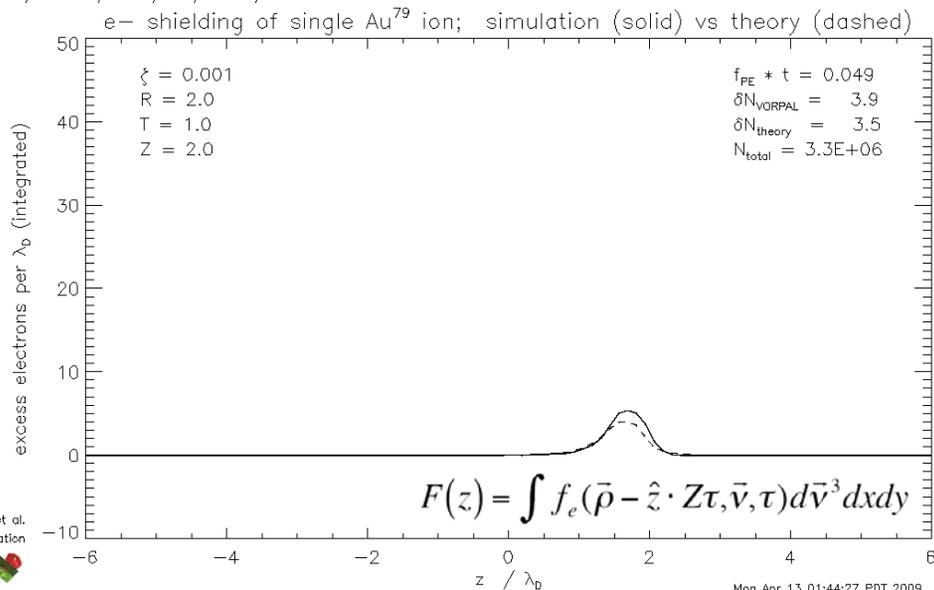
Simulated e- response to ion moving in x & z; anisotropic e- temp.

Data file: /scratch/scratchdirs/bruhwile/m327/k-es-4d/k-es-4d_SumRhoL1.h5



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Data file: /scratch/scratchdirs/bruhwile/m327/k-es-4d/k-es-4d_SumRhoL1.h5



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- Au⁷⁹ ion is moving along z-axis
 - R=2 (transverse e- temp. 4x larger)
 - T=1 ($v_x = -1 * v_{e,x,rms}$)
 - Z=2 ($v_z = 2 * v_{e,z,rms}$)
- Total e- shielding is shown in lower figure
 - peak response is seen after 1/2 of a plasma period
 - subsequent oscillation (for stationary ion) is not seen
- Deviations from theory ~30%
 - δf algorithm is not yet working in VORPAL for moving ion

Plans for future work

TECH

- “Conventional high-energy electron cooling
 - effect of magnetic field errors has not been fully explored
 - initial results from Sobol *et al.*; see poster Friday morning
 - confirmed speculation of S. Nagaitsev regarding wavelength dependence
- Coupled simulations of complete system
 - next, δf macro-particles from VORPAL coupled into FEL code
 - Planning to use GENESIS 3D
- Non-ideal modulator simulations (no theory)
 - consider effects of finite e- beam size
 - density gradients, vacuum interfaces, bulk space charge
 - distorted wakes; reflections from vacuum interface
 - no theory with which to compare
 - can't trust basic ES PIC → need algorithm for benchmarking
 - consider multiple ions (nonlinearities important?)

