Monte Carlo Mean Field Treatment of Microbunching Instability in the FERMI@Elettra First Bunch Compressor

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1. Self Consistent Vlasov-Maxwell Treatment
2. Field Calculation and Density Estimation
3. Microbunching Instability Studies
4. Discussion

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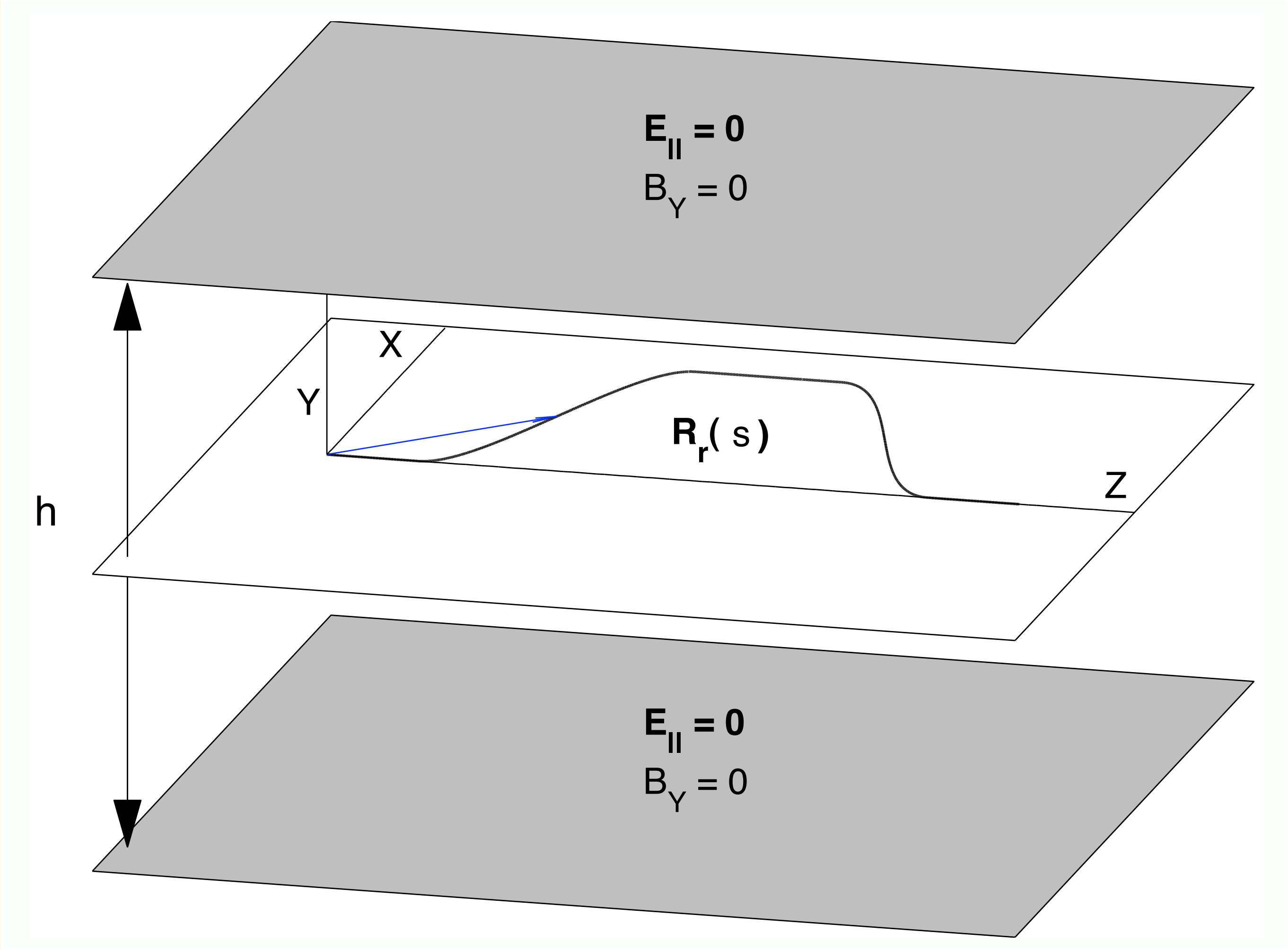
Basic Lab Frame Setup



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Self Consistent Vlasov-Maxwell Treatment

$3D$ Wave equation in lab frame with "$2D$" planar source:

$$(\partial_Z^2 + \partial_X^2 + \partial_Y^2 - \partial_u^2)\mathcal{E} = H(Y)\mathcal{S}(\mathbf{R}, u), \quad \mathcal{E}(\mathbf{R}, Y = \pm g, u) = 0.$$

where $u = ct$, $\mathcal{E}(\mathbf{R}, Y, u) = (E_Z, E_X, B_Y)$, $\mathbf{R} = (Z, X)$.

Vlasov equation in beam frame:

$$f_s - \kappa(s)xf_z + F_z f_{p_z} + p_x f_x + [\kappa(s)p_z + F_x]f_{p_x} = 0$$

where

$$\begin{aligned} F_z &= \frac{e}{v_r E_r}\mathbf{V}\cdot\mathbf{E}, \\ F_x &= \frac{e}{E_r\beta_r^2}\Big[-X_r'(s)E_Z + Z_r'(s)E_X - v_r B_Y)\Big], \end{aligned}$$

and $\mathbf{V} = v_r(\mathbf{t}(s) + p_x\mathbf{n}(s))$, $\mathbf{E} = (E_Z, E_X)$ and B_Y are evaluated at $\mathbf{R} = \mathbf{R}_r(s) + x\mathbf{n}(s)$ and $u = (s-z)/\beta_r$.

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Field Calculation and Density Estimation

Field formula:

$$\mathcal{E}(\mathbf{R},u) := \int_{-g}^{g} H(Y)\mathcal{E}(\mathbf{R},Y,u)dY \overset{H(Y)=\delta(Y)}{=} -\frac{1}{2\pi}\sum_{k=0}^{\infty} a_k \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta\ \mathcal{S}(\hat{\mathbf{R}},v,k)$$

where $\hat{\mathbf{R}} = \mathbf{R} + \sqrt{(u-v)^2-(kh)^2}(\cos\theta,\sin\theta)$ and $a_k = (-1)^k(1-\delta_{k0}/2)$.

- localization in θ for $v \ll u-kh \Longrightarrow \int d\theta$ with superconvergent trapezoidal rule
- non uniform behavior in $v \Longrightarrow \int dv$ with adaptive Gauss-Kronrod rule

Density estimation: from scattered beam frame points at s → smooth/global lab frame charge/current density via a 2D Fourier method.

1D Example: 1D orthogonal series estimator of $f(x)$, $x \in [0,1]$

$$f_J(x) := \sum_{j=0}^{J}\theta_j\phi_j(x), \quad \theta_j = \int_0^1 \phi_j(x)f(x)dx, \quad \phi_0(x)=1, \phi_j(x)=\sqrt{2}\cos(j\pi x), j=1,2,...$$

Since $f(x)$ is a probability density (X,X_n random variables distributed via f)

$$\theta_j = E\{\phi_j(X)\}, \quad \text{thus from Monte Carlo a natural estimate is} \quad \hat{\theta}_j := \frac{1}{N}\sum_{n=1}^{N}\phi_j(X_n)$$

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Beam to Lab Charge/Current Density Transformation

- To solve Maxwell equations in lab frame must express lab frame charge/current density in terms of beam frame phase space density
- To a good approximation lab frame charge/current densities are

$$\rho_L(\mathbf{R},Y,u) = H(Y)\rho_B(\mathbf{r},\beta_r u) \ ,$$
$$\mathbf{J}_L(\mathbf{R},Y,u) = \beta_r c H(Y)[\rho_B(\mathbf{r},\beta_r u)\mathbf{t}(\beta_r u+z) + \tau_B(\mathbf{r},\beta_r u)\mathbf{n}(\beta_r u+z)],$$

$$\rho_B(\mathbf{r},s) = Q\int dp_z dp_x f(\zeta,s), \quad \tau_B(\mathbf{r},s) = Q\int dp_z dp_x p_x f(\zeta,s),$$

where $\zeta = (z,p_z,x,p_x)$

Remark: subtlety in the change of independent variable u=ct $\rightarrow$ s

Derivation to be published in a forthcoming paper

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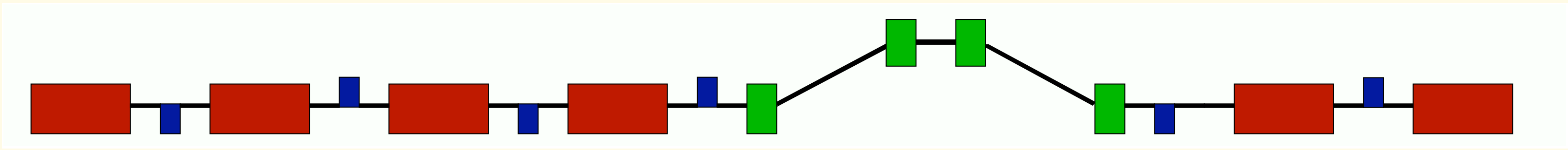
Microbunching in FERMI@Elettra First Bunch Compressor

Microbunching can cause an instability which degrades beam quality

This is a major concern for free electron lasers where very bright electron beams are required

FERMI@Elettra first bunch compressor system proposed as a benchmark for testing codes at the first Workshop on Microbunching Instability held in Trieste in 2007



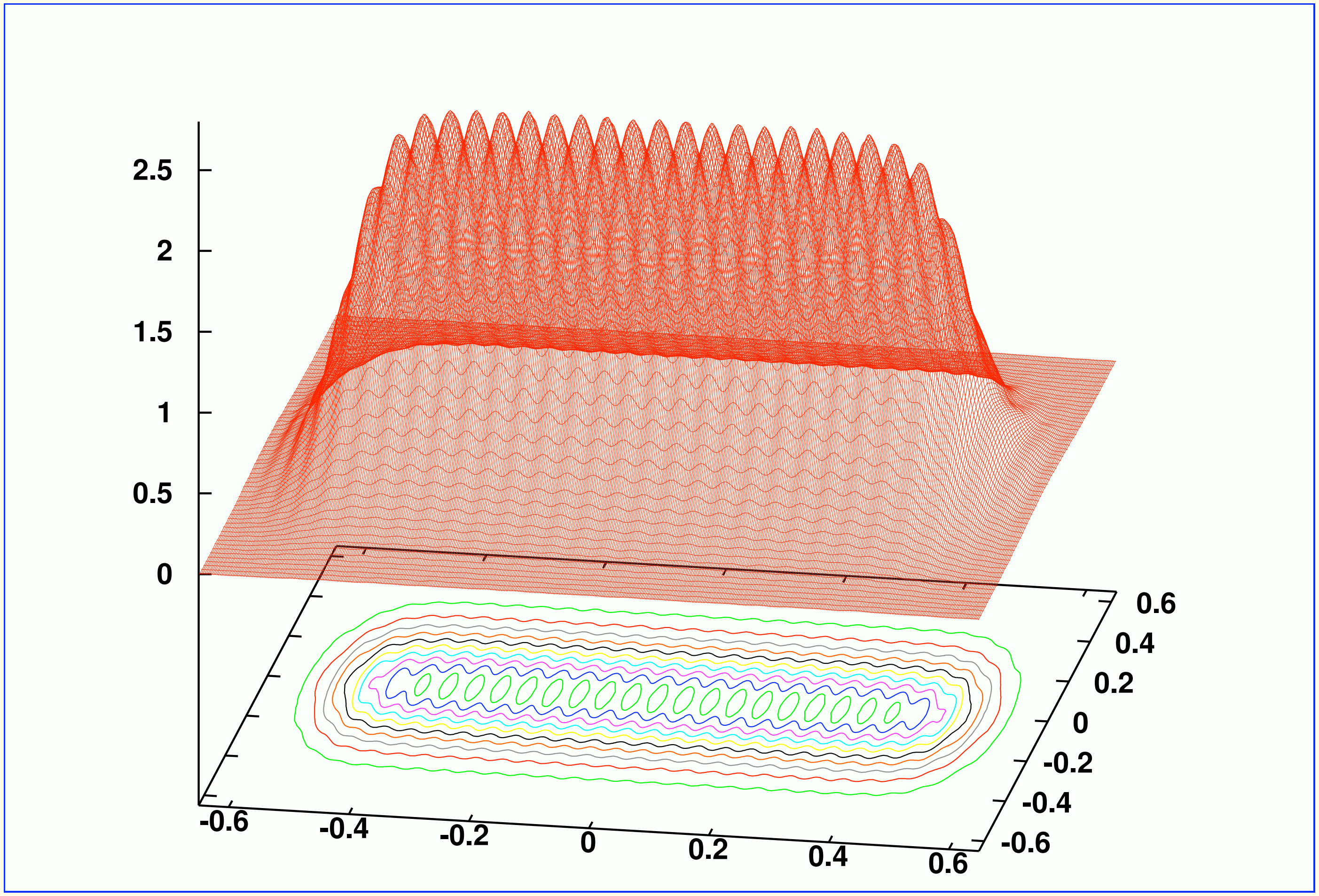
Layout first bunch compressor system

FERMI@Elettra First Bunch Compressor Parameters

Table 1: Chicane parameters and beam parameters at first dipole

Parameter	Symbol	Value	Unit
Energy reference particle	E_r	233	MeV
Peak current	I	120	A
Bunch charge	Q	1	nC
Norm. transverse emittance	$\gamma\epsilon_0$	1	μm
Alpha function	α_0	0	
Beta function	β_0	10	m
Linear energy chirp	h	-12.6	1/m
Uncorrelated energy spread	σ_E	2	KeV
Momentum compaction	R_{56}	0.057	m
Radius of curvature	ρ_0	5	m
Magnetic length	L_b	0.5	m
Distance 1st-2nd, 3rd-4th bend	L_1	2.5	m
Distance 2rd-3nd bend	L_2	1	m

Initial 2D Spatial Density



Initial spatial density in grid coordinates for A=0.05, $\lambda_0 = 100\mu$m.

Init. phase space density $= (1 + A\cos(2\pi z/\lambda_0))\mu(z)\rho_c(p_z - hz)g(x, p_x)$.

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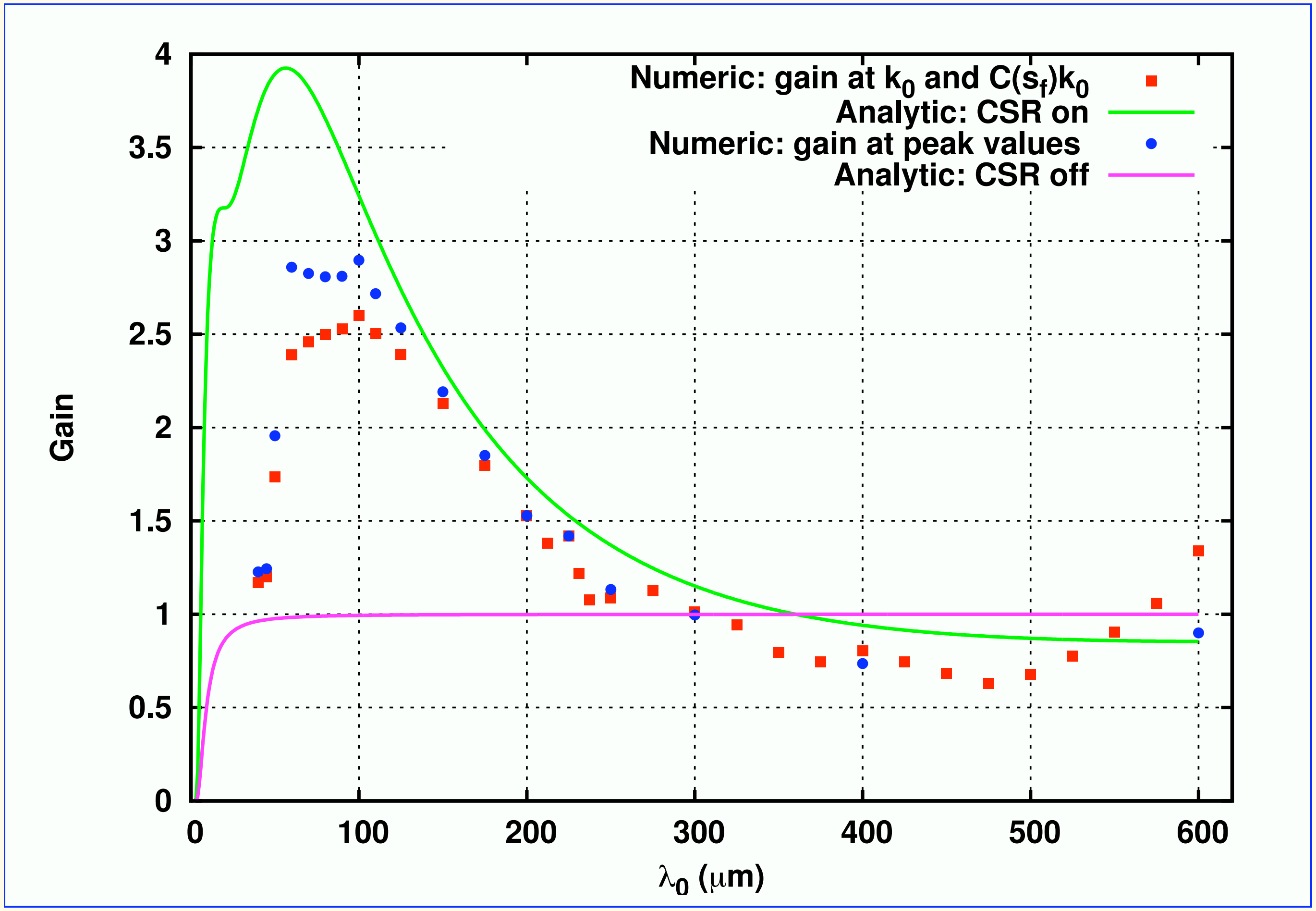
Gain factor



Gain $:= |\tilde{\rho}(k_f, s_f)/\tilde{\rho}(k_0, 0)|$, $\tilde{\rho}(k,s) = \int dz \exp(-ikz)\rho(z,s)$ and $k_f = C(s_f)k_0$ for $\lambda_0 = 2\pi/k_0$. Here $C(s_f) = 1/(1 + hR_{56}(s_f)) = 3.54$, $s_f = 8.029$m.

H.Huang and K.Kim, PRSTAB **5**, 074401, 129903 (2002); S.Heifets, G.Stupakov and S.Krinsky, PRSTAB **5**, 064401 (2002).

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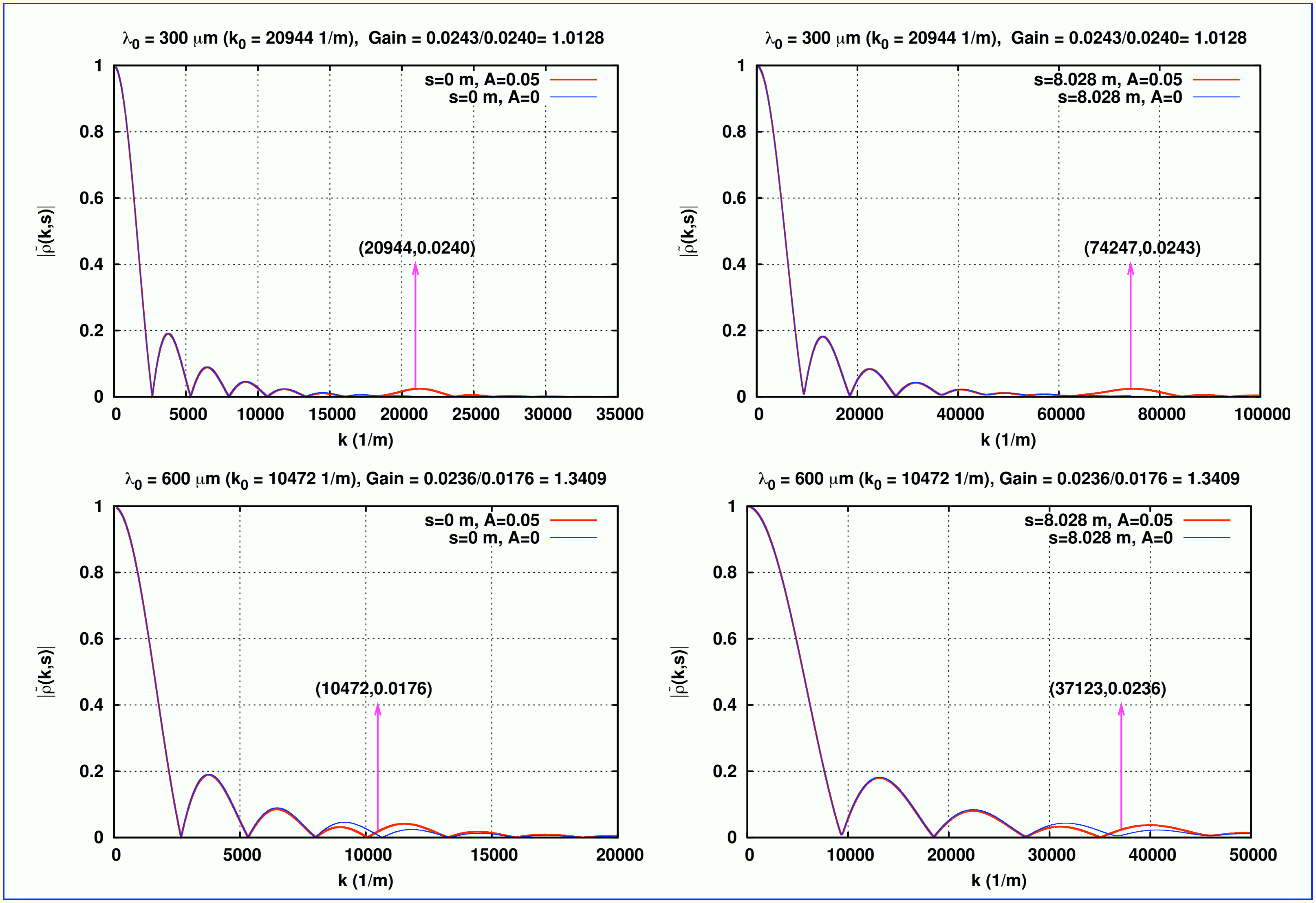
Spectra Longitudinal Density I



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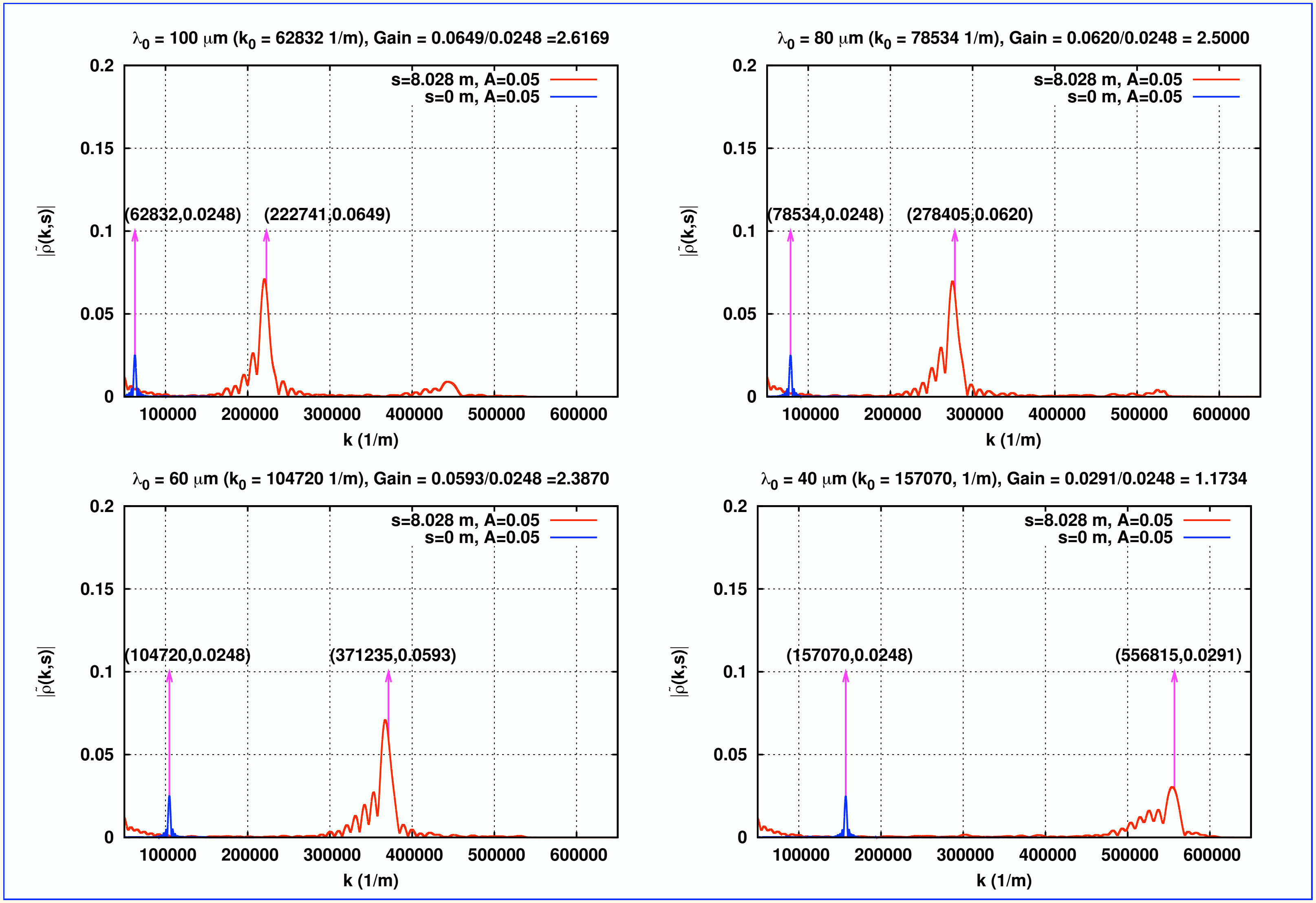
Spectra Longitudinal Density II



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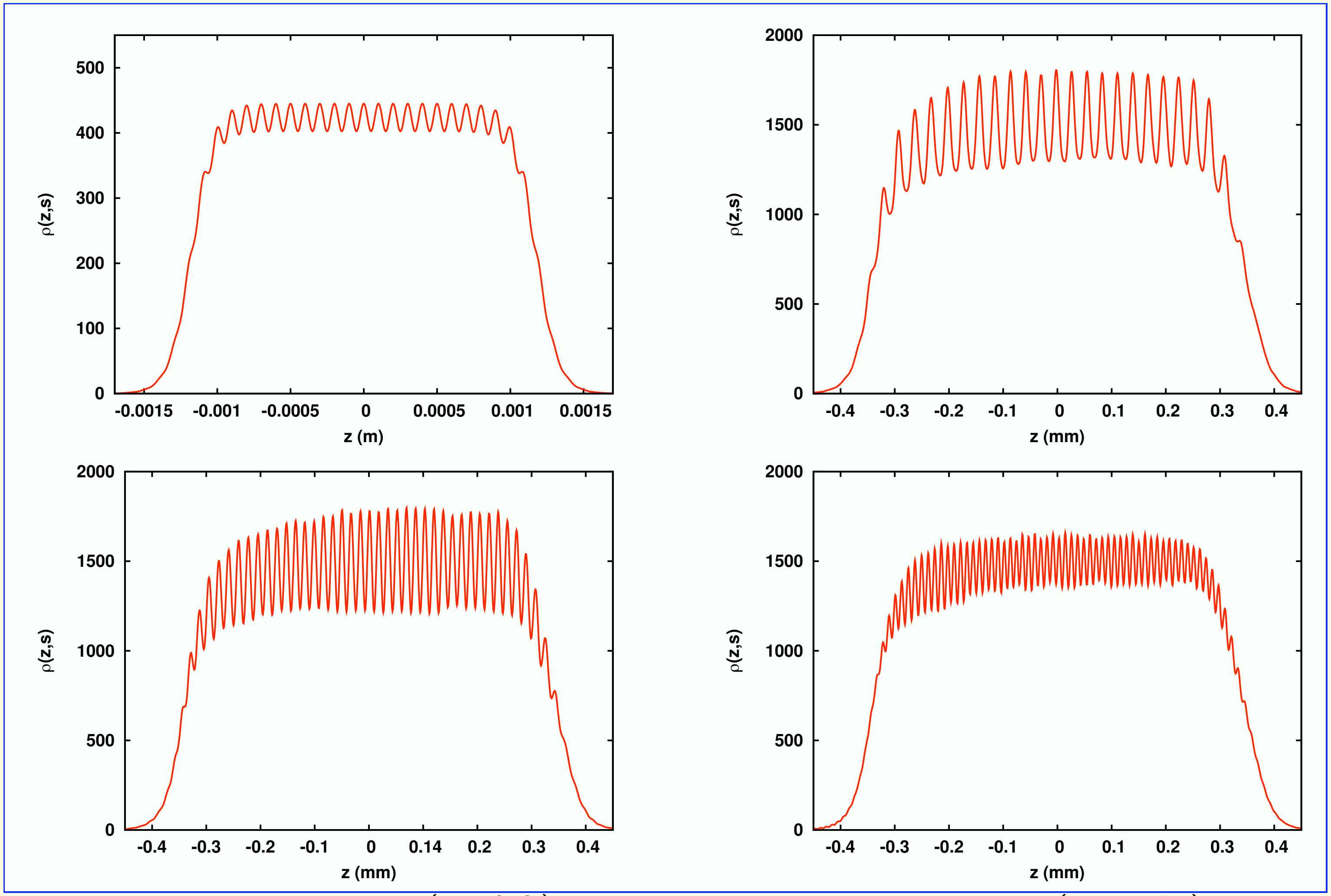
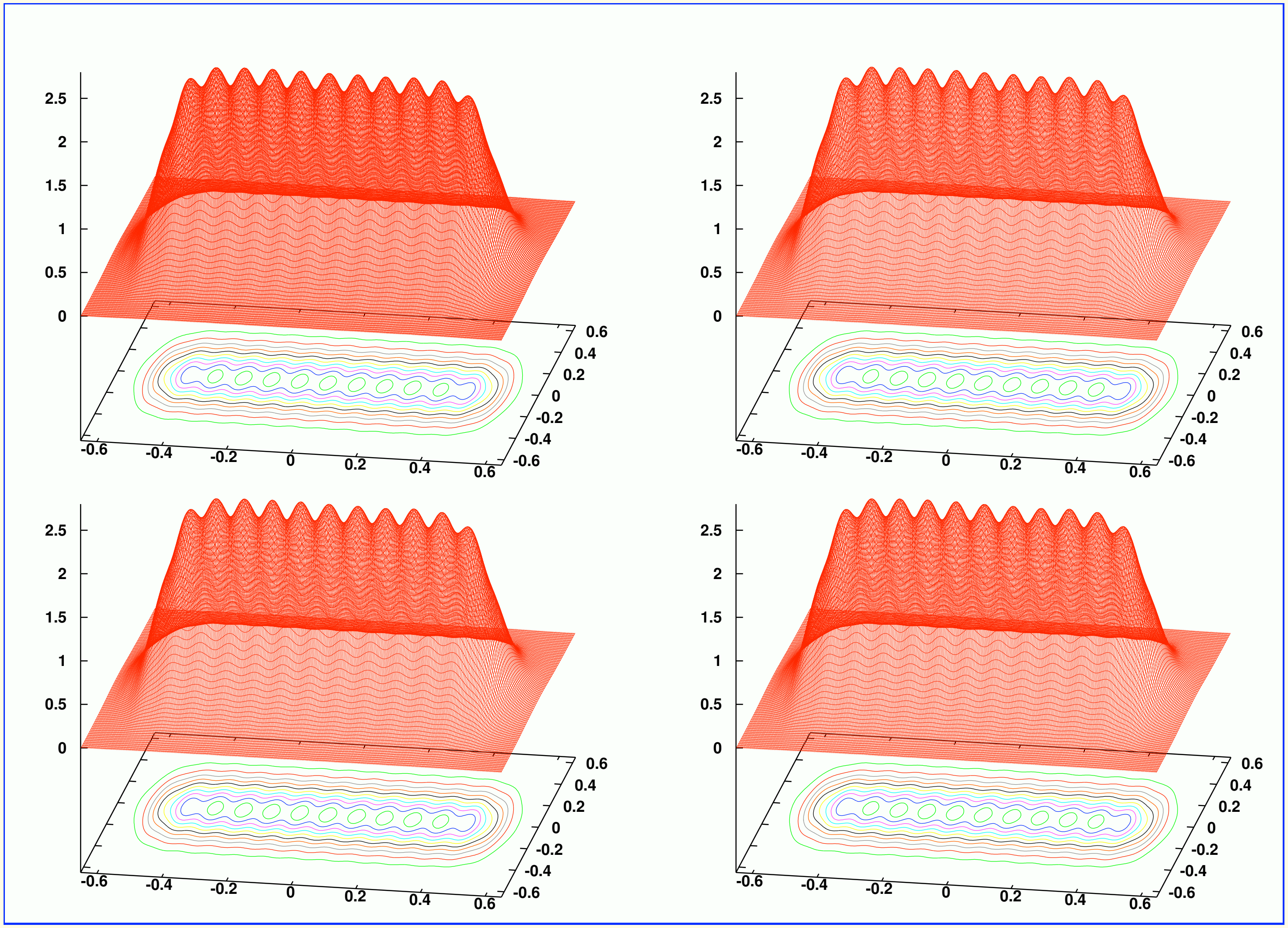
Longitudinal Density



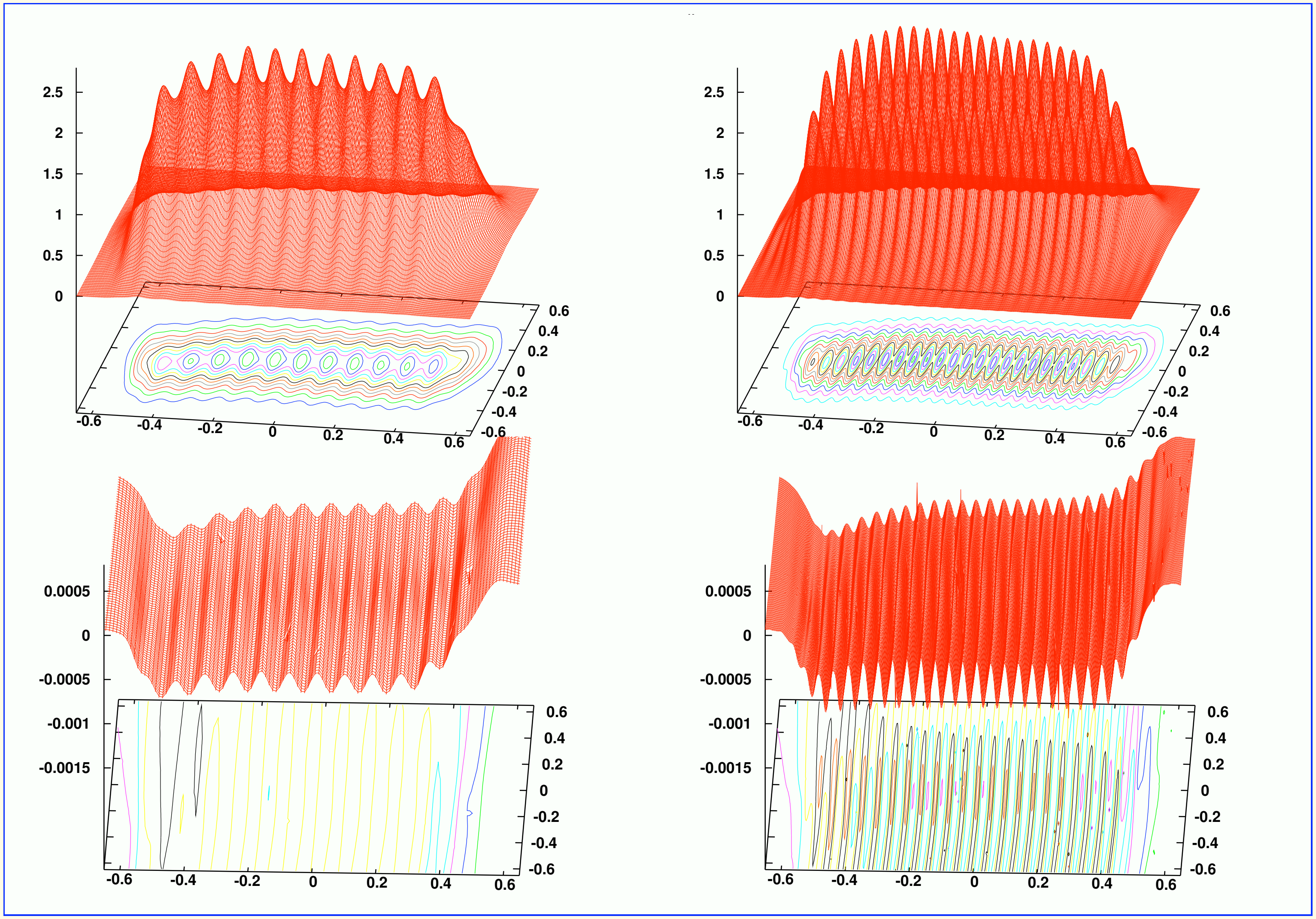
$\lambda_0 = 100\mu$m at $s = 0$ (top left), $\lambda_0 = 100\mu$m at $s = s_f$ (top right),
$\lambda_0 = 60\mu$m at $s = s_f$ (bottom left), $\lambda_0 = 40\mu$m at $s = s_f$ (bottom right).

Stationary Grid



λ_0=200μm. s=0.25s_f (top left), s=0.5s_f (top right), s=0.75s_f (bottom left), s=s_f (bottom right).

2D spatial density and longitudinal force at $s = s_f$



$\lambda_0 = 200\mu$m (top left), $\lambda_0 = 100\mu$m (top right), $\lambda_0 = 200\mu$m (bottom left), $\lambda_0 = 100\mu$m (bottom right)

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Discussion

- FERMI@Elettra microbunching studies at $\lambda_0 \geq 40\mu$m:
  - Very small effect of μBI on mean power and transverse emittance
  - Gain factor at long wavelengths shows breakdown coasting beam assumption
  - Gain factor at short wavelengths indicates deviations from analytical gain formula
  - A paper has been submitted to PRSTAB
- Work in progress and future work:
  - Study wavelengths shorter than $\lambda_0 = 40\mu$m
  - Study dependence on the amplitude of the initial modulation and on the uncorrelated energy spread
  - Study initial perturbation with more than one frequency
  - Complete studies for benchmark microbunching instability including RF cavities

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Computational Issues

- Intensive memory requirement and expensive computational cost:
  - Typical simulations done on the parallel clusters ENCANTO in New Mexico and NERSC at LBNL: N procs = 200-1000, N particles = 2×10^7-5×10^8, few hours of CPU time
  - Memory requirement: for $\lambda_0 = 50\mu$m store 3D array of dimension $1500 \times 128 \times 200$ on master processor (to avoid massive communications between slave processors)
- To reduce storage/computational cost:
  - Analytical work + state of the art numerical techniques: integration, interpolation, density estimation
  - Parallel computing

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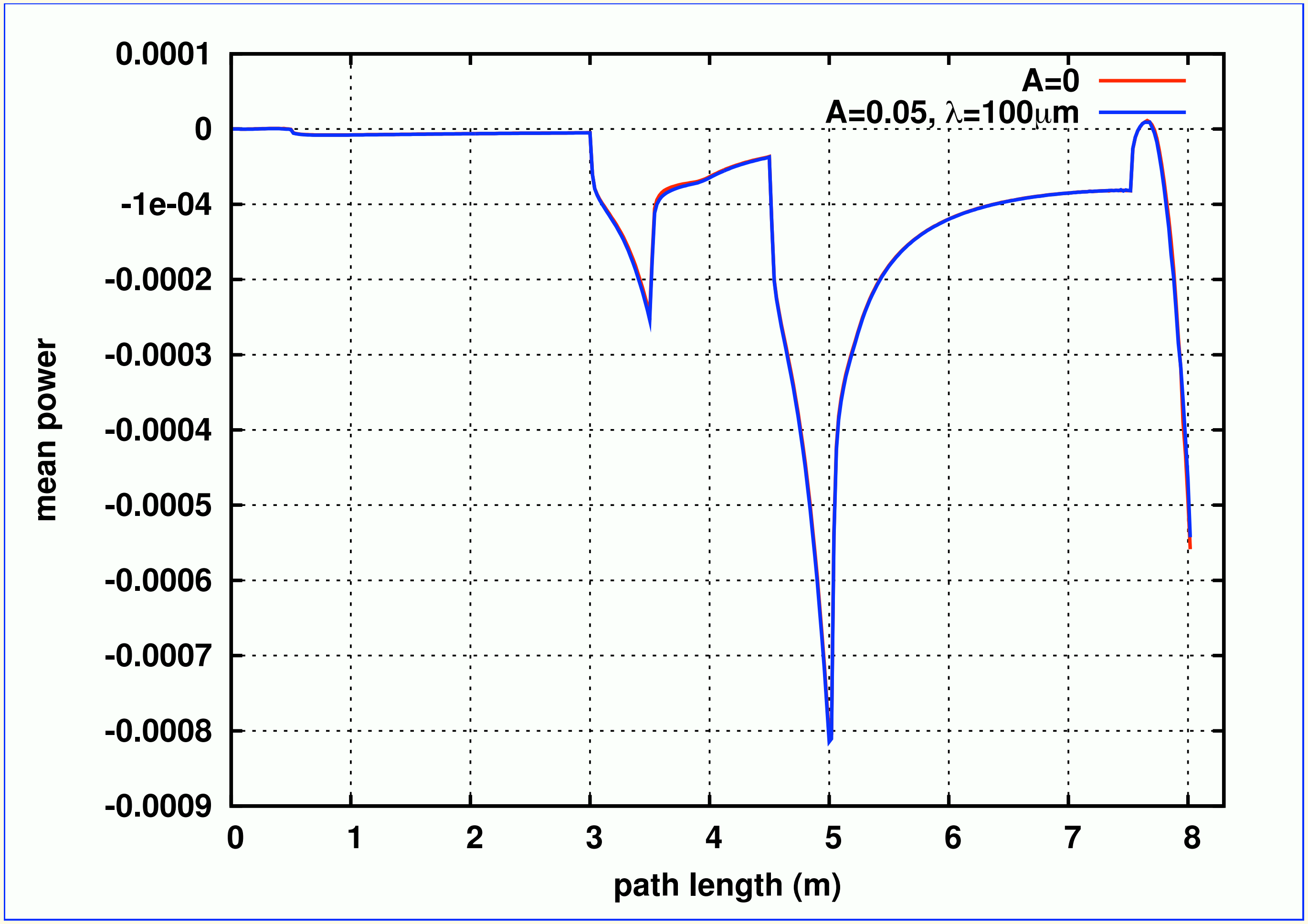
FERMI@Elettra First Bunch Compressor II



Mean power

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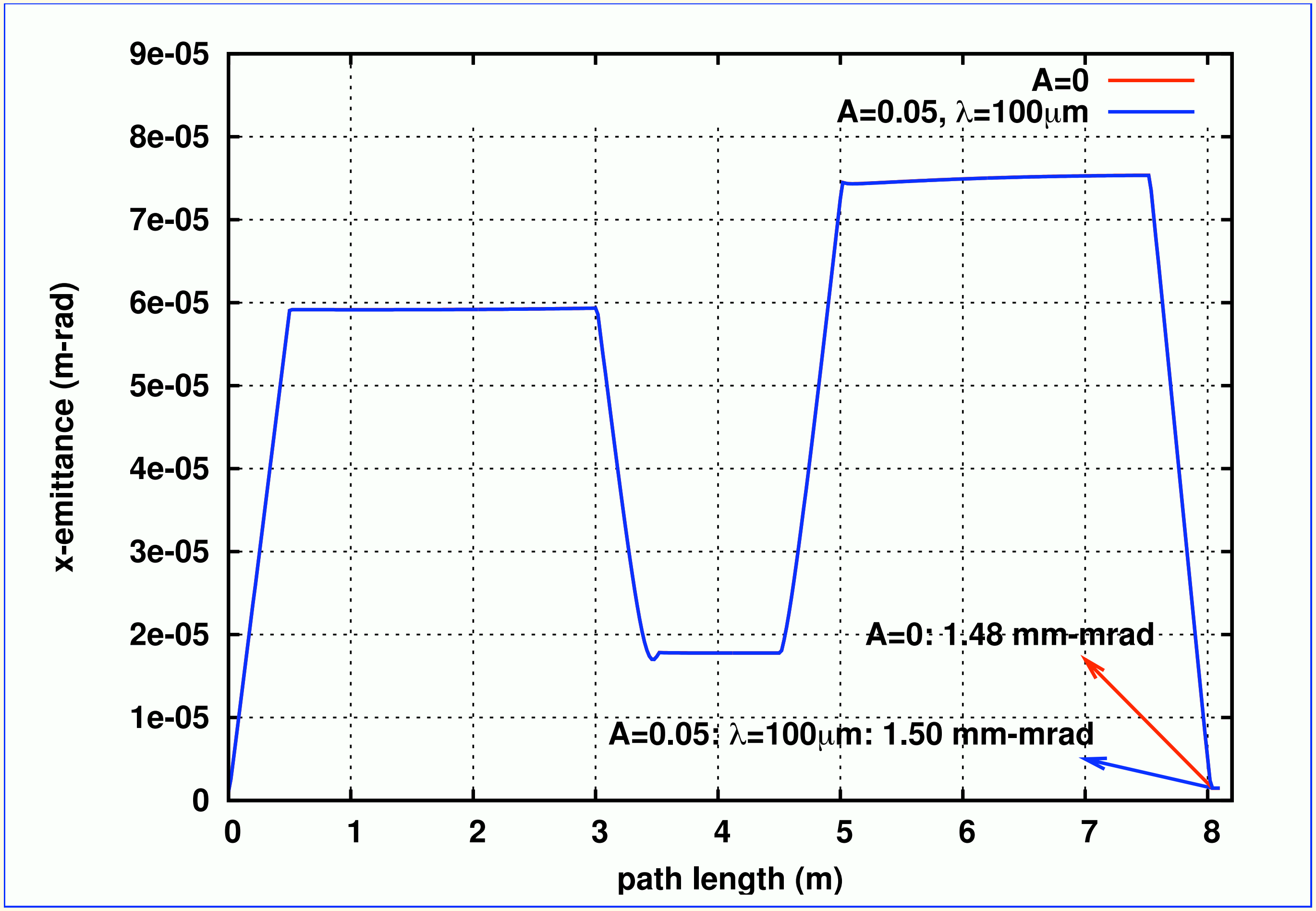
FERMI@Elettra First Bunch Compressor III



x-emittance

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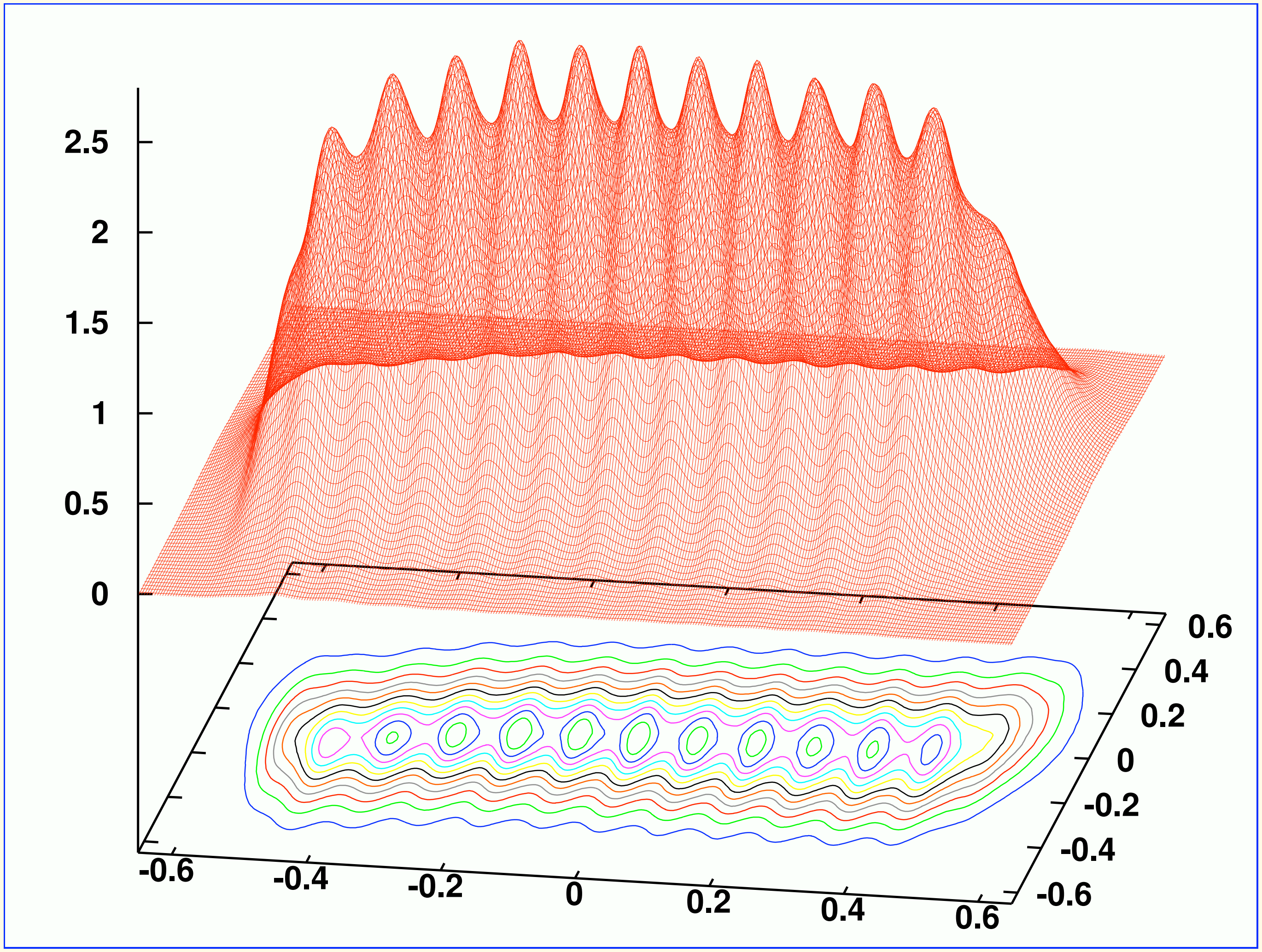
2D spatial density in grid coordinates at $s = s_f$ for $\lambda_0 = 200\mu$m



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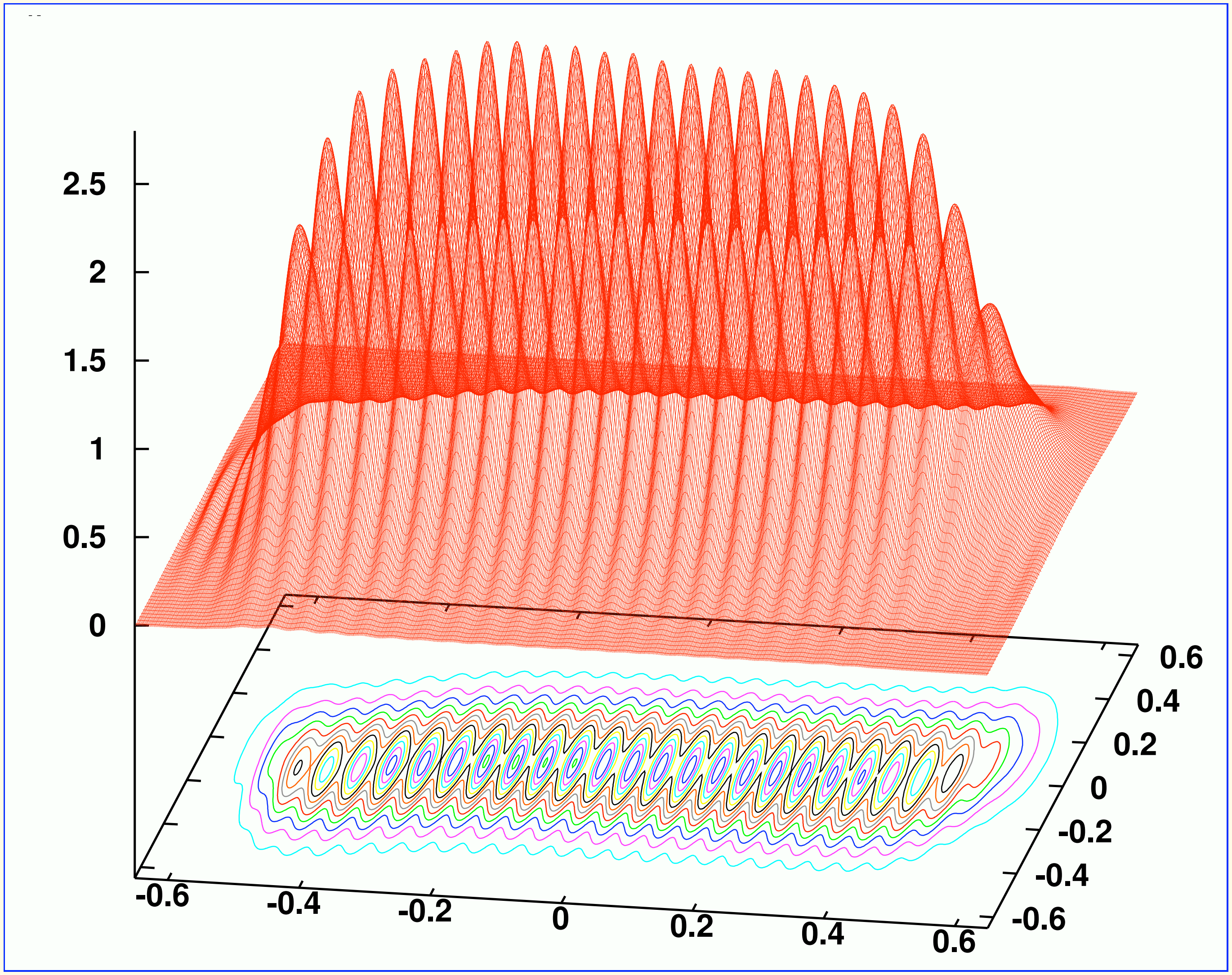
2D spatial density in grid coordinates at $s = s_f$ for $\lambda_0 = 100\mu$m



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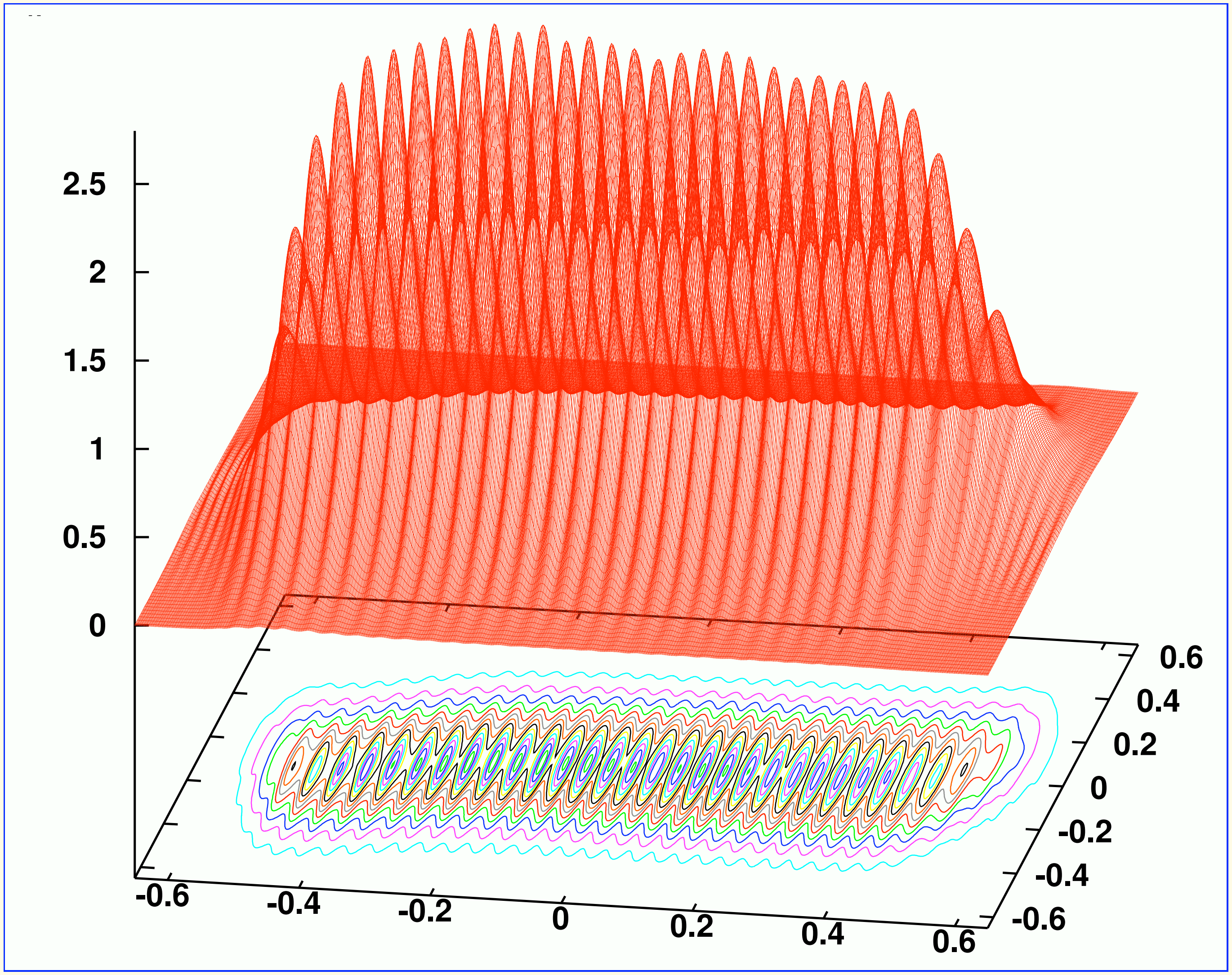
2D spatial density in grid coordinates at $s = s_f$ for $\lambda_0 = 80\mu$m



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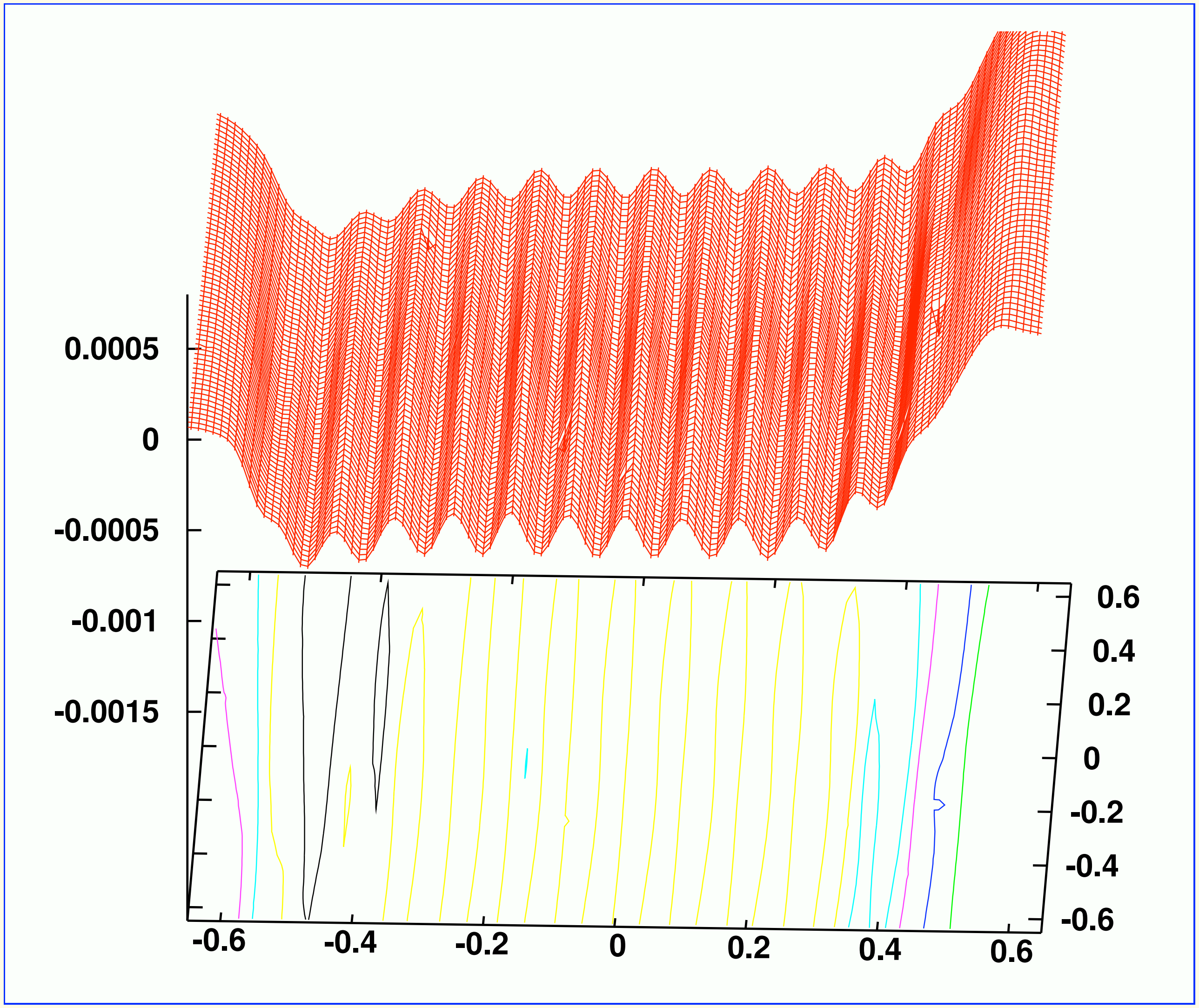
Longitudinal force in grid coordinates at $s = s_f$ for $\lambda_0 = 200\mu$m



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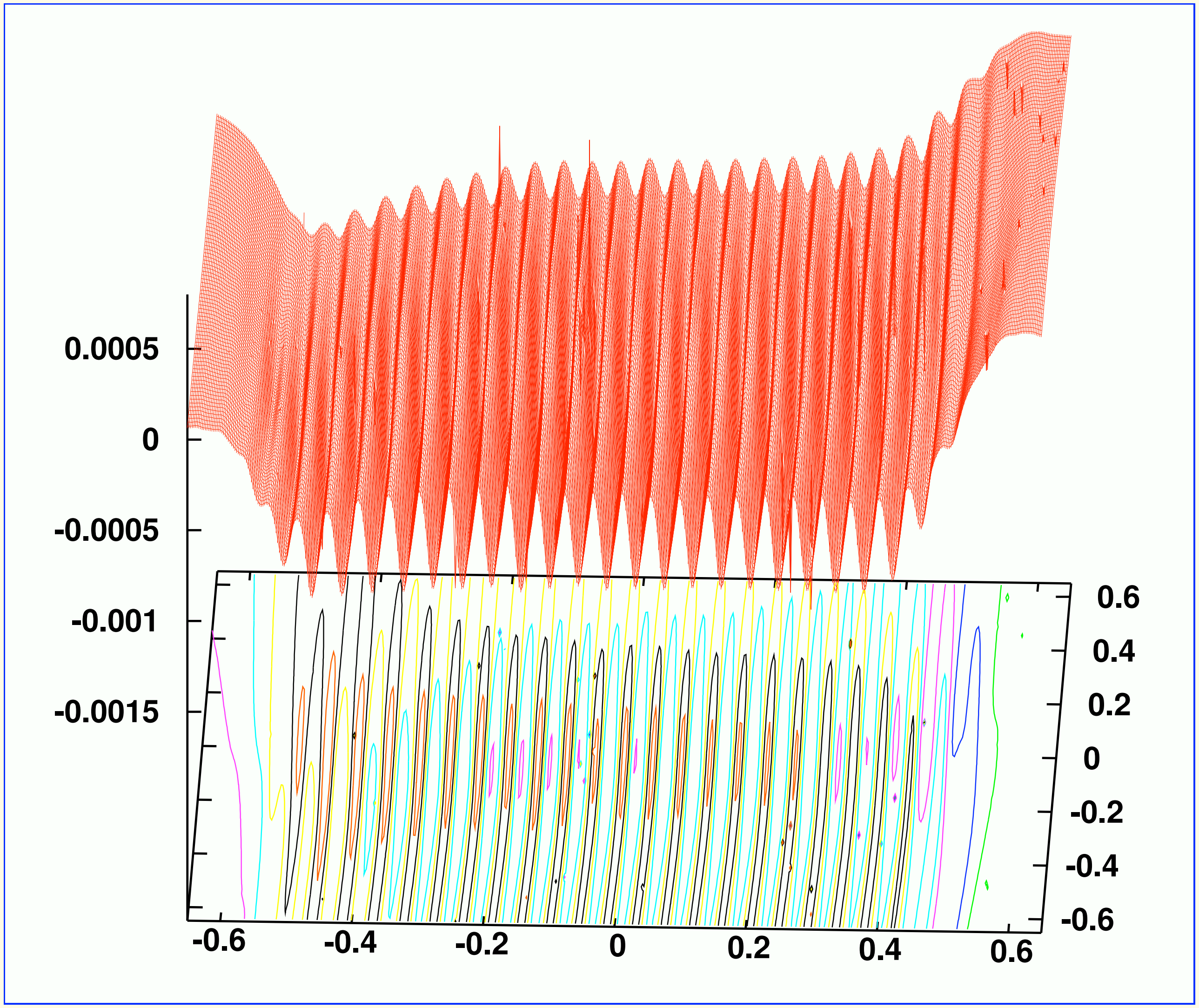
Longitudinal force in grid coordinates at $s = s_f$ for $\lambda_0 = 100\mu$m



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Longitudinal force in grid coordinates at $s = s_f$ for $\lambda_0 = 80\mu$m