Application of the reduction of scale range in a Lorentz boosted frame to the numerical simulation of particle acceleration devices.

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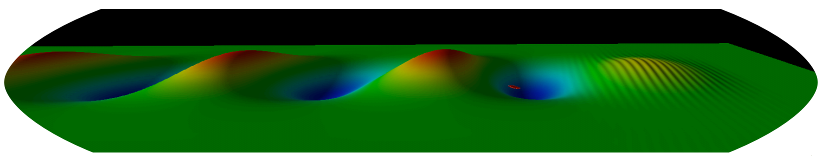
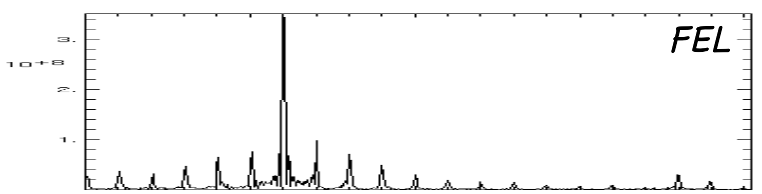
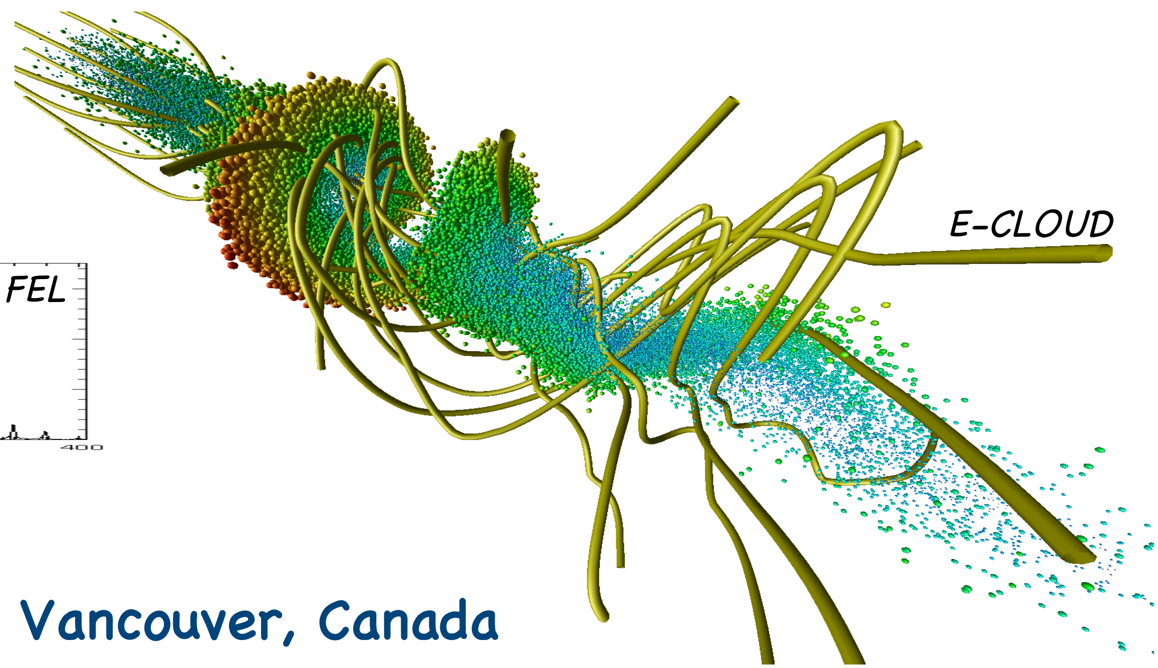


LWFA





May 4-8, 2009 - Vancouver, Canada

Outline

- Concept
- Difficulties
- Examples of application
  - electron cloud effects
  - laser wakefield acceleration
  - free electron laser
- Conclusion

Special relativity

Lorentz transformation (LT) for v along x

$$t' = \gamma\,(t - vx/c^2) \qquad \gamma = (1 - v^2/c^2)^{-1/2}$$
$$x' = \gamma\,(x - vt)$$
$$y' = y$$
$$z' = z$$

Time dilation/space contraction

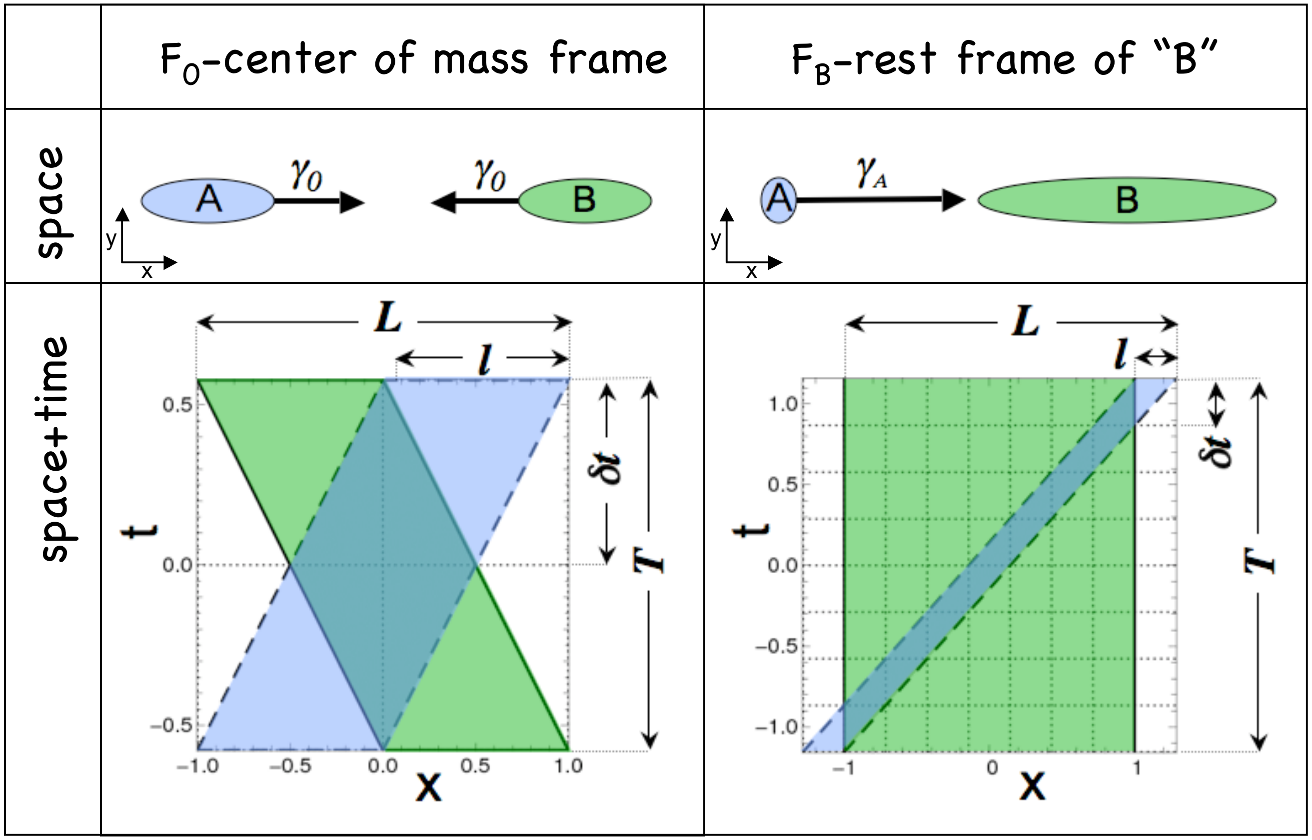
at rest: $\Delta t,\ \Delta x = 0$ → in motion: $\Delta t' = \gamma \Delta t$

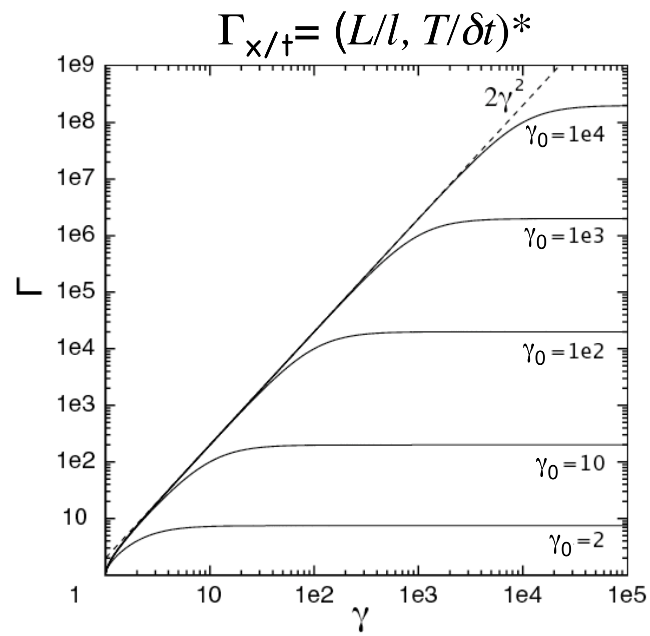
$\Delta x,\ \Delta t = 0$ → $\Delta x' = \Delta x/\gamma$

Lorentz invariant (invariant to change of reference frame)

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2 \Delta t'^2$$

Range of space and time scales spawned by two identical beams crossing each other





- Γ is not invariant under the Lorentz transformation: $\Gamma_{x/t} \propto \gamma^2$.
- There exists an "optimum" frame which minimizes it.
- Result is general and applies to light beams too.

*J.-L. Vay, *Phys. Rev. Lett.* **98**, 130405 (2007)

Consequence for computer simulations

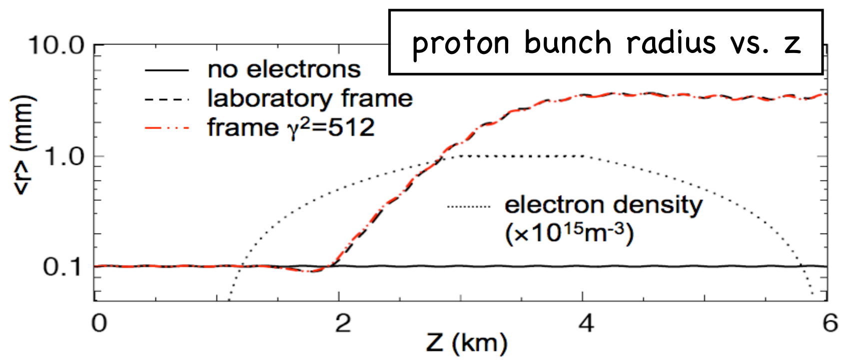
of computational steps grows with the full range of space and time scales involved

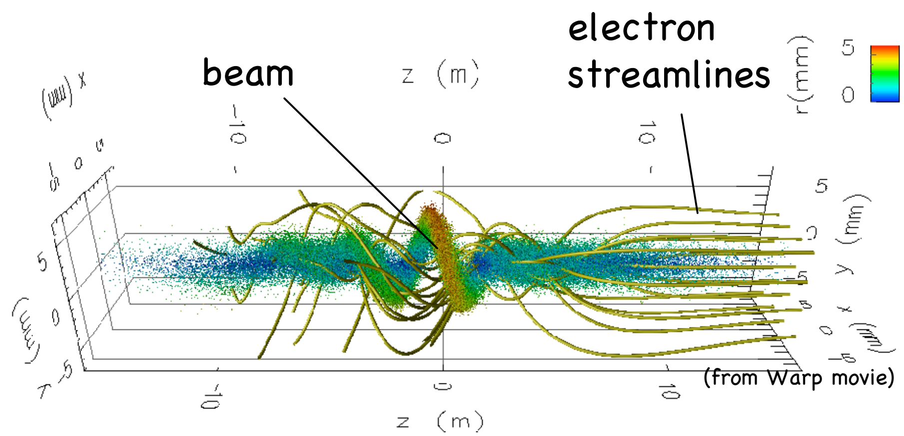
Choosing optimum frame of reference to minimize range can lead to dramatic speed-up for relativistic matter-matter or light-matter interactions.

Calculation of e-cloud induced instability of a proton bunch*

- Proton energy: γ=500 in Lab
- L=5 km, continuous focusing

Code: Warp (Particle-In-Cell)





CPU time (2 quad-core procs):

- lab frame: >2 weeks
- frame with γ^2=512: <30 min

Speedup x1000

*J.-L. Vay, *Phys. Rev. Lett.* **98**, 130405 (2007)

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Seems simple but ⚠. Algorithms which work in one frame may break in another. Example: the Boris particle pusher.

- Boris pusher ubiquitous
  - In first attempt of e-cloud calculation using the Boris pusher, the beam was lost in a few betatron periods!
  - Position push: $\mathbf{X}^{n+1/2} = \mathbf{X}^{n-1/2} + \mathbf{V}^{n}\,\Delta t$ -- no issue
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^{n}\mathbf{V}^{n} + \frac{q\,\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \frac{\gamma^{n+1}\mathbf{V}^{n+1} + \gamma^{n}\mathbf{V}^{n}}{2\,\gamma^{n+1/2}} \times \mathbf{B}^{n+1/2}\right)$

    issue: $\mathbf{E}+v\times\mathbf{B}=0$ implies $\mathbf{E}=\mathbf{B}=0$ => large errors when $\mathbf{E}+v\times\mathbf{B}\approx 0$ (e.g. relativistic beams).

- Solution
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^{n}\mathbf{V}^{n} + \frac{q\,\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \frac{\mathbf{V}^{n+1} + \mathbf{V}^{n}}{2} \times \mathbf{B}^{n+1/2}\right)$

- Not used before because of implicitness. We solved it analytically*

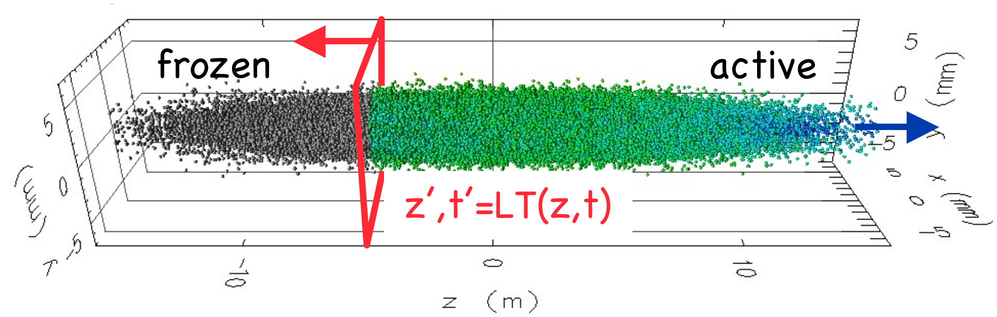
$$\begin{cases} \gamma^{i+1} = \sqrt{\dfrac{\sigma + \sqrt{\sigma^2 + 4(\tau^2 + u^{*2})}}{2}} \\ \mathbf{u}^{i+1} = [\mathbf{u}' + (\mathbf{u}'\cdot\mathbf{t})\mathbf{t} + \mathbf{u}'\times\mathbf{t}]/(1+t^2) \end{cases}$$

(with $\mathbf{u} = \gamma\mathbf{v}$, $\mathbf{u}' = \mathbf{u}^{i} + \frac{q\Delta t}{m}\left(\mathbf{E}^{i+1/2} + \frac{\mathbf{v}^{i}}{2}\times\mathbf{B}^{i+1/2}\right)$, $\boldsymbol{\tau} = (q\Delta t/2m)\mathbf{B}^{i+1/2}$; $u^{*} = \mathbf{u}'\cdot\boldsymbol{\tau}/c$, $\sigma = \gamma'^2 - \tau^2$, $\gamma' = \sqrt{1+u'^2/c^2}$, $\mathbf{t} = \boldsymbol{\tau}/\gamma^{i+1}$).

*J.-L. Vay, *Phys. Plasmas* **15**, 056701 (2008)

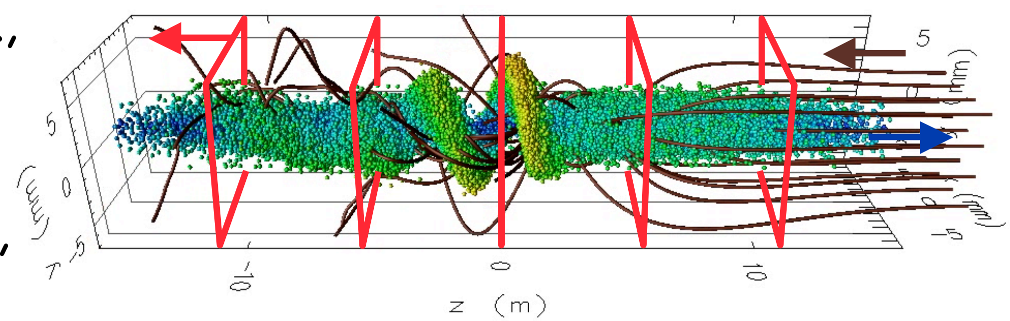
Other possible complication: inputs/outputs

- Often, initial conditions known and output desired in laboratory frame
  - relativity of simultaneity => inject/collect at plane(s) ⊥ to direction of boost.
- Injection through a moving plane in boosted frame (fix in lab frame)
  - fields include frozen particles,
  - same for laser in EM calculations.



- Diagnostics: collect data at a collection of planes
  - fixed in lab fr., moving in boosted fr.,
  - interpolation in space and/or time,
  - already done routinely with Warp

  for comparison with experimental data, often known at given stations in lab.



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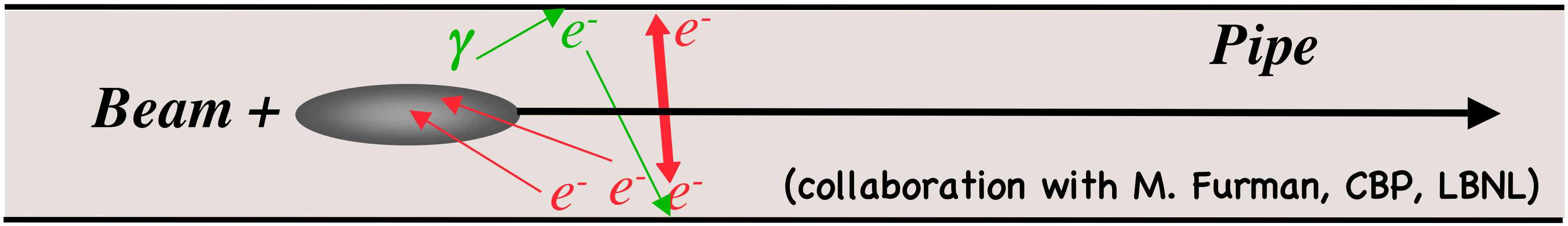
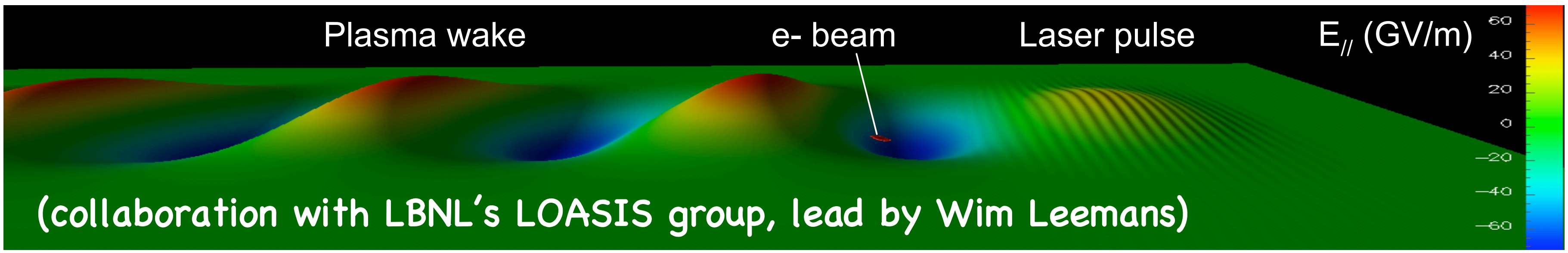
Several areas in which simulations in a boosted may be beneficial were identified

Electron cloud driven beam instabilities



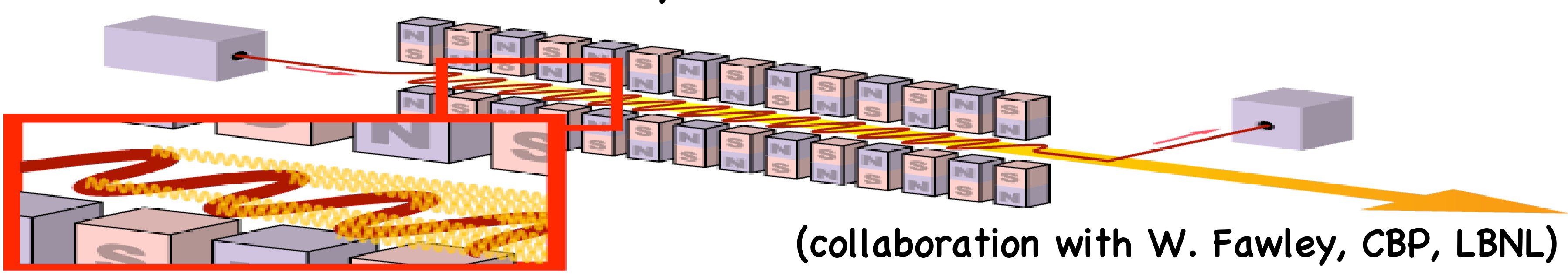
(collaboration with M. Furman, CBP, LBNL)

Laser-plasma wakefield accelerators



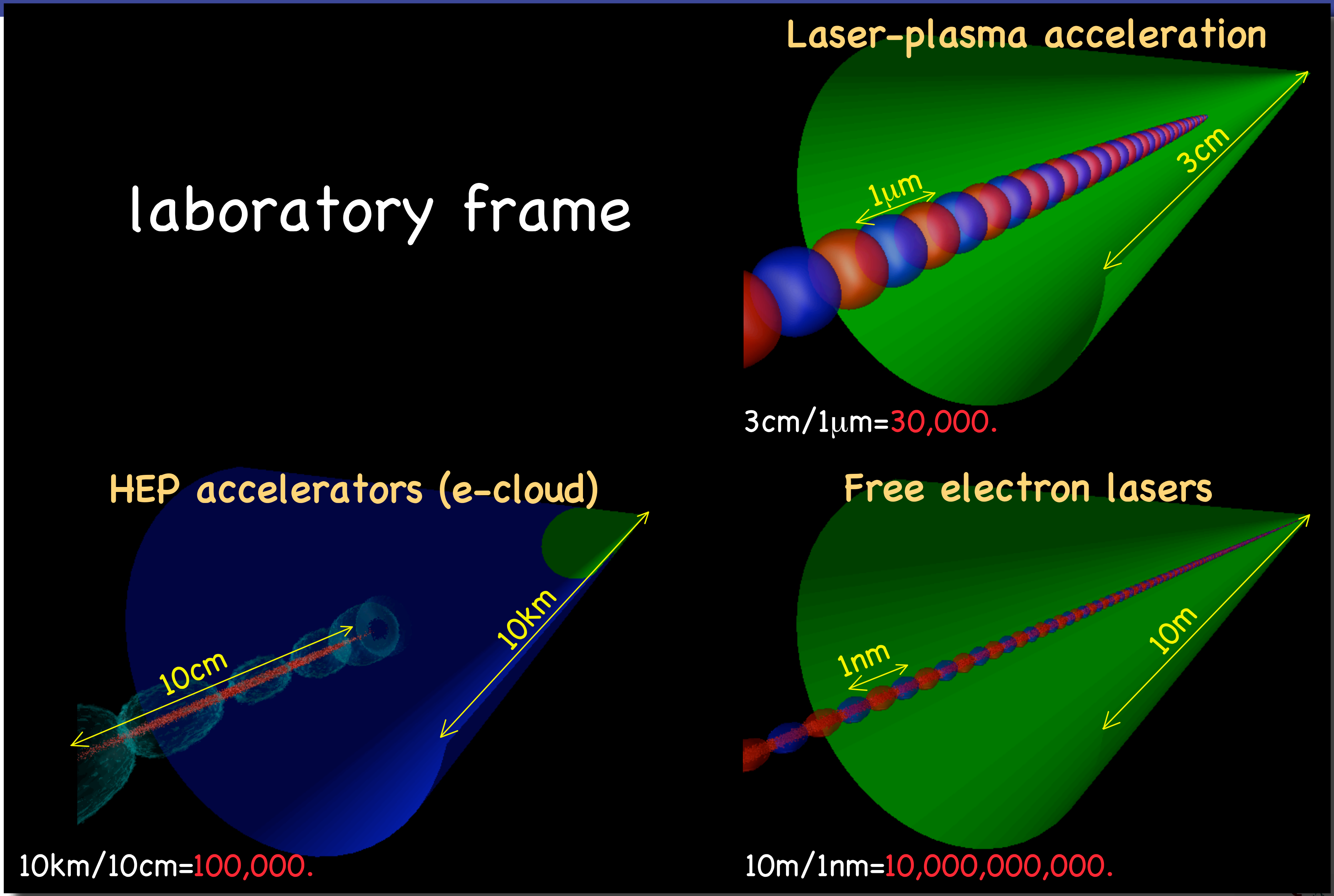
(collaboration with LBNL's LOASIS group, lead by Wim Leemans)

Free electron lasers/coherent synchrotron radiation

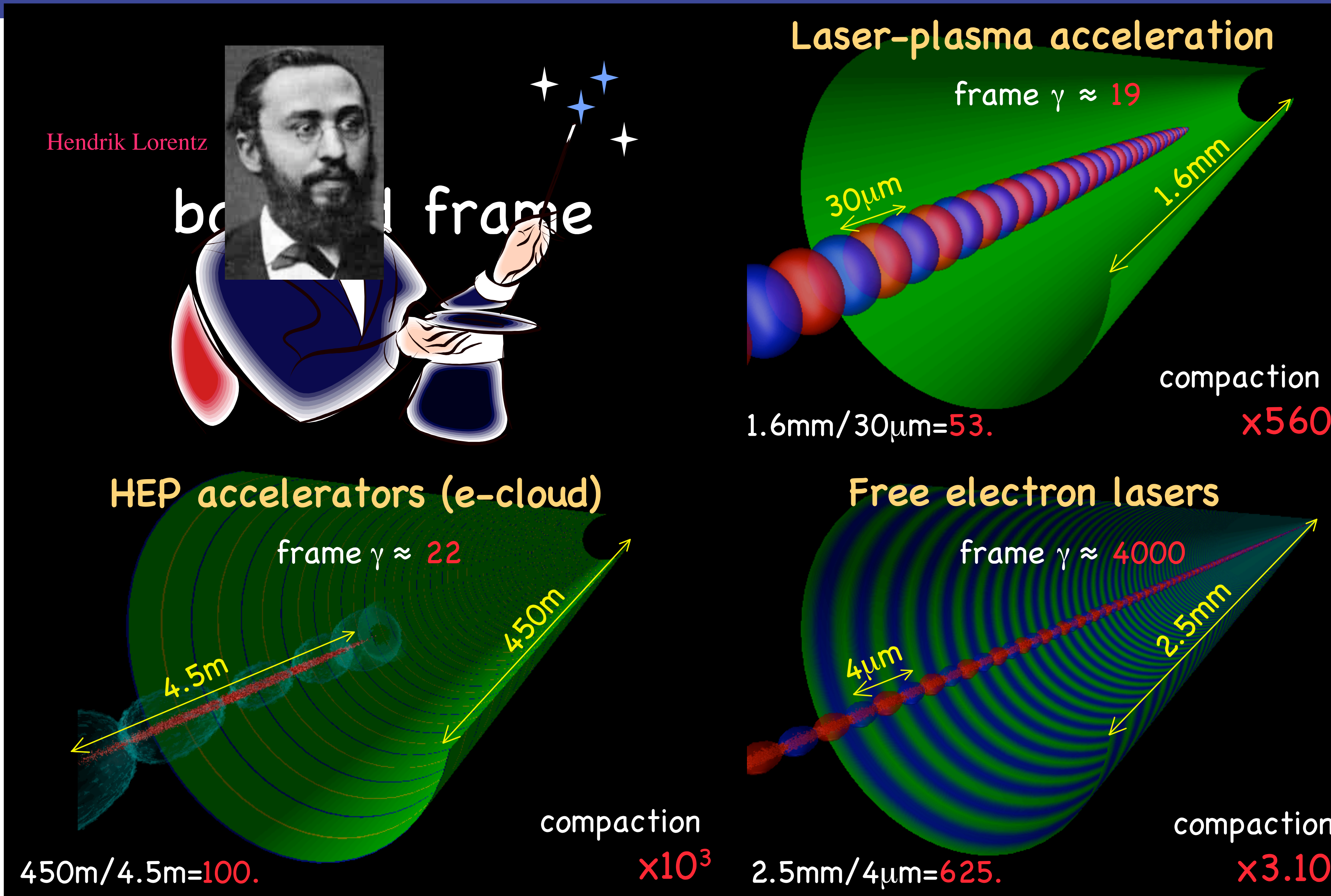


(collaboration with W. Fawley, CBP, LBNL)

Large scale range renders simulation difficult, if not impractical, in lab frame



Lorentz transformation => large level of compaction of scales range



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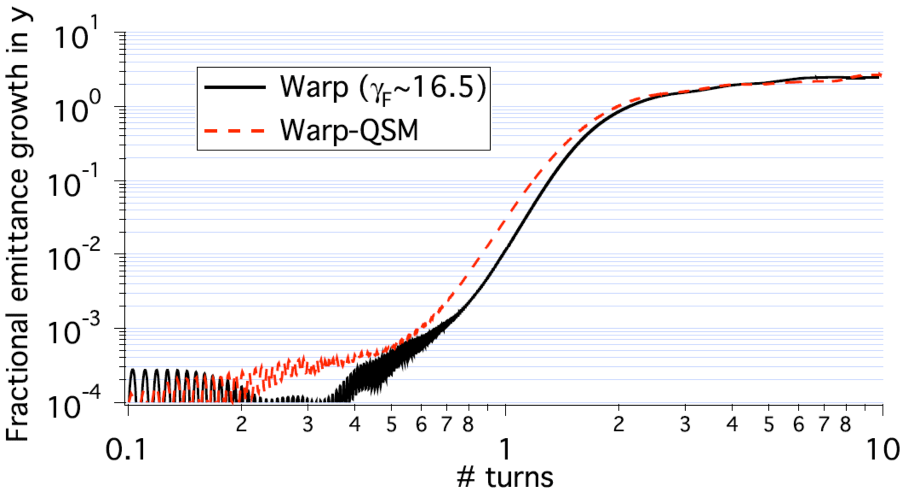
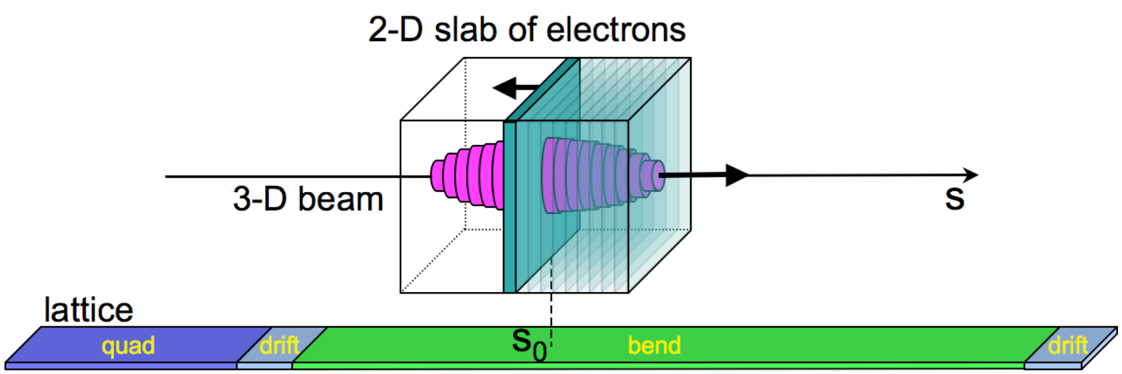
E-cloud: benchmarking against quasistatic model for LHC scenario

Excellent agreement on emittance growth between boosted frame full PIC and "quasistatic" for e-cloud driven transverse instability in continuous focusing model of LHC

Electron cloud density	ρ_e	10^{14} m^{-3}
Bunch population	N_b	1.1×10^{11}
Beta functions	$\beta_{x,y}$	66.0, 71.54 m
rms bunch length	σ_z	0.13 m
rms beam size	$\sigma_{x,y}$	0.884 mm
rms momentum spread	δ_{rms}	0
Circumference	C	26.659 km
Nominal tunes	$Q_{x,y}$	64.28, 59.31
Relativistic factor	γ	479.6
Pipe radius	R_p	2.2 cm (with flat tops at ±1.8 cm)
Initial beam position offset	δy	0.1 σ_y
Dipole field (electrons only)	B_{y0}	8.39 T



The "quasistatic" approximation uses the separation of time scales for pushing beam and e-cloud macro-particles with different "time steps"



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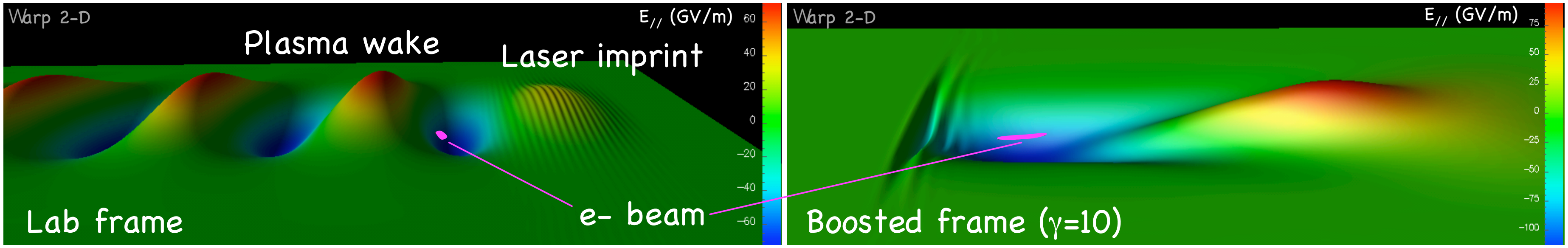
Laser-plasma wakefield accelerator simulation in a boosted frame
(λ=0.8μm, a_0=1, k_pL=2, L_p=1.5mm in lab)

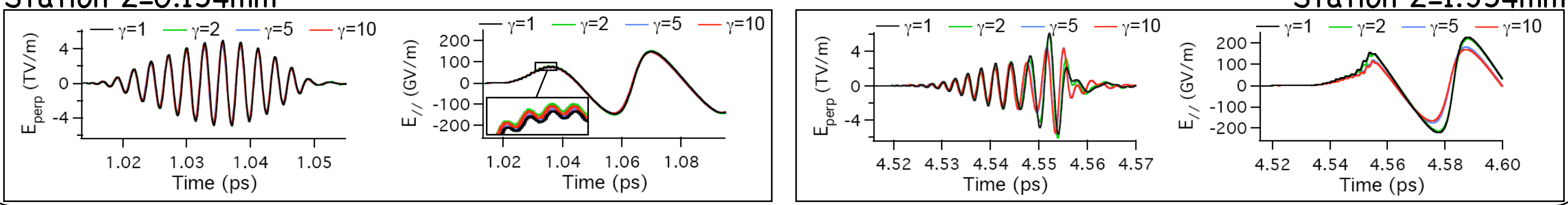
Snapshots of surface plot of // electric field in lab frame and boosted frame at γ=10



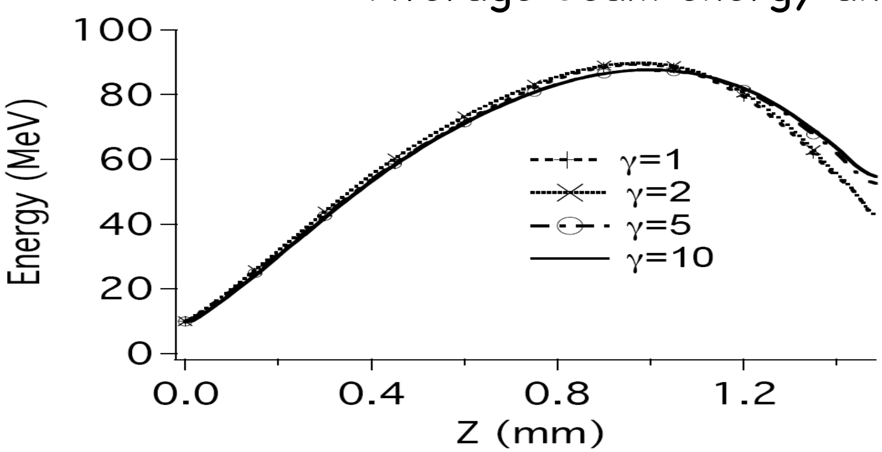
⊥ and // electric field history in lab frame

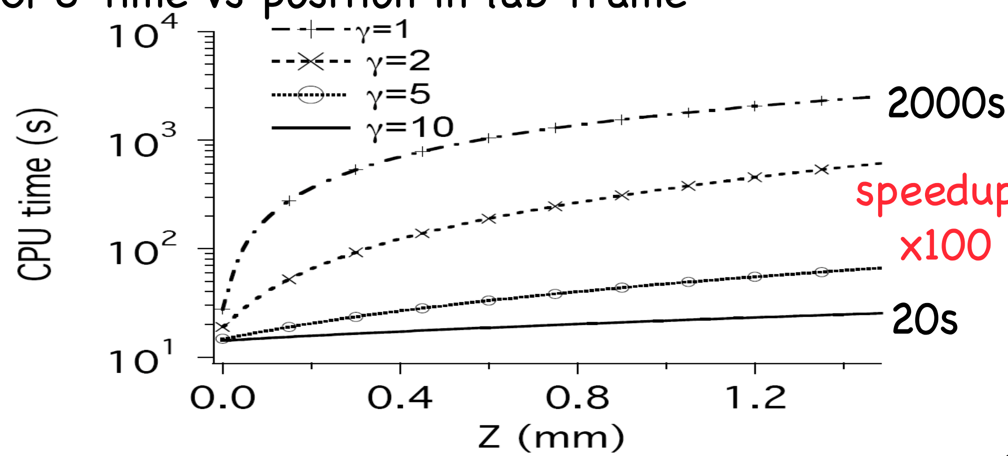
Station z=0.154mm

Station z=1.354mm



Average beam energy and CPU time vs position in lab frame

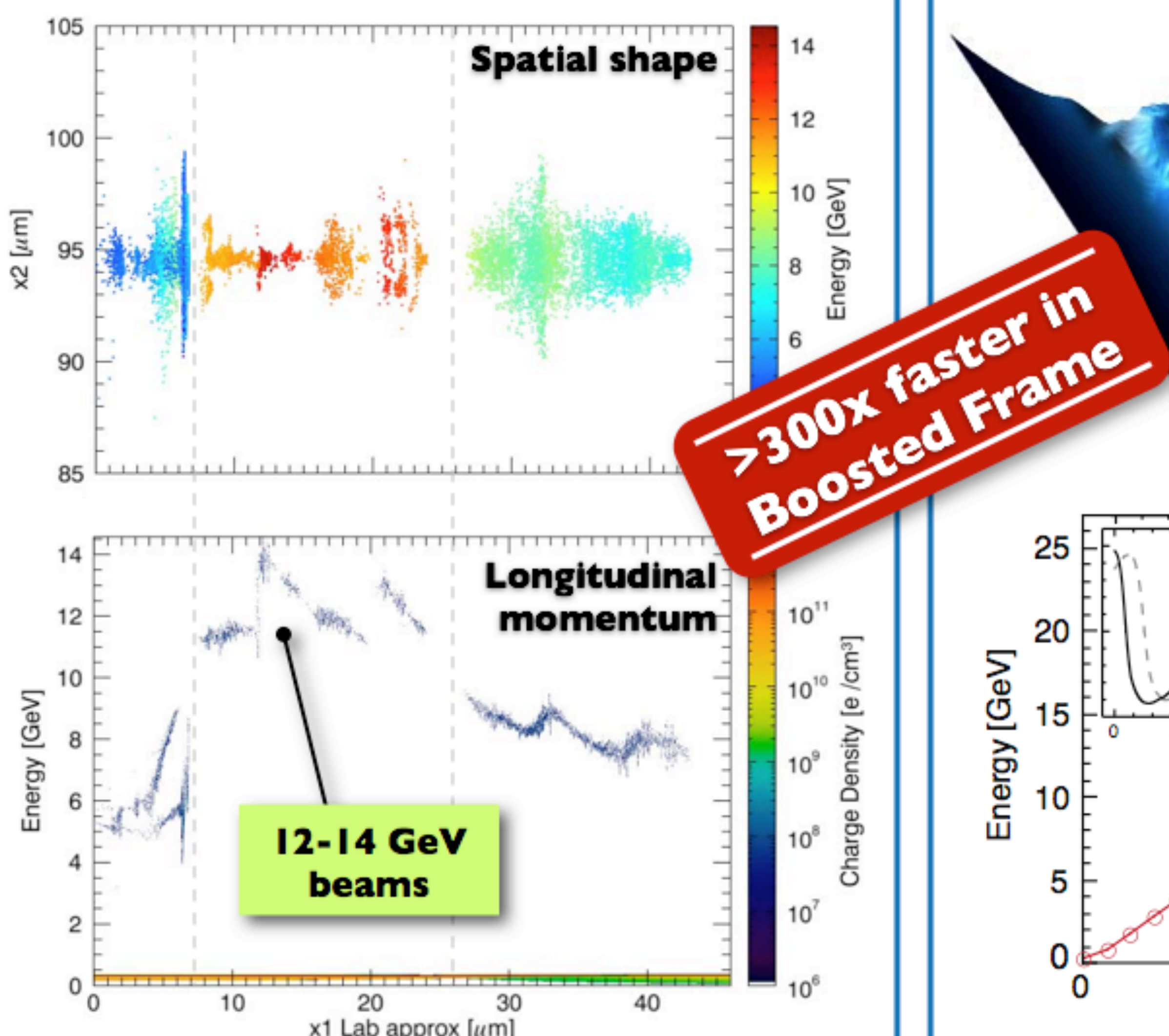




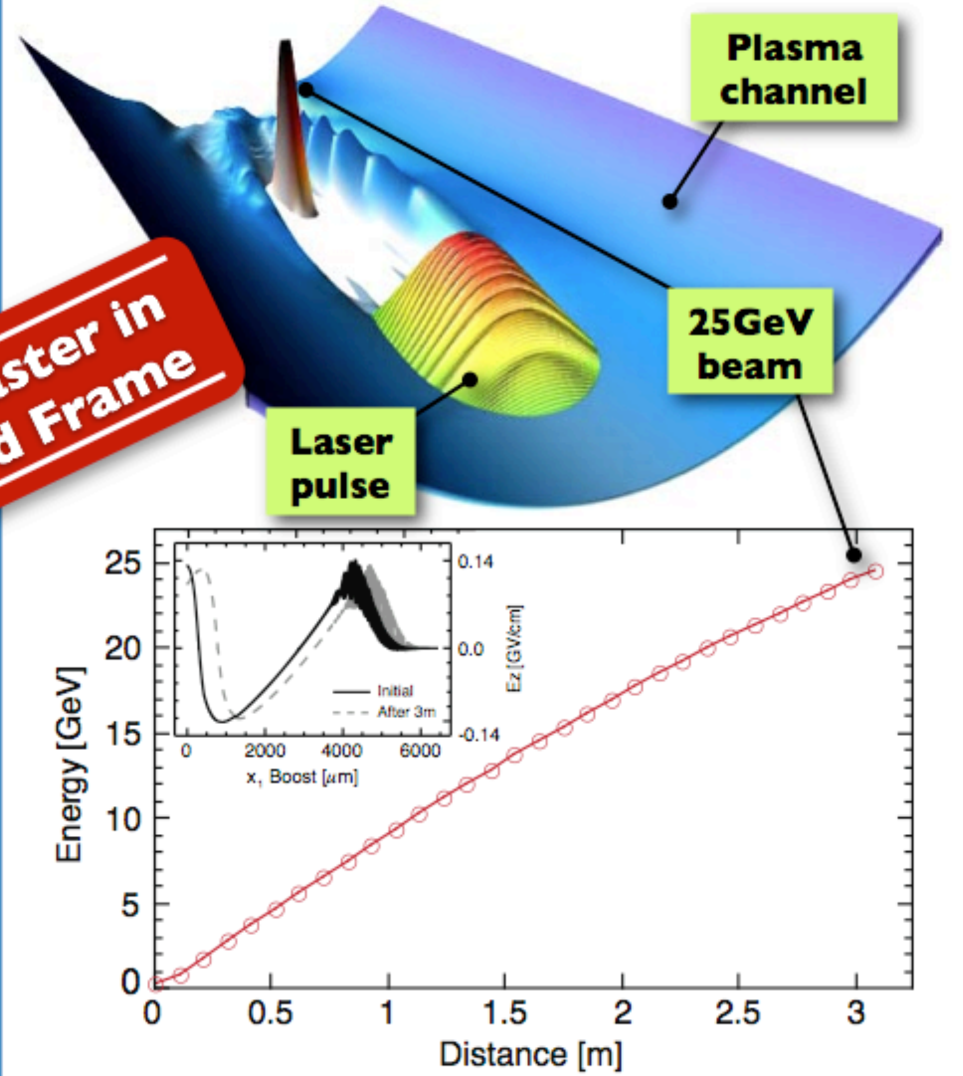
Full 3D ultra-fast boosted frame simulations for next generation lasers using OSIRIS

S. F. Martins et al. *Thursday 5:30pm (TH4GBC05)*

Self-injection: >10 GeV

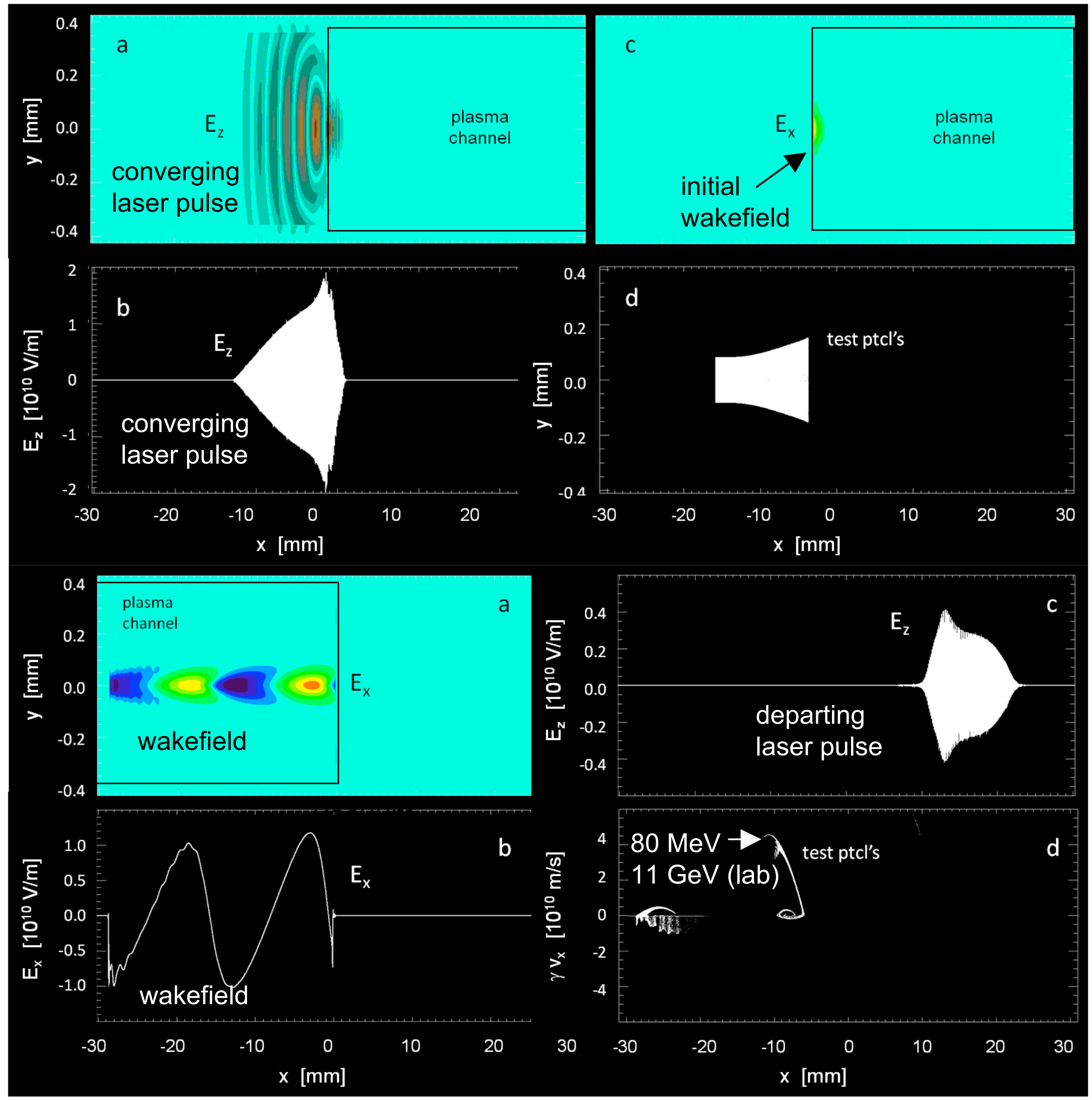


External-injection: 25GeV & counting...



Boosted-frame LWFA proof-of-principle in VORPAL shows >2,000x speed-up in 1D, 2D

- Test cases are 10 GeV quasilinear LWFA stages
  - test particles used to sample wake
- Grid size & resolution are same as standard lab frame runs
- 2D example shown to right: n_e=6 x 10^{16} cm^{-3}; L_{deph}~2.4 m; a_0=1; E_{peak}~11 GeV (lab)
  - agrees with scaling calculations
  - 2,000x speed-up observed
- More work required
  - improved noise reduction
  - automated set-up, diagnostics
  - validation and testing



Bruhwiler, Cary, Cowan, Paul, Geddes, Mullowney, Messmer, Esarey, Cormier-Michel, Leemans & Vay, Proc. AAC Workshop (2008); supported by DOE/HEP SBIR & SciDAC.

TECH-X CORPORATION

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FEL in Boosted-Frame E&M Code

poster W. Fawley WE5RFP029, Wednesday morning

Physics ignored by Eikonal codes but accessible to boosted frame approach:

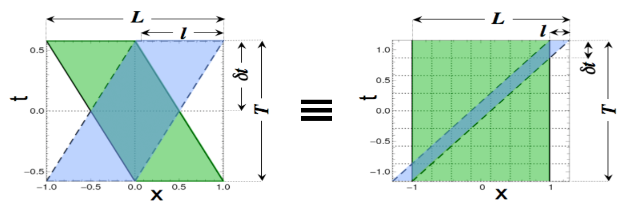
- Backward wave emission
- Wide-angle emission (generally highly red-shifted)
- CSE for all undulator, e-beam configurations
  - Emission from very short beams
  - Emission from beams with rapidly-varying envelope properties
  - Emission from beams bunched with "multiple colors"
- Properties of *very* high gain systems ($L_G/\lambda_u < 5$)
- FEL emission from beams in multiple harmonic undulators
  - Biharmonic (or triharmonic undulators)
  - Effects of adiabatic match sections
- FEL emission in waveguides where v_{group} strongly varying with ω (normally relevant to microwave FEL's operating near cutoff)

Overall computational speed impressive compared to *full* E&M *but much slower than standard eikonal method*: Not likely to become dominant paradigm for short wavelength FEL's but *might* be useful for very high gain microwave/far-IR devices or situations with wideband spectral output

WM Fawley & J-L Vay, Electromagnetic Simulation of Free-Electron Amplifier Physics via the Lorentz-Boosted Frame Approach, PAC09, May 2009

Conclusion and outlook

- The range of scales of a system is not a Lorentz invariant ($\propto\gamma^2$), and there exists an optimum frame minimizing it => orders of magnitude speedup predicted for some simulations.
- Calculating in a boosted frame more demanding, eventually:
  - developed new particle pusher for e-cloud problems,
  - added capabilities for injection/diagnostics in boosted frame.
- Orders of magnitude speedup demonstrated for a class of first-principle simulations of multiscale problems: laser-plasma acceleration, e-cloud in HEP accelerators, free electron lasers.
- Explore other applications: CSR, astrophysics,…
- Can we develop methods which costs do not depend on frame?



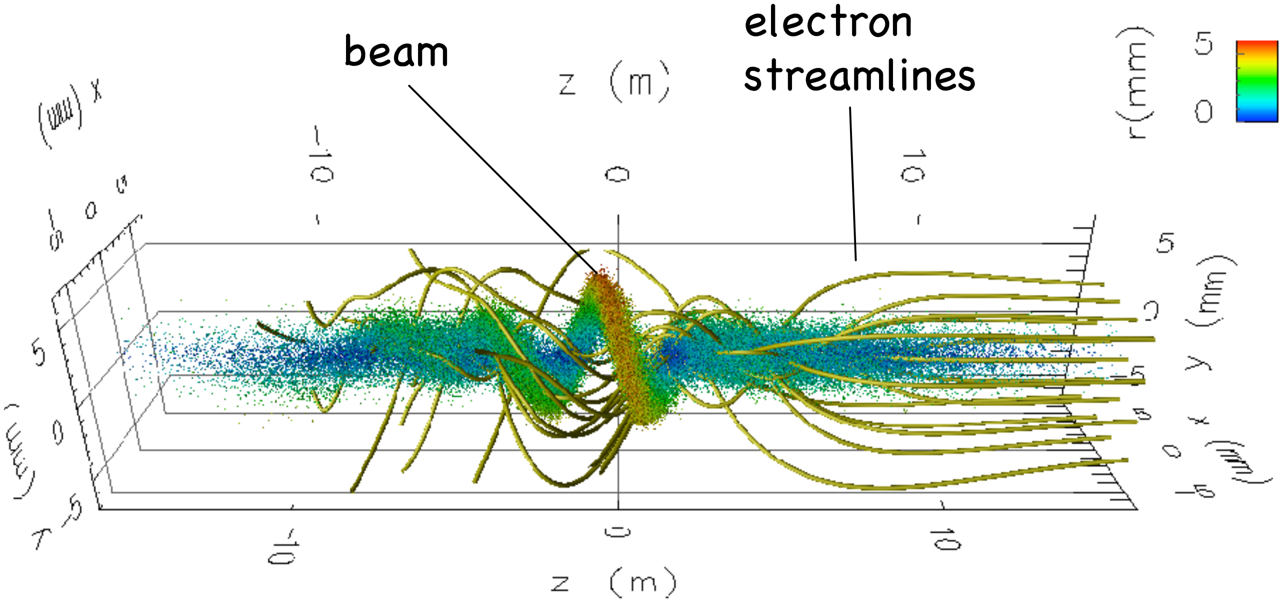
BACKUPS

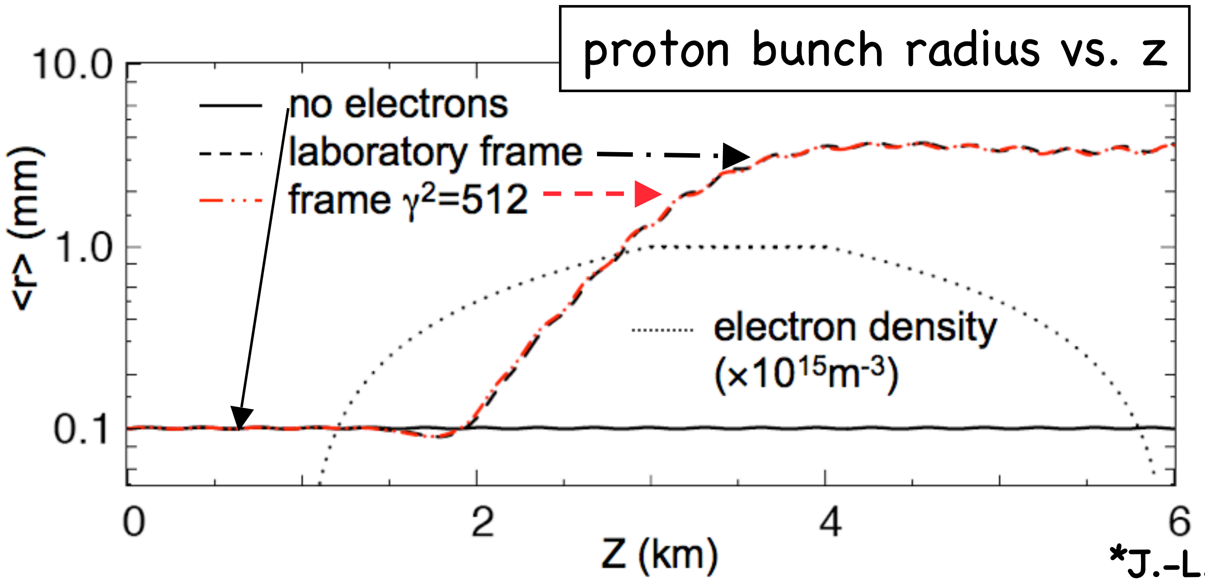
Boosted frame calculation sample
proton bunch through a given e^- cloud*

This is a proof-of-principle computation: TMC instability of a proton bunch

Proton energy: γ=500 in Lab
- L= 5 km, continuous focusing

Code: Warp (Particle-In-Cell)





CPU time:
- lab frame: >2 weeks
- frame with γ^2=512: <30 min

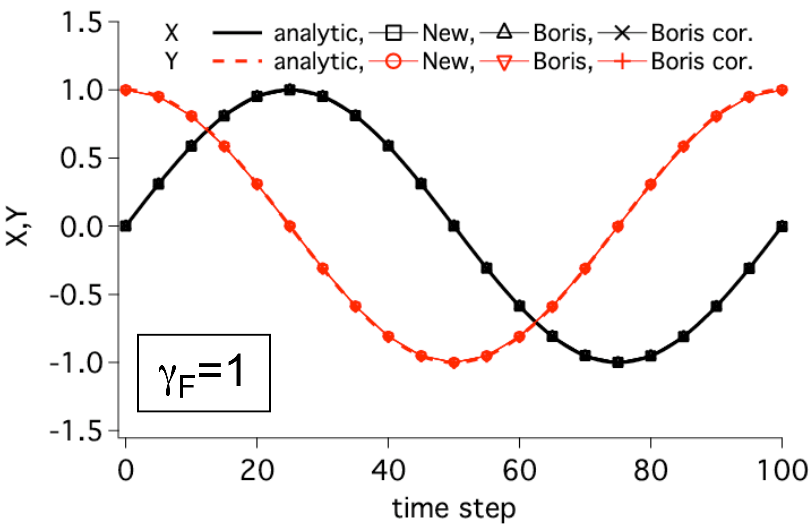
Speedup x1000

*J.-L. Vay, *Phys. Rev. Lett.* **98**, 130405 (2007)

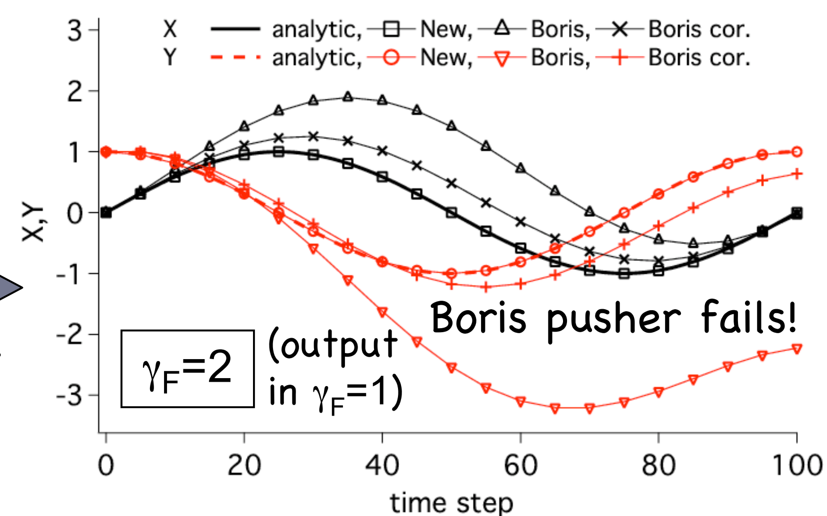
Seems simple but ⚠. Algorithms which work in one frame may break in another. Example: the Boris particle pusher.

- Boris pusher ubiquitous
  - Position push: $\mathbf{X}^{n+1/2} = \mathbf{X}^{n-1/2} + \mathbf{V}^{n}\,\Delta t$ -- no issue
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^{n}\mathbf{V}^{n} + \frac{q\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \frac{\gamma^{n+1}\mathbf{V}^{n+1} + \gamma^{n}\mathbf{V}^{n}}{2\,\gamma^{n+1/2}} \times \mathbf{B}^{n+1/2}\right)$
- New pusher*
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^{n}\mathbf{V}^{n} + \frac{q\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \frac{\mathbf{V}^{n+1} + \mathbf{V}^{n}}{2} \times \mathbf{B}^{n+1/2}\right)$
- Test one particle in constant **B**

*J.-L. Vay, *Phys. Plasmas* **15**, 056701 (2008)



becomes ExB drift in boosted frame



Lorentz boosted simulations applied to various problems

3-D electron driven TMC instability (Warp-LBNL), **x1000**

2-D free electron laser toy problem (Warp-LBNL), **x45,000***/**

3-D coherent synchrotron emission (Warp-LBNL), **x350***

2-D laser-plasma acceleration (Warp-LBNL), **x100***

1-D laser-plasma acceleration (Vorpal-Tech-X), **x1,500**

laser-plasma acceleration (Osiris-IST, Portugal) **x150** 2-D, **x75** 3-D

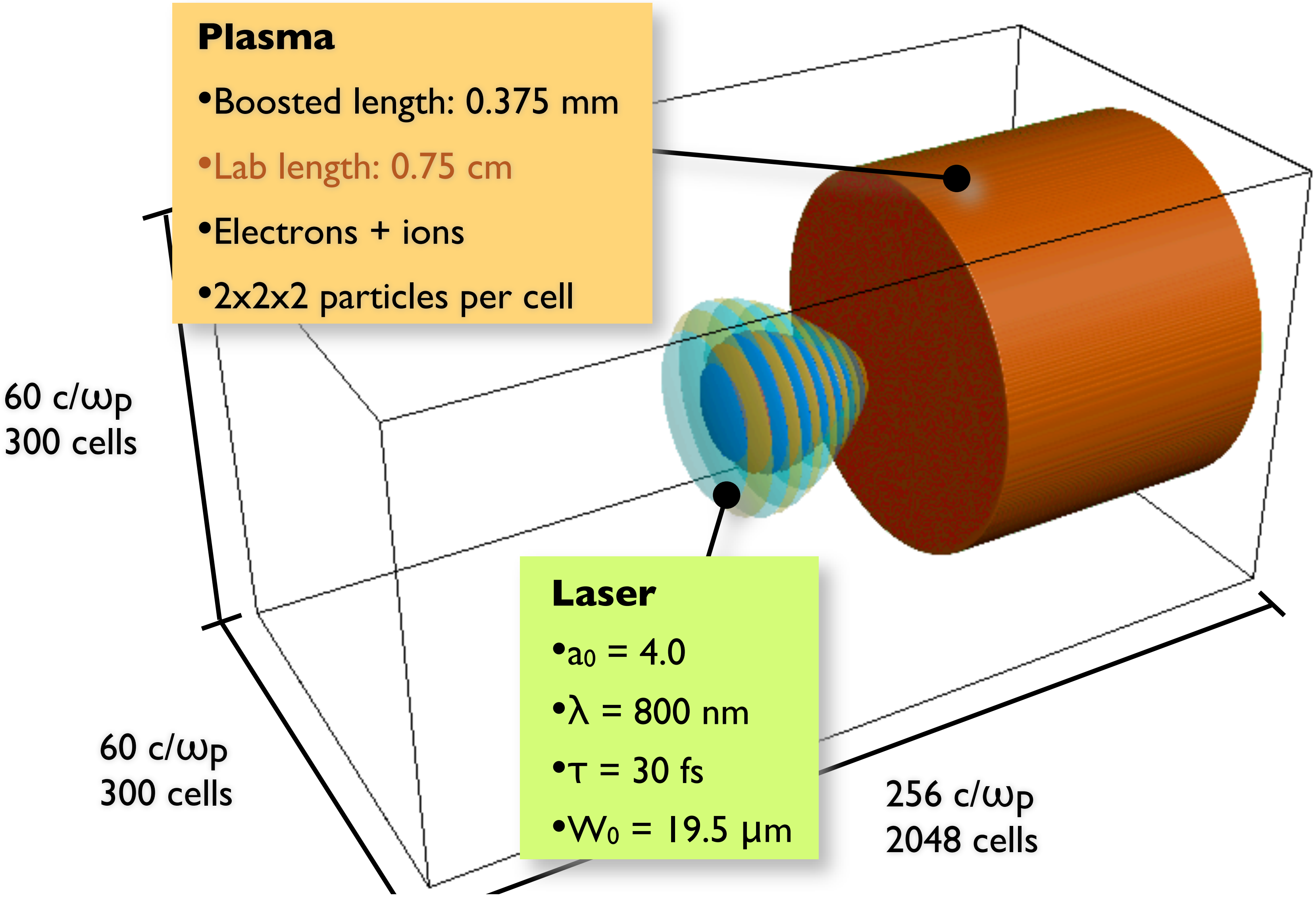
*estimated

**compared to PIC simulation in lab frame. PIC in boosted frame slower than Eikonal codes but allows study of matching ramp and sub-harmonic bunching which are not accessible to Eikonal codes.

Other applications: astrophysics,...?

3D Laser Wakefield Accelerator*

Boosted frame simulation apparatus



*Courtesy S. F. Martins, Instituto Superior Técnico, Lisboa, Portugal

3D Laser Wakefield Accelerator*

Example of Boost performance gains vs. standard laboratory frame

Theoretical gain

Boost velocity $\gamma = 20$

Standard gain $\gamma^4 (1+\beta)^3 = 1.28 \times 10^6$

Moving window Gain $= 800$

Standard gain $\gamma^2 (1+\beta)^2 = 1.6 \times 10^3$

Specific considerations

Gain decrease factor

Particles per cell **x4**

Plasma resolution **x2**

Moving ions **x2**

Transverse size **x1.5**

Total Time & Time step **x6**

"Real gain"

Simulation duration

Lab (estimate) **40** days, **1.4x10⁵** CPU h.

Boost **12** hours, **2x10³** CPU h.

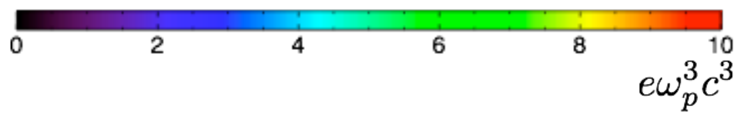
Boost Gain **~75**

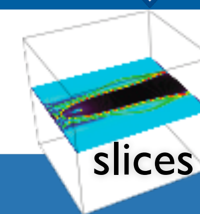
*Courtesy S. F. Martins, Instituto Superior Técnico, Lisboa, Portugal

3D Laser Wakefield Accelerator*

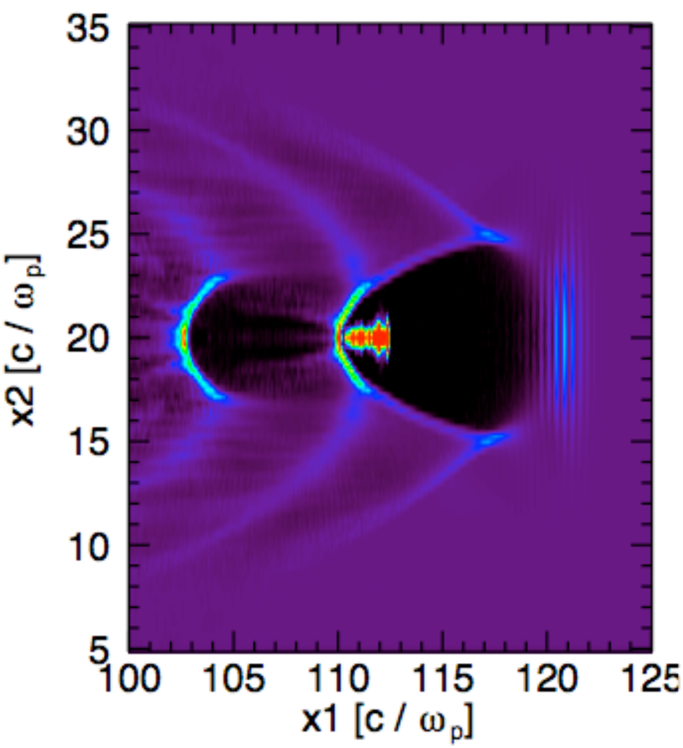
Wake structure comparison

Electron density



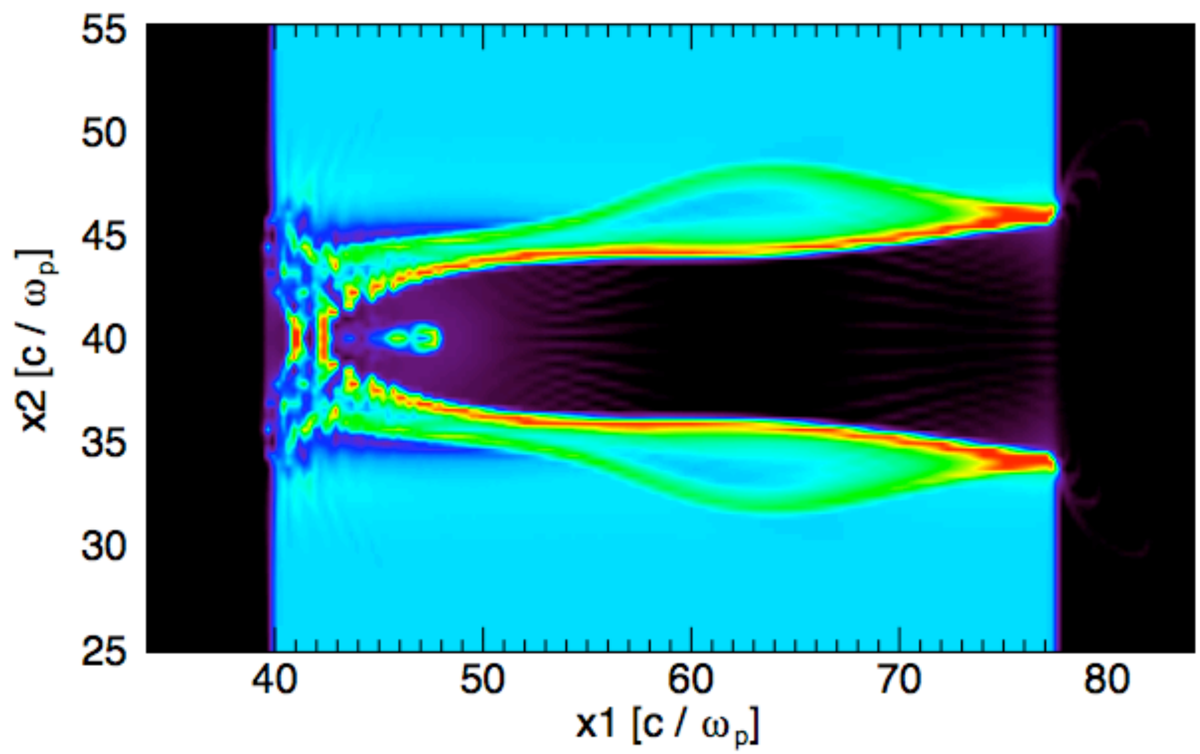


Laboratory frame



- Several simultaneous buckets
- Moving window focuses on specific plasma region

Boosted frame

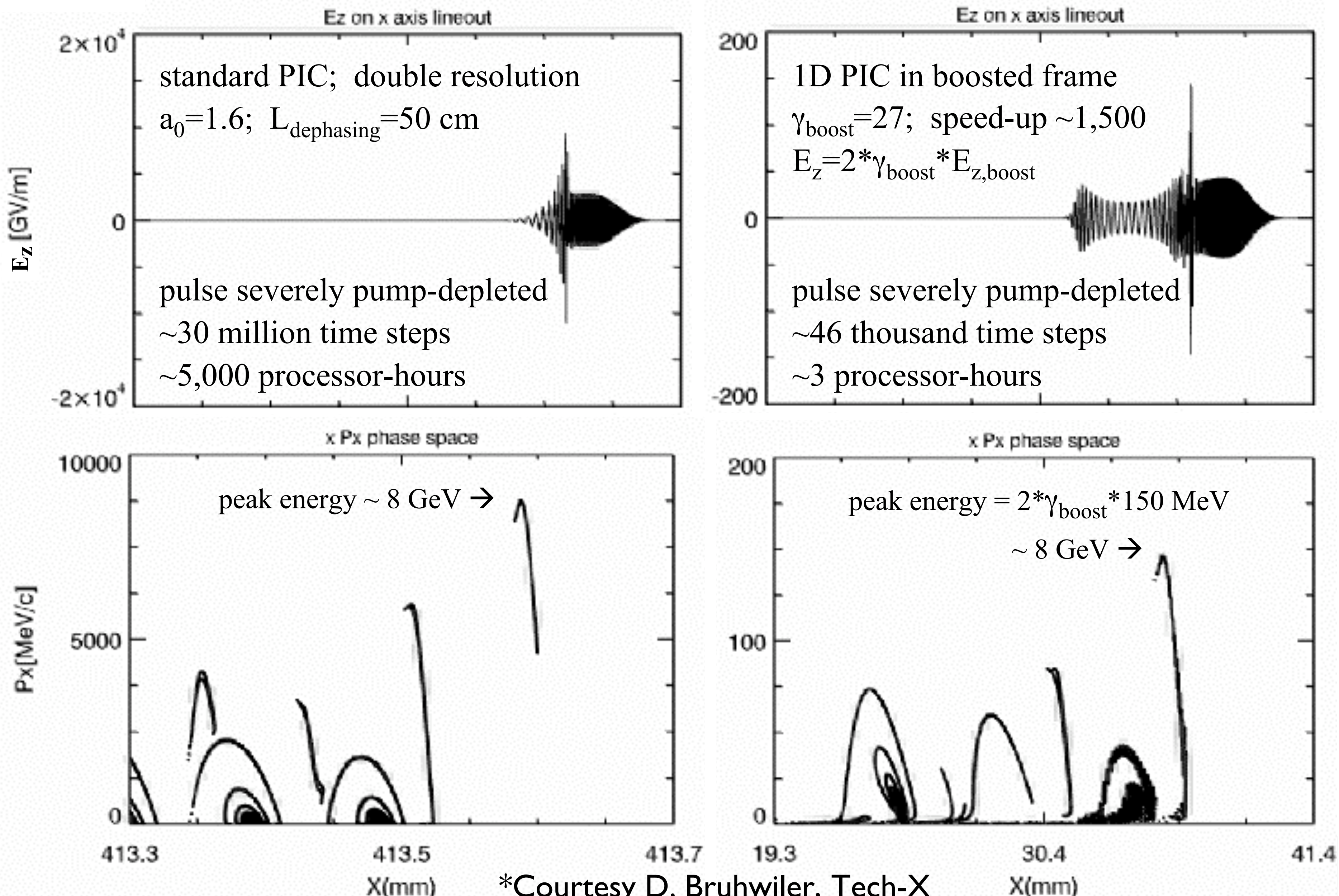


- Single spatial bucket
- Full evolution of complete plasma region

$\gamma = 4$

*Courtesy S. F. Martins, Instituto Superior Técnico, Lisboa, Portugal

Boosted-frame Simulations of LBNL* *BELLA* Concept are 1,500x faster



*Courtesy D. Bruhwiler, Tech-X