

THEORY AND APPLICATIONS OF LATTICE WITH NEGATIVE MOMENTUM COMPACTION FACTOR

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In which cases the structure with such properties is necessary?

- to exclude the transition energy crossing

$$\eta = -1/\gamma_{tr}^2 - 1/\gamma^2$$

- to have the higher collective instability threshold

$$\left(\frac{\delta\omega}{\omega}\right)^2 = \eta^2 \left(\frac{\sigma}{p}\right)^2 \geq \frac{eI_{peak}|\eta|}{2\pi m_0 c^2 \gamma \beta^2} \left| \frac{Z_L(n)}{n} \right|$$

- to match the different accelerators in longitudinal plane

$$\Delta\phi_{\max} = \pm W \sqrt{\frac{2\pi h |\eta| \Omega_{rev}}{eV p_s R \cos\phi_s}}$$

- to adjust the different local slip factor for the optimized stochastic cooling
- to avoid the sextupoles in the synchrotron light sources

The lattice having the imaginary gamma transition meets to all requirements!!!

History of lattice with imaginary gamma-transition

- in 1955 Vladimirsky and Tarasov** suggested method to get the imaginary γ_{tr} and did it by increasing number of “compensating magnets” with a reversed field but the same gradients, as would be called for in a design with no compensating magnets and where is slightly more than the tune.
- In 1958 Courant and Snyder** quantitatively described this idea of the negative momentum compaction factor.

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Later many authors tried to realize this idea of imaginary transition energy in different lattices:

- In 1972 Lee Teng** suggested the modular method;
- In 1974 Bruck** developed the regular focusing structure with the “missing” magnet cell in **Saturne II**;
- In 1983 Franczak, Blasche, Reich** excited superperiodically the quadrupoles for **SIS-18**;
- In 1985 Gupta, Botman, Craddock** at an initial design stage of the **TRIUMF KF** used missing magnet;

History of lattice with imaginary gamma-transition

- In **1989** **Senichev, Golubeva, Iliev** suggested the “resonant” lattice for **Moscow Kaon Factory**;
- In **1992** **Ng, Trbojevic, Lee** applied the modular method of Lee Teng for **MB (FNAL)**;
- In **1992** **U.Wienands, N.Golubeva, A.Iliev, Yu.Senichev, R.Servranckx** adopted the “resonant” lattice for Kaon Factory (**TRIUMF**);
- In **1993** **E. Courant , A. Garen and U. Wienands** took the “resonant” lattice for **LEB (SSC)**;
- In **1995** **Y. Senichev** wrote the “resonant” lattice theory and applied it for **Main Ring (JPARC)**
- In **2000** **H. Schönauer, Yu. Senichev et al.**, The “resonant” lattice for Proton driver for a **Neutrino Factory (CERN)**
- In **2007** **Y.Senichev et al.**, The “resonant” lattice for **Super-Conducting option of HESR (FAIR)**
- In **2008** The “resonant” lattice is one of the candidate for **PS2 (CERN)**

Regular and Irregular lattices

Momentum Compaction factor (MCF):

$$\alpha = \frac{1}{2\pi} \int_0^{2\pi} \frac{D(\vartheta)}{\rho(\vartheta)} d\vartheta$$

where the dispersion $D(\theta)$ is:

$$D'' + K(\vartheta)D = \frac{1}{\rho(\vartheta)}$$

If in the optics with eigen frequency ν the curvature $1/\rho(\vartheta)$ is modulated with frequency ω

$$1/\rho(\vartheta) \sim \boxed{Be^{i\omega\vartheta}} + 1/\bar{R}$$

the dispersion solution and Momentum Compaction Factor are:

$$D(\vartheta) \sim Ae^{i\nu\vartheta} + \boxed{\frac{B}{\nu^2 - \omega^2} e^{i\omega\vartheta}} + \bar{D} \quad \alpha = \frac{\bar{D}}{\bar{R}} + \frac{\overline{\tilde{D}(\vartheta) \cdot \tilde{r}(\vartheta)}}{\bar{R}}$$

Regular lattice

In conventional regular FODO lattice $\omega \gg \nu$.

Therefore the dispersion oscillates with eigen frequency (tune) ν :

$$D(\mathcal{G}) \approx A e^{i\nu\mathcal{G}} + \bar{D}$$

Then Momentum Compaction Factor (MCF) is determined by average values ratio:

$$\alpha = \frac{\langle D(\mathcal{G}) \rangle}{\langle \rho(\mathcal{G}) \rangle} = \frac{\bar{D}}{\bar{R}} \approx \frac{1}{\nu^2}$$

and the maximum energy of accelerator without the transition energy crossing is determined by $\gamma_{\max} \approx \nu$ or for the $\pi/2$ phase advance FODO lattice $\gamma_{\max} \approx N_{\text{cell}}/4$

Irregular lattice with curvature modulation (missing magnet lattice)

In case of eigen frequency ν is enough close to the curvature oscillation with the superperiodicity frequency $S = \nu + \delta$, the dispersion oscillates with the forced frequency $\omega = S$:

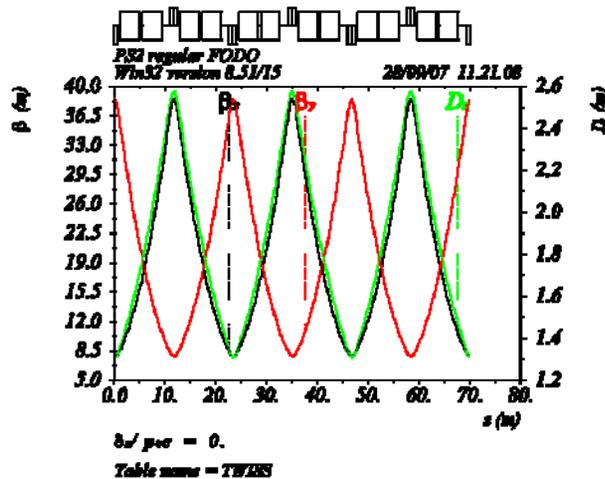
$$D(\mathcal{G}) \sim \frac{B}{\nu^2 - S^2} e^{i\mathcal{G}S} + \bar{D}$$

In irregular structure MCF depends on the curvature modulation B and detuning $\delta = S - \nu \ll \nu$:

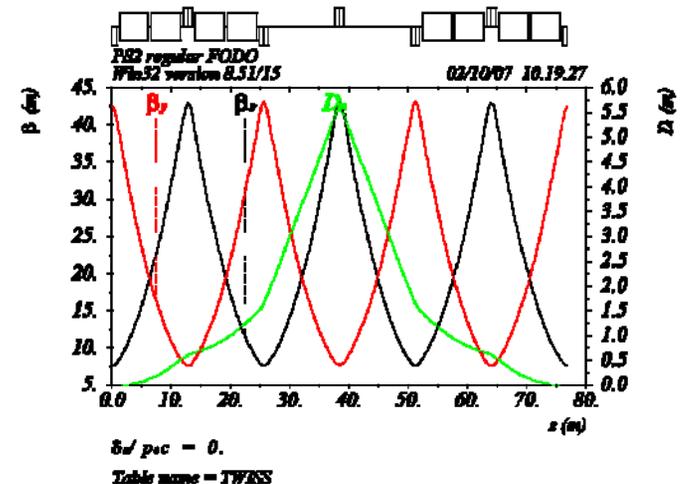
$$\alpha \approx \frac{1}{\nu^2} \left[1 - \frac{B^2}{4 \frac{\delta}{\nu}} \right]$$

Irregular lattice with curvature modulation ("missing" magnet lattice)

3 regular FODO cells with total length
 $3 \times 23.21 \text{ m} = 69.63$
 $L_{\text{mag}} = 3.7 \text{ m}$



3 irregular FODO missing cells
 with total length 76.8 m
 $L_{\text{mag}} = 4.9 \text{ m}$



Conclusion

"Missing magnet" lattices has advantages, practically does not perturb β -functions and disadvantages, requires the large phase advance value, significantly increases the arc length.

Results of “Resonant” lattice theory:

From the article: *Yu. Senichev, A “resonant” lattice for a synchrotron with a low or negative compaction factor, KEK Preprint 9740, 1997*

and *Journal Experimental and Theoretical Physics, vol. 105, n. 5, p.988,*

<http://www.springerlink.com/content/g411774815735702/fulltext.pdf>.

The solution of equation $D'' + K(\vartheta)D = 1 / \rho(\vartheta)$

with modulation of gradient $K(\vartheta) + \varepsilon k(\vartheta)$

$$\varepsilon \cdot k(\vartheta) = \sum_{k=0}^{\infty} g_k \cos k\vartheta \quad \text{and curvature:} \quad \frac{1}{\rho(\vartheta)} = \frac{1}{R} \left(1 + \sum_{n=1}^{\infty} r_n \cos n\vartheta \right)$$

gives the expression for MCF:

$$\alpha_s = \frac{1}{\nu^2} \left\{ 1 + \frac{1}{4 \cdot (1 - kS/\nu)} \cdot \left[\left(\frac{\bar{R}}{\nu} \right)^2 \frac{g_k}{[1 - (1 - kS/\nu)^2]} - r_k \right]^2 \right\}$$

The “resonant” lattice is based on the resonantly correlated curvature $1/\rho(\theta)$ and gradient modulations $K(\theta)$ in arcs with integer tunes in horizontal plane.

“Resonant” Lattice with minimum circumference and control of gamma transition in a wide region

The lattice has the remarkable feature:

The gradient and the curvature modulation amplify each by other if they have opposite signs,

$$g_k \cdot r_k < 0$$

The ratio between them is desirable to have:

$$|r_k| \leq \left(\frac{\bar{R}}{\nu} \right)^2 \left| \frac{g_k}{1 - (1 - kS)^2} \right| \quad \text{and} \quad \frac{1}{4(kS/\nu - 1)} \cdot \left(\frac{g_k}{[1 - (1 - kS/\nu)^2]} - r_k \right)^2 \approx 2, \text{ or } 0$$

On the contrary they can compensate each other when they have the same sign.



The gamma transition varies in a wide region from $\gamma_{tr} \sim \nu_x$ to $\gamma_{tr} \sim i\nu_x$

with quadrupole strength variation only!!!

In “resonant” lattice

-to provide a **dispersion-free straight section**, the arc consisting of S_{arc} superperiods must have a 2π integer phase advance or have the special dispersion suppressor.

- in order to drive the momentum compaction factor, the horizontal betatron tune ν_{arc} must be less than the resonant harmonic of perturbation kS_{arc} , and the difference between them has to be of a minimum integer value. We take $\nu_{arc} - kS_{arc} = -1$

Thus, the arc superperiodicity S_{arc} has to be even and ν_{arc} is odd.

The “golden” ratio between S_{arc} and v_{arc}

To fulfill all mentioned conditions we have to have the strictly fixed sets of S_{arc} and v_{arc} :

4:3; 6:5; 8:6; 8:7,..... and so on.

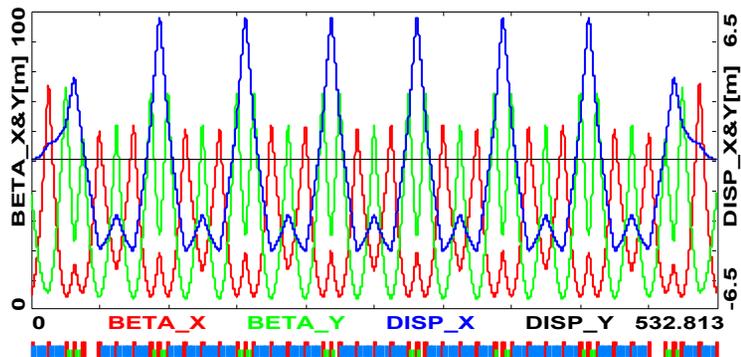


4:3 + 4:3

With and w/o the special dispersion suppressor

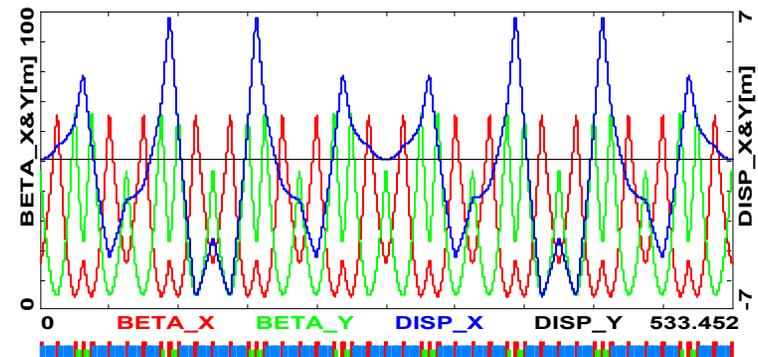
with suppressor (9 quads families)

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w/o suppressor (3 quads families)

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the lattice w/o suppressor has **3 families** of quadrupoles only
 the special suppressor requires additionally **5-6 families**

Compensation of sextupole non-linearity

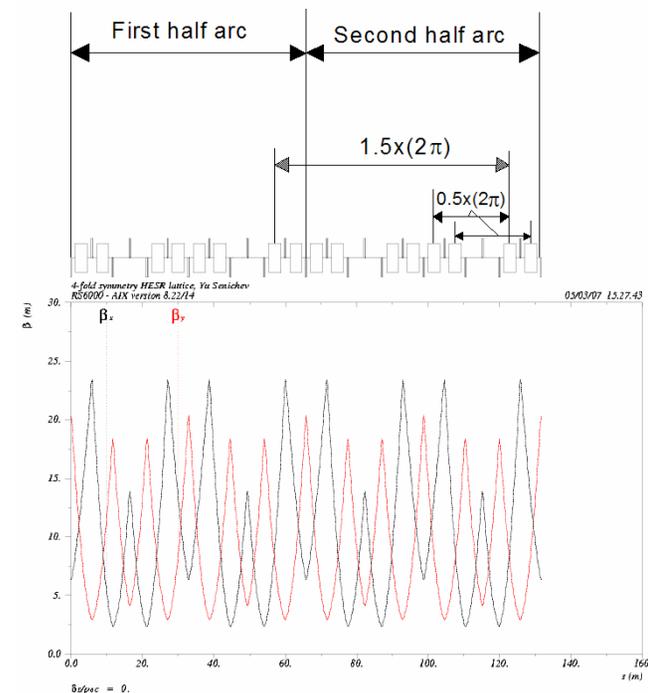
- In that case the phase advance between any two cells located in the different half arcs and separated by $\frac{S_{arc}}{2}$ number of

$$\frac{v_{arc}}{S_{arc}} \cdot \frac{S_{arc}}{2} = \frac{v_{arc}}{2} = \pi + 2\pi n$$

superperiods is then equal to

- the total multipole of third order is canceled:

$$M_3^{total} = \sum_{n=0}^N S_{x,xy} \beta_x^{l/2} \beta_y^{m/2} \exp in(l\mu_x + m\mu_y) = 0$$



The higher order resonance excitation and non-linear tune shifts

the coefficients $\zeta_x, \zeta_y, \zeta_{xy}$ are the
non-linear tune shifts:

$$\zeta_x = \zeta_x^{sex} + \zeta_x^{oct}$$

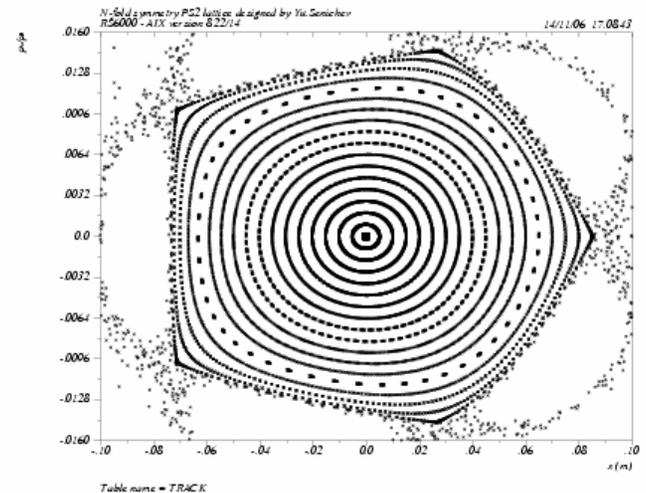
$$\zeta_{xy} = \zeta_{xy}^{sex} + \zeta_{xy}^{oct}$$

$$\zeta_y = \zeta_y^{sex} + \zeta_y^{oct}$$

as example

$$\zeta_x^{sex} = -\frac{3}{4} \left[\sum_{p=-\infty}^{\infty} \frac{|h_{3010p}|^2}{\nu_x - p} + \sum_{\substack{p=-\infty \\ p \neq p_0}}^{\infty} \frac{3|h_{3030p}|^2}{3\nu_x - p} \right]$$

$$\zeta_x^{oct} = \frac{1}{32\pi\Delta^2} \int_0^{2\pi} \beta_x^2 O_x R d\theta$$

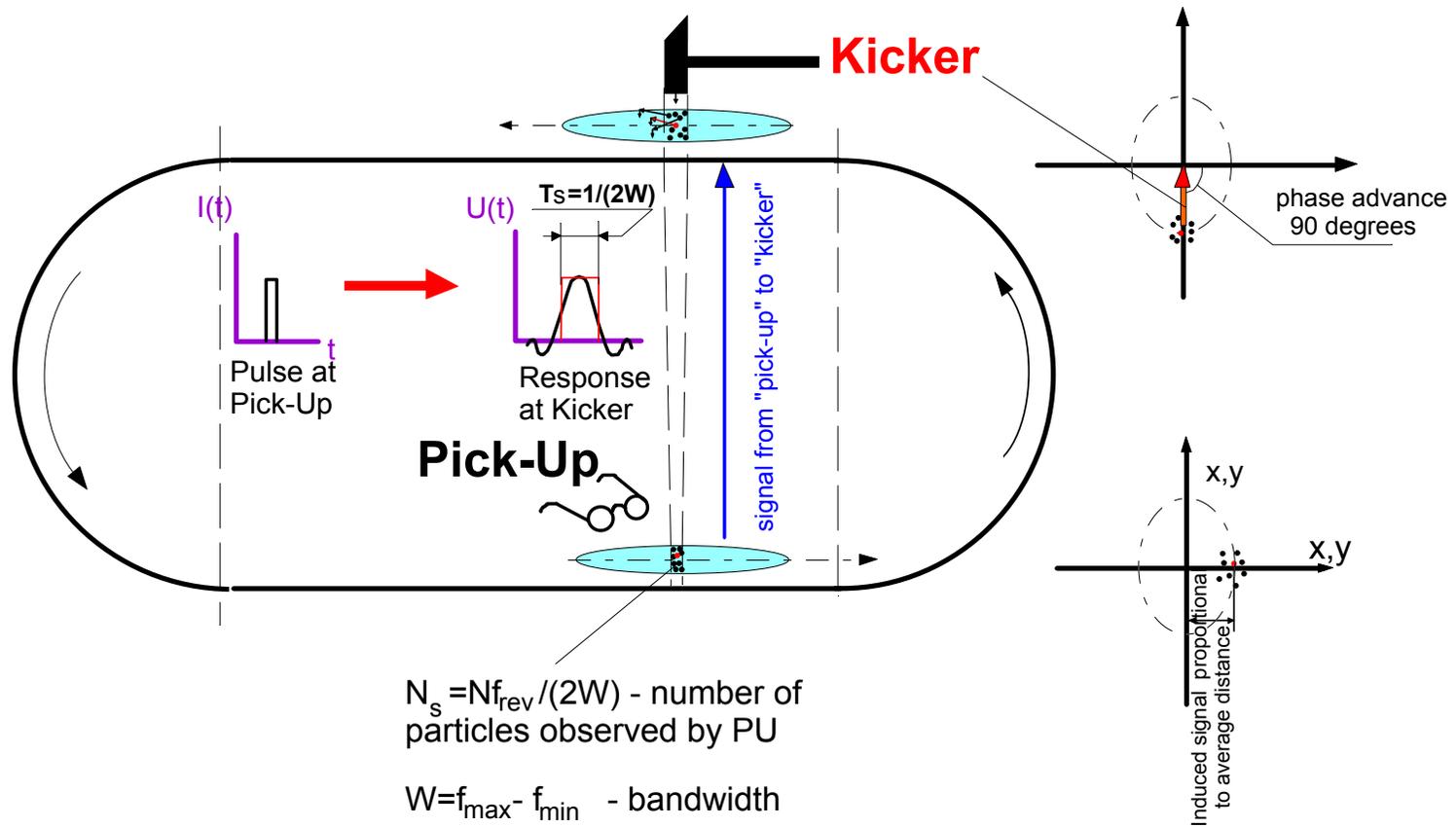


Dynamic aperture after
chromaticity compensation (for
PS2)

Thus, the “Resonant” structure has the features:

1. Ability to achieve the negative momentum compaction factor with minimum circumference and control of gamma transition in a wide region;
2. Dispersion-free straight section without special suppressor;
3. Low sensitivity to multipole errors and sufficiently large dynamic aperture.
4. Minimum families of focusing and defocusing quadrupoles and separated adjustment of gamma transition, horizontal and vertical tunes;
5. Convenient sextupole chromaticity correction scheme;
6. Independent optics parameters of arcs and straight sections

Stochastic cooling principle and requirements to the optics



Real and Imaginary arcs for Stochastic Cooling:

The momentum compaction factor in imaginary and real arcs takes the

meaning: $\alpha_{pk} = \frac{1}{4v_{arc}^2}$ $\alpha_{kp} = -\frac{1}{4v_{arc}^2}$

and slip factors:

$$\eta_{pk} = \frac{1}{\gamma^2} - \frac{1}{4v_{arc}^2}$$

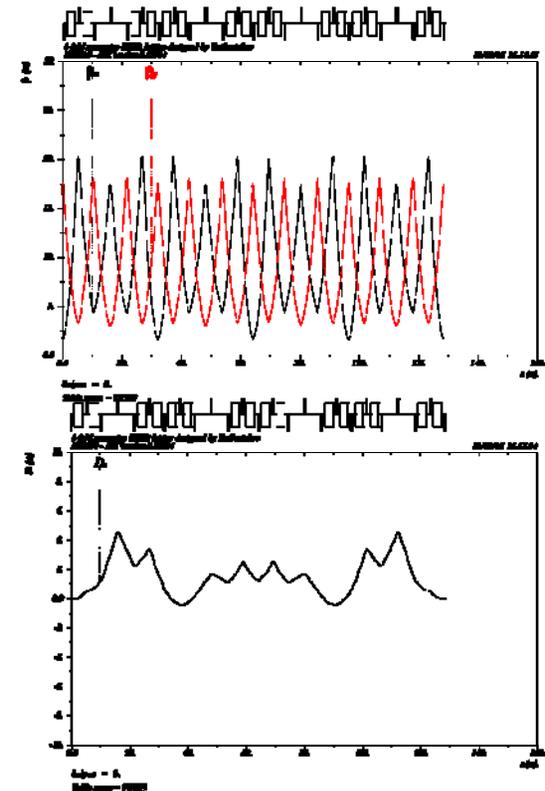
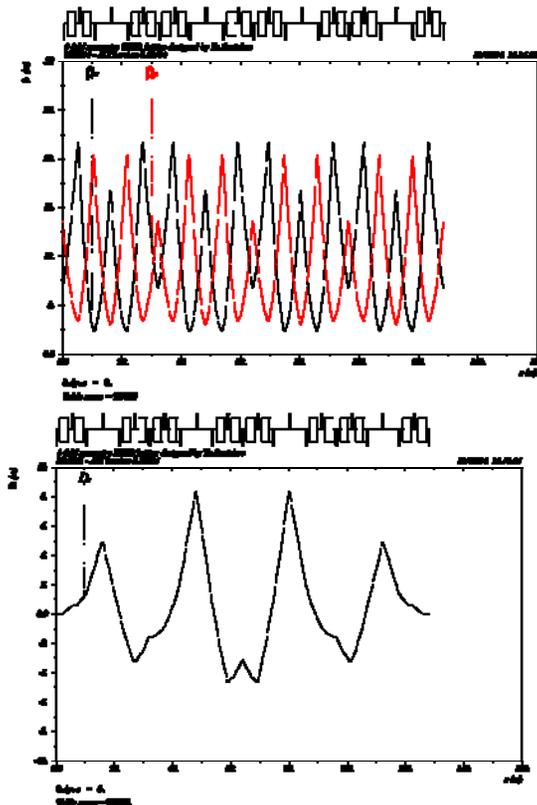
$$\eta_{kp} = \frac{1}{\gamma^2} + \frac{1}{4v_{arc}^2}$$

In case $\gamma \approx 2v_{arc}$ the **real** arc is isochronous $\eta_{pk} \approx 0$

the **imaginary** arc has a slip factor $\eta_{kp} \approx 1/2v_{arc}^2$

Twiss parameters of the real and imaginary arcs of SC option for HESR (FAIR)

The β -function and dispersion on the imaginary, the real 4-fold symmetry arcs



What can we do for Synchrotron Light Source Optics?

Almost all **Synchrotron Light Sources** work higher of the transition energy, therefore chromaticity must be $\xi > 0$

Since the horizontal emittance depends upon the horizontal dispersion function,

as $\varepsilon_x \propto \langle H \rangle_{dipole}$, where $H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta_x'^2$

In order to get $\varepsilon_{x, min}$ the dispersion \rightarrow minimum value

Stronger sextupoles are required \rightarrow the dramatic decreasing of DA

There are two methods:

1. Sextupoles have to be compensated
2. Lattice w/o sextupole with imaginary γ_{tr} when chromaticity should be $\xi < 0$

Sextupole compensation in SLS optic

For 3-d integer resonance the influence of the non-linearity is specified by the discriminant in the expression:

$$\hat{I}_x^{1/2} = -\frac{3h_{30p} \cos 3\hat{\vartheta}_x}{8\zeta_x} \pm \frac{1}{4\zeta_x} \sqrt{\frac{9}{4}h_{30p}^2 - 8\zeta_x(\Delta + \zeta_{xy}I_y)}$$

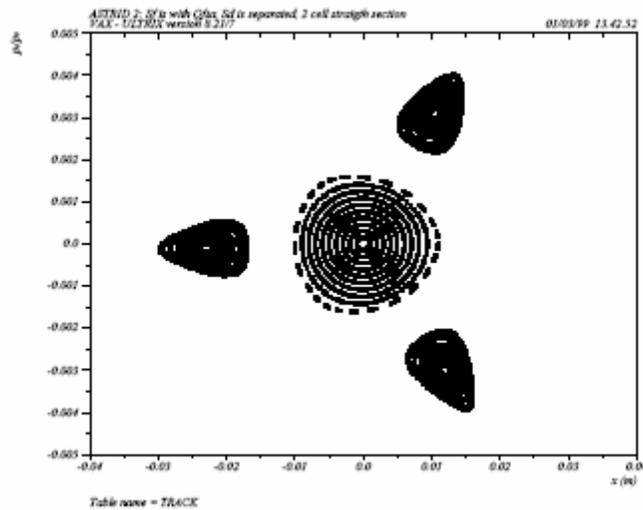
The lattices with $\zeta_x \gg h_{30p}$ have to be classified as a special lattice, since it is a case, when the value of h_{30p} is effectively suppressed, but the non-linearity remain to be under control and strong.

If the sign of the detuning Δ coincides with the sign of the tune shift, the discriminant is negative and the system has only one centre at

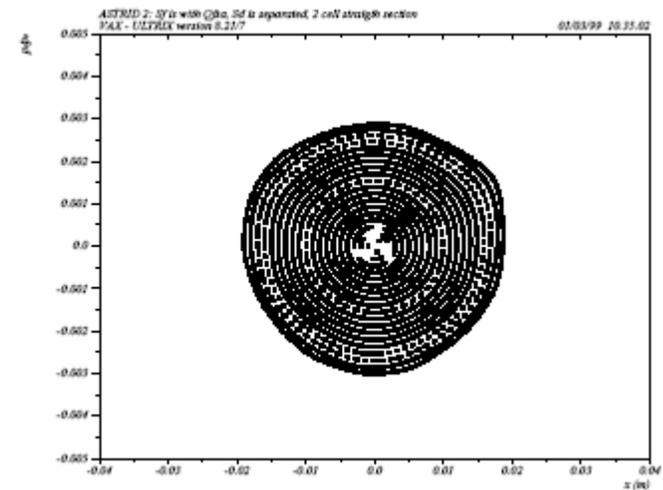
Dynamic aperture tracking

negative and positive detune

$$\zeta > 0; \Delta < 0$$



$$\zeta > 0; \Delta > 0$$



Conclusion

“Resonant” lattice was developed with features:

- ability to achieve the negative momentum compaction factor using the resonantly correlated curvature and gradient modulations;
- gamma transition variation in a wide region from $\gamma_{tr} = v_x$ to $\gamma_{tr} = i v_x$ with quadrupole strength variation only;
- integer odd 2π phase advance per arc with even number of superperiod and dispersion-free straight section;
- independent optics parameters of arcs and straight sections;
- two families of focusing and one of defocusing quadrupoles;
- separated adjustment of gamma transition, horizontal and vertical tunes;
- convenient chromaticity correction method using sextupoles;
- first-order self-compensating scheme of multipoles and as consequence low sensitivity to multipole errors and a large dynamic aperture

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The author is grateful to IKP colleagues for taking part in development on the super-conducting lattice for High Energy Storage Ring of FAIR project.

The second order non-linearity

-After some canonical transformation we can get the second order approach of Hamiltonian in the next view:

$$H(J_x, \vartheta_x, \theta_x, J_y, \vartheta_y, \theta_y) = \nu_x J_x + \nu_y J_y + \sum g(M, N, n_1, n_2, p) J_x^{M/2} J_y^{N/2} \exp i(n_1 \vartheta_x + n_2 \vartheta_y - p \theta)$$

-Now let us suppose that we are some where around of the third order resonance:

$$\begin{aligned} 3\bar{\nu}_x &= p_0, \\ \bar{\nu}_x &= \nu_x + \Delta \end{aligned}$$

-the Hamiltonian takes a view

$$H_1(J, \psi, \theta) = \nu_x J_x + \nu_y J_y + \frac{1}{2} J_x^{3/2} \left\{ h_{3030p_0} \exp i(3\psi_x - p_0 \theta) + c.c. \right\} + \zeta_x J_x^2 + \zeta_{xy} J_x J_y + \zeta_y J_y^2$$

6. Independent optics parameters of arcs and straight sections

- Tune arc does not depend on the transition energy and is kept constant;
- Special insertion on the straight section allows to match the $\beta_{x,y}$ -functions between arcs and straight sections;
- Dispersion-function on the straight sections always equal zero;
- All high order non-linearities are compensated inside each arc.

Nekhoroshev's criterium:

the non-linearity in both planes have to have the same sign and $4\zeta_x \zeta_y \geq \zeta_{xy}^2$

The lattices with $\zeta_x \gg h_{30p}$ have to be classified as a special lattice, since it is a case, when the value of h_{30p} is effectively suppressed, but the non-linearity remain to be under control and strong.

If the sign of the detuning Δ coincides with the sign of the tune shift ζ_x , the discriminant is negative and the system has only one centre at $I_x = 0$

The quasi-isochronism condition by Nekhoroshev is fulfilled, when

$$k_x (2\zeta_x I_x^r + \zeta_{xy} I_y^r) + k_y (2\zeta_y I_y^r + \zeta_{xy} I_x^r) = 0$$

$$\zeta_x k_x^2 + \zeta_{xy} k_x k_y + \zeta_y k_y^2 = 0$$



Convex or concave
resonant surface with
maximum stable region

Minimum families of focusing and defocusing quadrupoles and separated adjustment of gamma transition, horizontal and vertical tunes

Convenient sextupole chromaticity correction scheme

