Transverse Schottky noise with space charge

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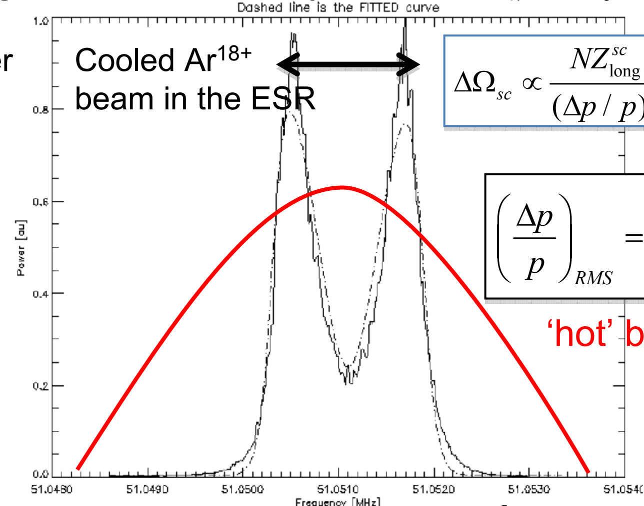
Schottky noise measurements

Low vs. high intensity beams

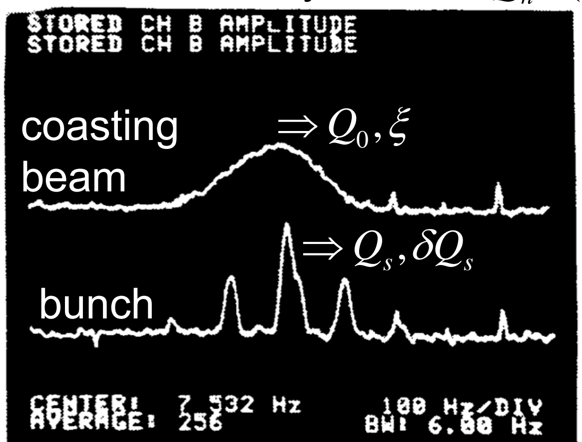
Longitudinal Schottky band at $f_n = n f_0$ (51 MHz)



$$\Delta\Omega_{sc} \propto \frac{N Z^{sc}_{\text{long}}}{(\Delta p / p)^2}$$

$$\left(\frac{\Delta p}{p}\right)_{RMS} = \frac{\Delta f_{RMS}}{n \eta f_0}$$

Transverse Schottky band at $Q_n = (n \pm Q_0)$

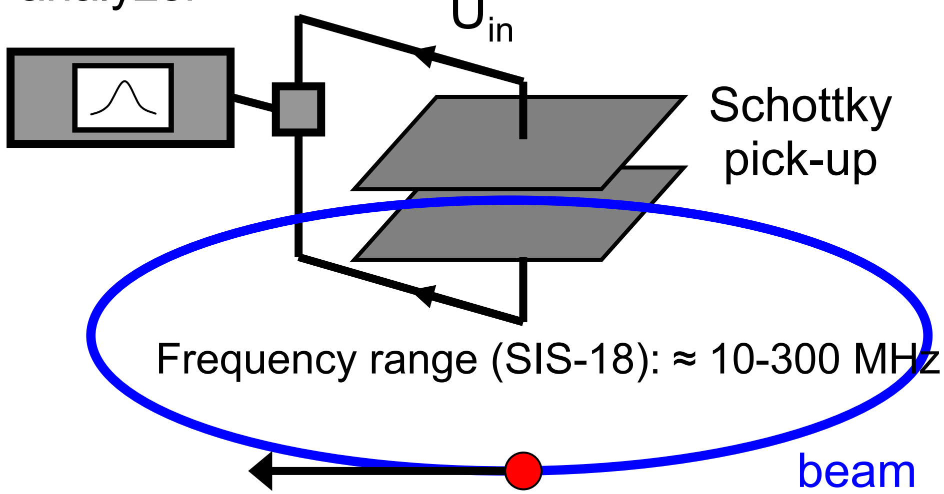


CERN/SPS measurement (Linnecar, PAC1981)

Sum signal U_Σ: beam current fluctuations ΔI

Difference signal U_Δ: beam 'offset' fluctuations $\Delta\bar{x}$

set-up in the SIS-18 heavy ion synchrotron at GSI



Effect of space charge on transverse Schottky bands ?

Transverse Schottky spectrum with space charge

theory/simulation for **coasting beams**

Space charge tune shift:

$$Q = Q_0 - \Delta Q_{sc} \quad \text{with} \quad \Delta Q_{sc} \propto \frac{q^2 NR}{m \beta_0^2 \gamma_0^3 \varepsilon}$$

Transverse equation of motion ('rigid beam'):

$$\frac{d^2 x}{dt^2} + Q_0^2 \omega_0^2 x - 2\omega_0^2 Q_0 \Delta Q_{sc} \left(x - \overline{x} \right) = 0$$

Gaussian momentum distribution:

$$f(\delta) = \frac{1}{\sqrt{2\pi}\delta_{rms}} \exp\left(-\frac{\delta^2}{2\delta_{rms}^2} \right)$$

Chromatic tune spread: $\delta Q_\xi = S \delta_{\mathrm{RMS}}$

Dispersion function for dipole oscillations:

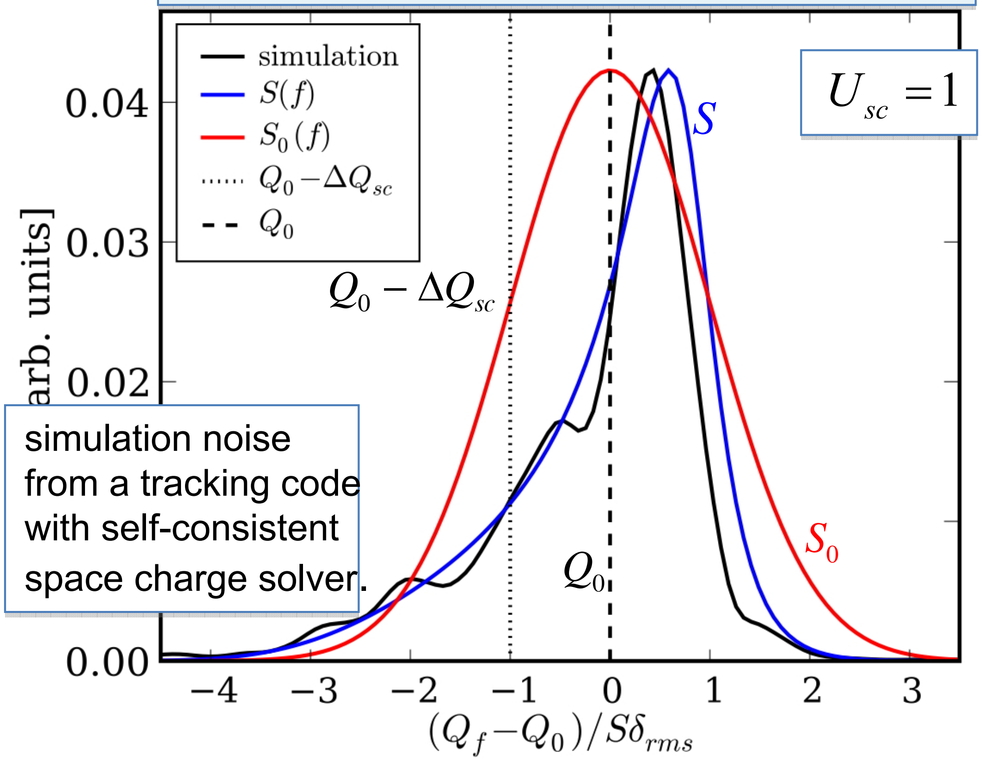
$$D(u) = U_{sc} \int_{-\infty}^{\infty} \frac{f(\delta) d\delta}{u - \delta} = i \sqrt{\frac{\pi}{2}} U_{sc} w\left(u / \sqrt{2} \right)$$

with the normalized tune $u = \dfrac{Q_f - Q_0 + \Delta Q_{sc}}{\delta Q_\xi}$

Space charge parameter: $U_{sc} = \dfrac{\Delta Q_{sc}}{\delta Q_\xi}$

Modified transverse Schottky band:

$$S(Q_f) = \frac{d\left(\overline{x} I \right)^2}{dQ_f} = \frac{S_0(u)}{\left| 1 - D(u) \right|^2}$$



O. Boine-Frankenheim, V. Kornilov, S. Paret, Phys. Rev. ST Accel. Beams (2008)
A. Burov, V. Lebedev, Phys. Rev. ST Accel. Beams (2009)

Transverse Schottky spectrum with space charge
SIS-18 experiments with **coasting beams**

Fit of the measured Schottky band to:

$$S(f) = \frac{A\exp\left(-u^2\right)}{\left|1 - BU_{sc}w(u)\right|^2}$$

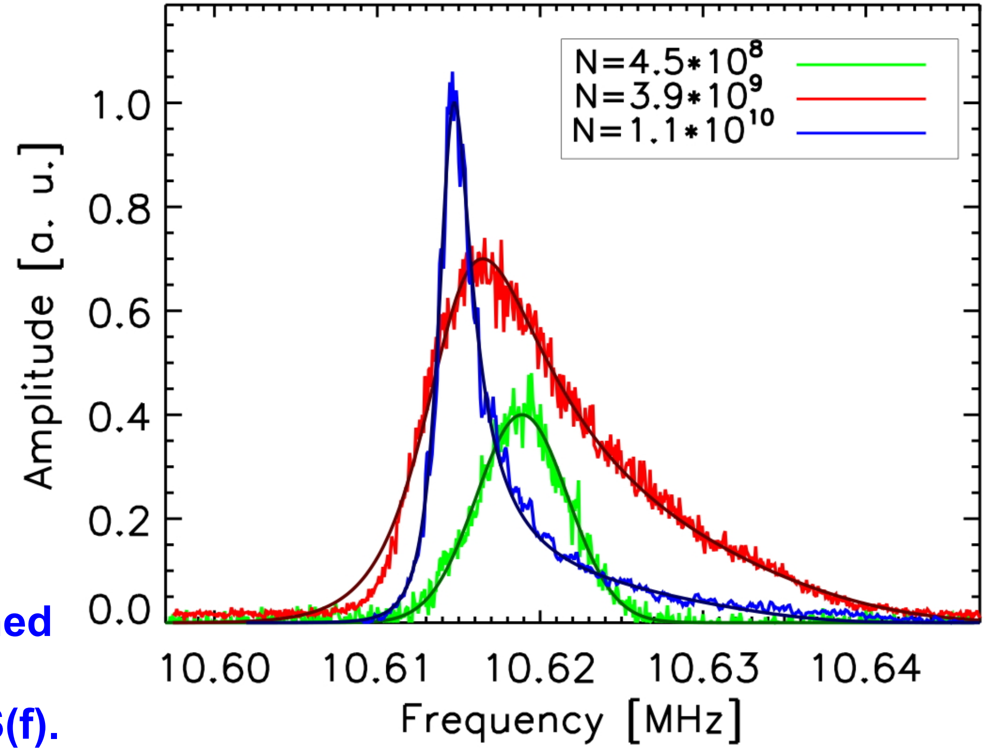
Result of the fit:

$N/10^9$	$\delta_{\mathrm{rms}}\ [10^{-4}]$	δQ	ΔQ_{sc}	U_{sc}
0.45	2.8	0.013	0.0	0.0
3.9	6.7	0.033	0.03	0.86
11.0	7.8	0.031	0.06	1.84

The space charge tune shift ΔQ_{sc} and the space charge parameter U_{sc} can be obtained from the measured Schottky spectrum using a fit to the the analytic expression S(f).

Measured transverse (lower) Schottky side-band for different numbers of ions in the ring N.

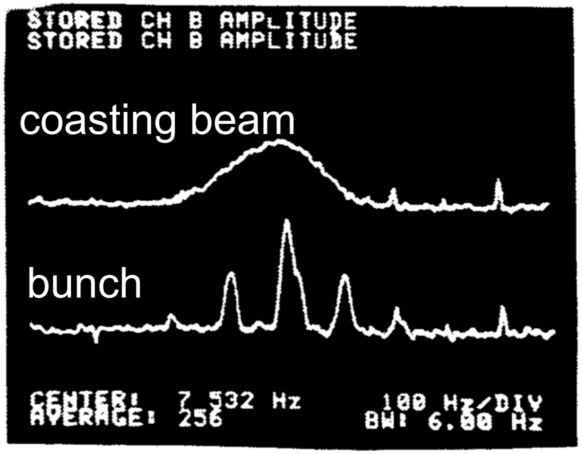
Beam parameter: Ar^{18+}, 11.4 MeV/u, f_0=215 kHz



Transverse Schottky noise from **bunched beams**

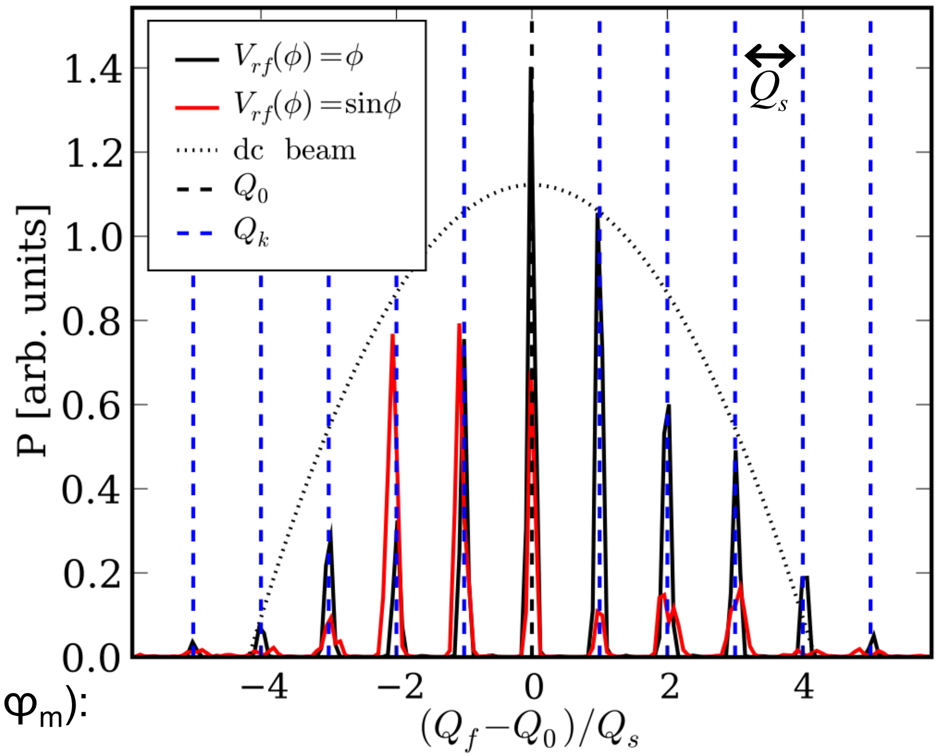
low intensity

Transverse Schottky band at $Q_n = (n \pm Q_0)$



CERN/SPS measurement
(Linnecar, PAC 1981)

Transverse 'offset' noise from the particle tracking code PATRIC. Bunching factor B_f=0.35.



Synchrotron satellites (synchrotron tune Q_s):

$$Q_{n,k} = (n \pm Q_0) + kQ_s$$

Tune spread in a rf bucket (bunch half-length φ_m):

$$\delta Q \approx kQ_s \frac{\phi_m^2}{16} << S\delta_{\mathrm{RMS}}$$

Simulation time: $T = 10\dfrac{Q_s}{f_0}$ (10 synchrotron periods)

Head-tail modes with space charge

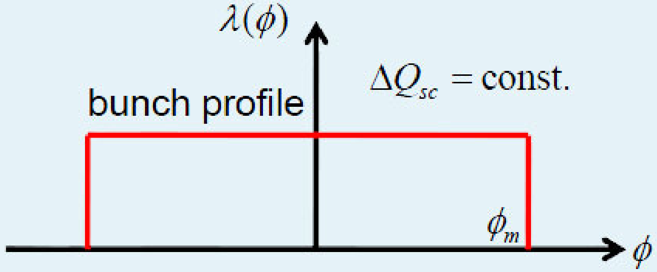
Barrier airbag distribution

M.Blaskiewicz, Phys.Rev.ST Accel.Beams 1, 044201 (1998)

Longitudinal bunch distribution (velocity v_m):

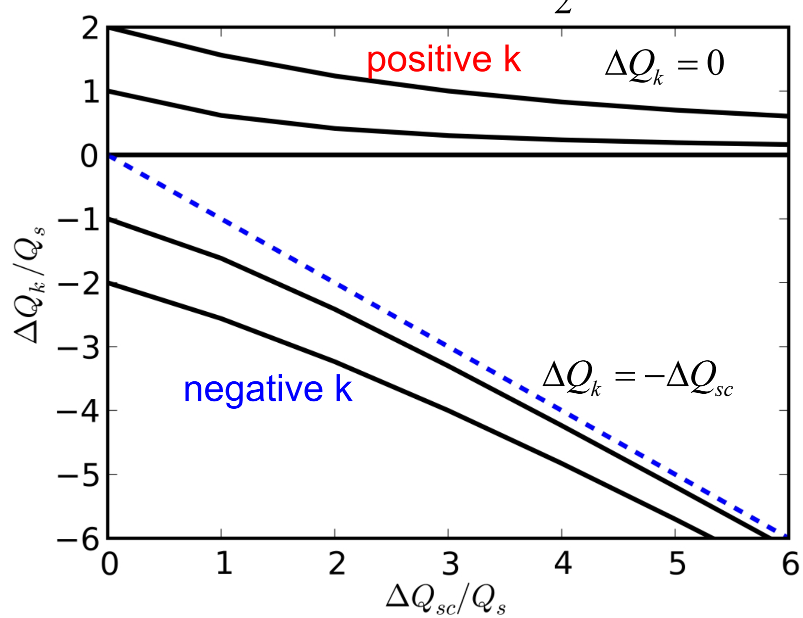
$$f(v,\phi) = A\left[\delta(v_m - v) + \delta(v_m + v)\right]$$

Synchrotron tune: $Q_s = \frac{1}{f_0}\frac{v_m}{2l}$ (bunch length l)



$x_\pm(\phi,t)$: transverse offset of particles with $v = \pm v_m$

head-tail tune shifts: $\Delta Q_k = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{(\Delta Q_{sc}/2)^2 + (kQ_s)^2}$



space charge parameter

Strong space charge $\Delta Q_{sc} >> Q_s$

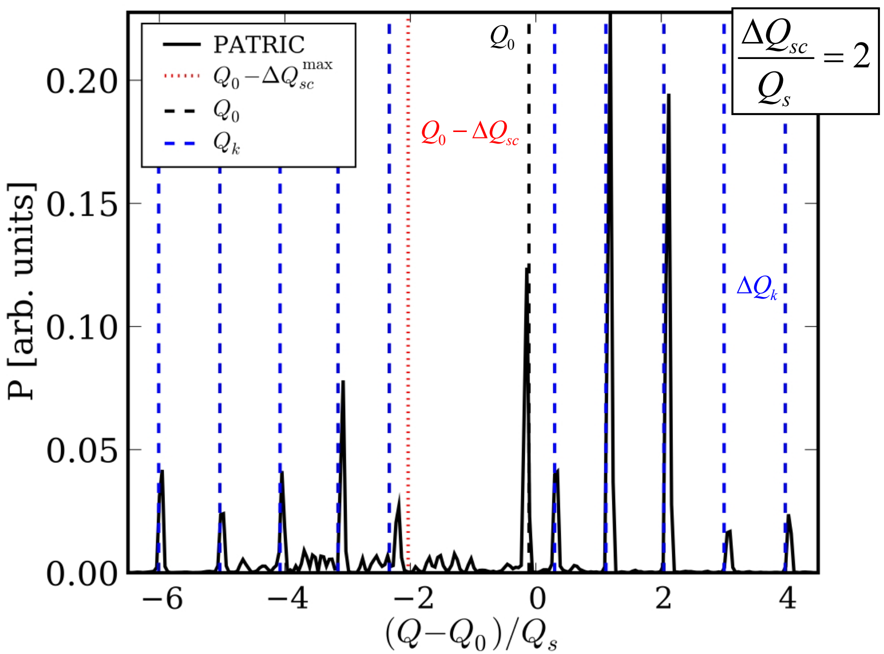
positive k: $\Delta Q_k = \frac{(kQ_s)^2}{\Delta Q_{sc}} \to 0$ dipole moment dominates $\bar{x}_k = (x_{+,k} + x_{-,k})/2 = \cos(k\pi\phi/\phi_m)$

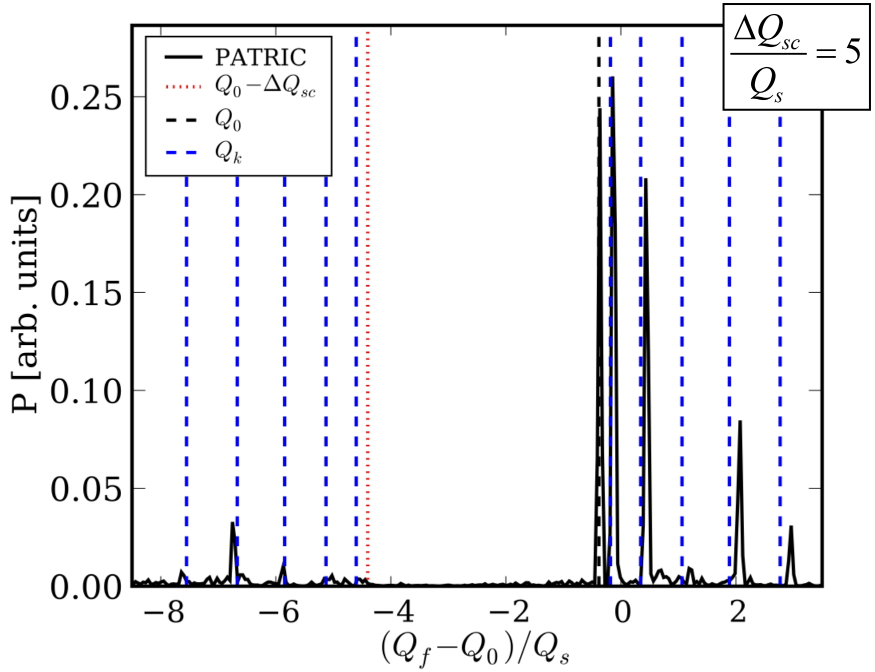
negative k: $\Delta Q_k = -\Delta Q_{sc}$ quadrupole moment dominates $\Delta x_k = (x_{+,k} - x_{-,k}) = (i\Delta Q_k / kQ_s)\sin(k\pi\phi/\phi_m)$

Transverse simulation noise spectrum with space charge

barrier airbag distribution

'Offset ' noise spectrum from the particle tracking code PATRIC with a 2.5D self-consistent space charge solver.

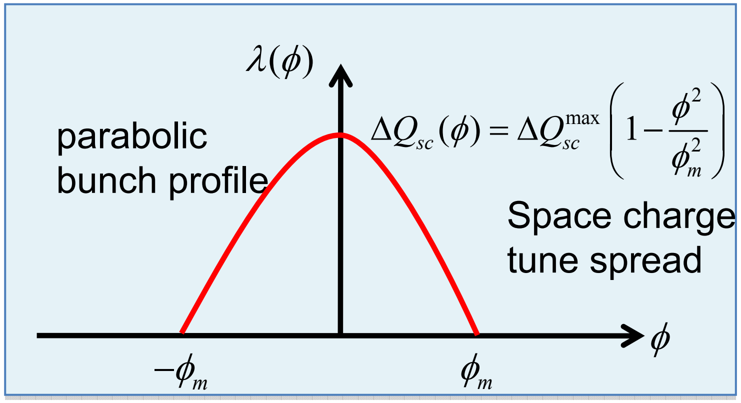




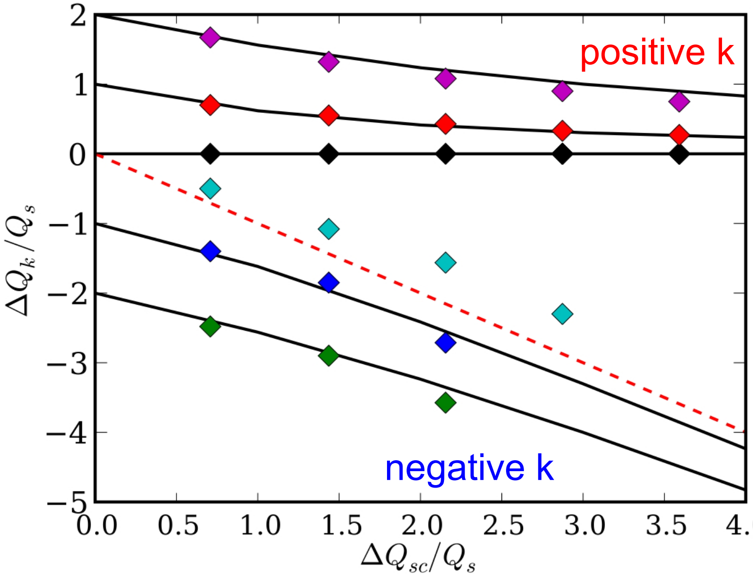
- Good agreement with the analytic ΔQ_k for moderate space charge (code validation).
- Satellites with negative k disappear for strong space charge.

Transverse simulation noise spectrum with space charge

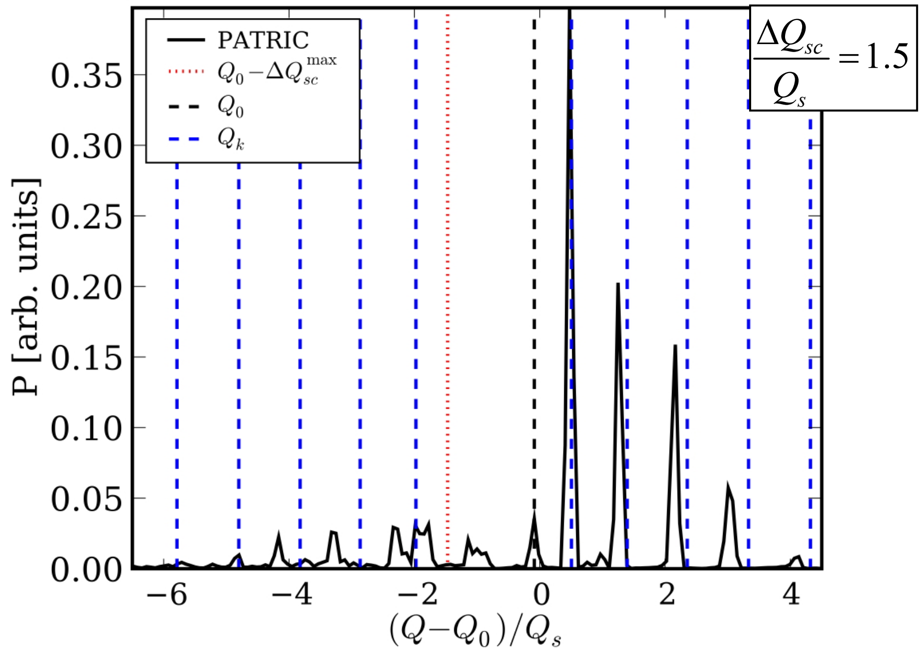
Elliptic bunch distribution



Head-tail modes from the airbag model (analytic vs. parabolic bunch (simulation noise satellites)



'Offset ' noise spectrum from the particle tracking code PATRIC with a 2.5D self-consistent space charge solver



Simulation studies for moderately strong space charge

Positive k: good agreement with airbag model

Negative k: satellites are strongly damped with space charge.

Measure the space charge tune shift in bunches $\Delta Q_{sc} = \frac{(kQ_s)^2}{\Delta Q_k}$

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Conclusions

Transverse Schottky noise spectrum from coasting beams:

○ For moderately strong space charge we find a very good agreement between measured transverse Schottky bands and the analytic expression for the fluctuation spectrum of 'rigid' dipole oscillations.

○ From a fit to the expression one can retrieve the space charge tune shift and the Landau damping rate.

Transverse Schottky noise spectrum from bunched beams :

○ We obtain good agreement between the positive k synchrotron satellites from the simulation noise spectrum and the analytic head-tail tune shifts from the barrier airbag model.

○ Negative k satellites are strongly suppressed due to the effect of space charge.

Possible application: **Measure space charge tune shift ΔQ_{sc} from the positive k Schottky noise satellites ΔQ_k.**