Circularly Inclined Solenoid Channel for 6D Ionization Cooling of Muons

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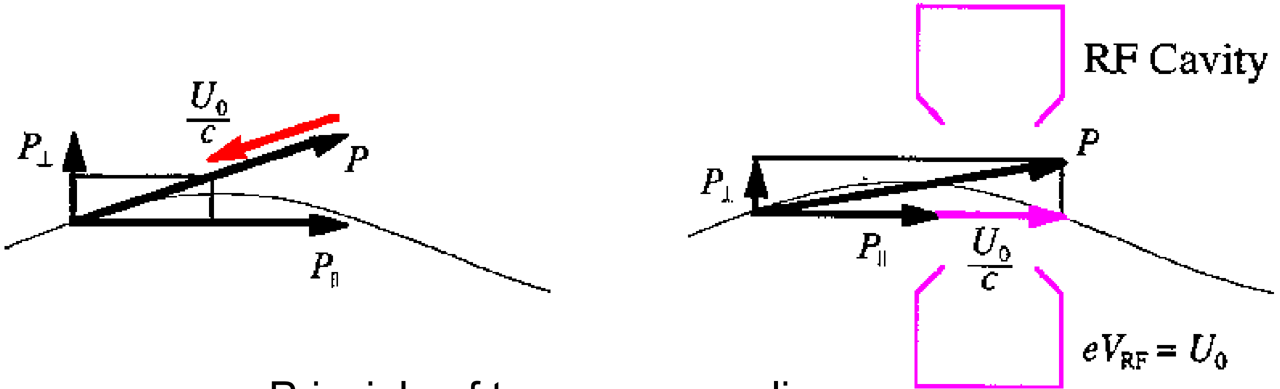
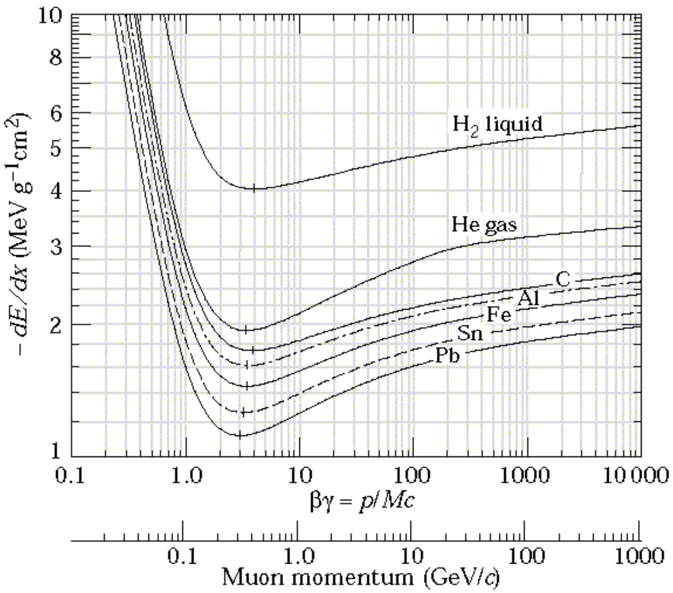
Ionization Cooling Basics



Principle of transverse cooling



There is no longitudinal cooling in the most suitable range 2-300MeV/c.

With higher momentum *p* > 300MeV/c it is difficult to obtain small β-function which is necessary for small equilibrium emittance:

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta\gamma\ \beta_\perp}{2}\frac{d\left\langle \theta_{rms}{}^2 \right\rangle}{ds}$$

Damping Re-partition for Longitudinal Cooling

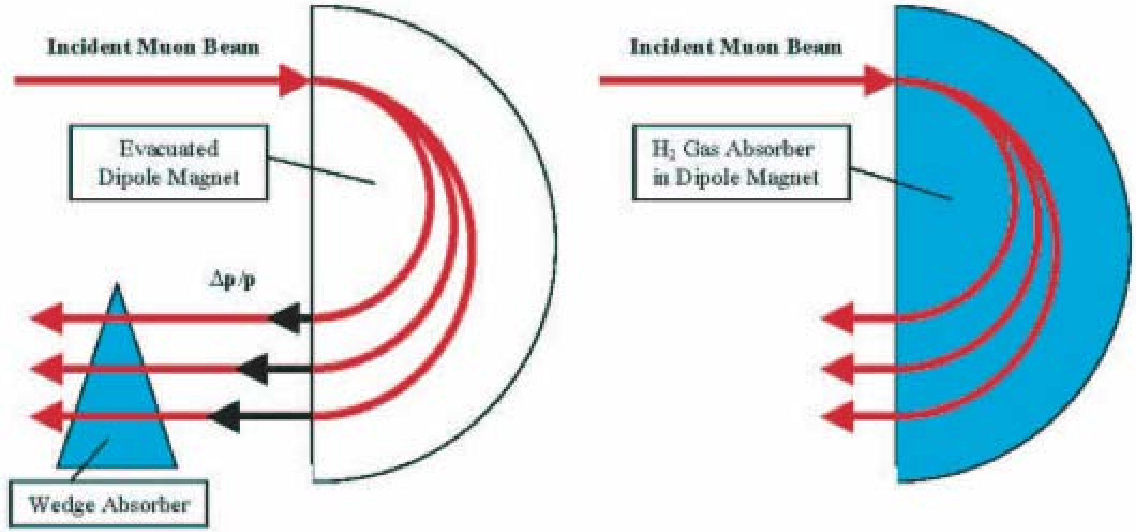


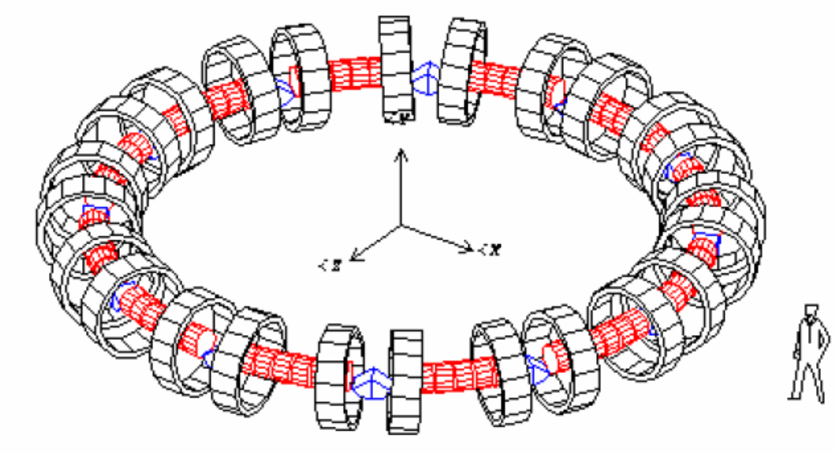
Figure 1. Use of a Wedge Absorber for Emittance Exchange

Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

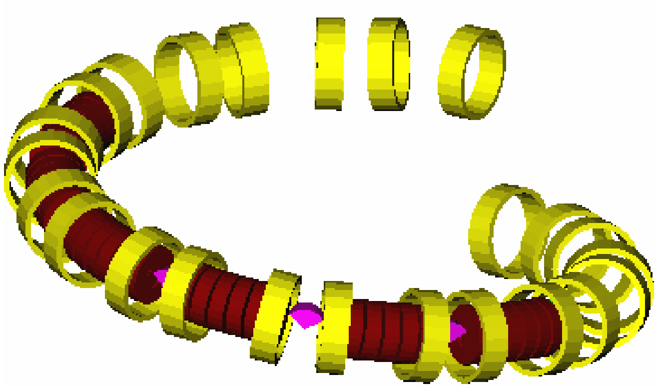
Dispersion and/or large positive momentum compaction ⇒ higher momentum muons make longer path in the absorber ⇒ lose more energy ⇒ longitudinal cooling

6D Cooling Schemes

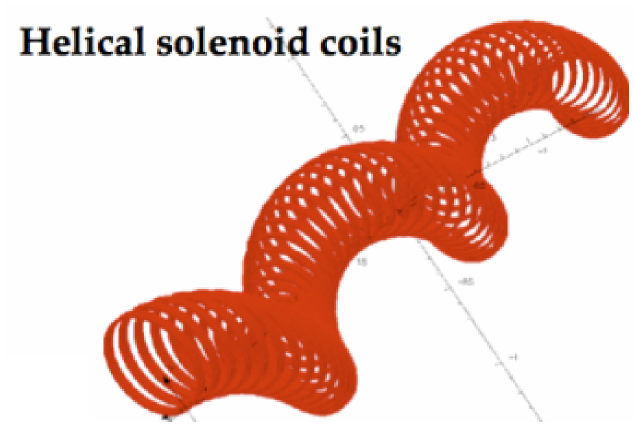
RFOFO ring



“Guggenheimed”



Helical Cooling Channel



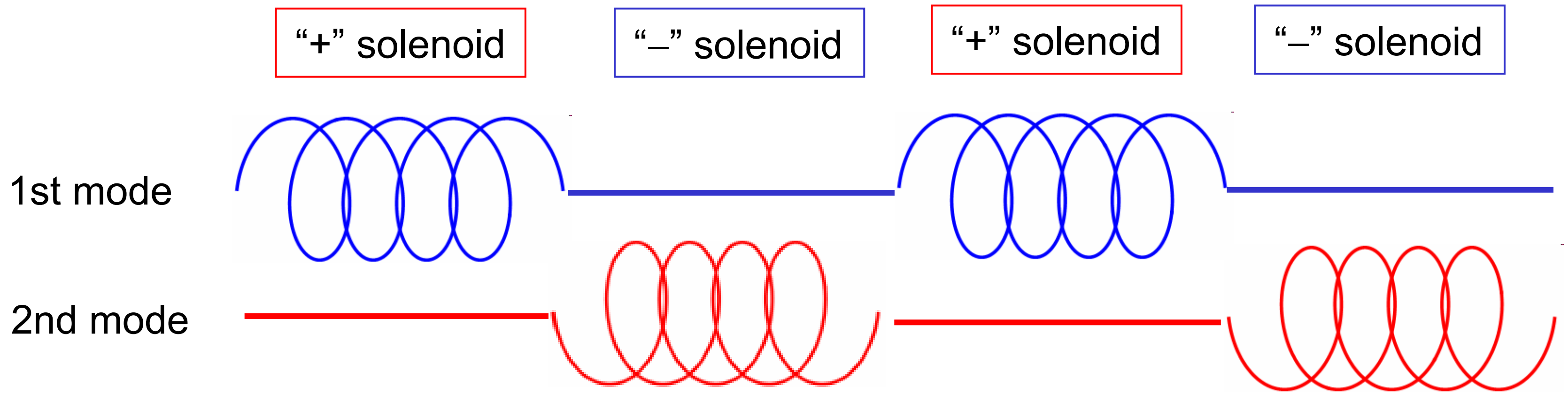
“Guggenheim” : too long ⇒ muon decay; problem with RF in magnetic field.

HCC: no viable solution yet for RF inside coils.

Both channels are selective to muon sign, it is either $\mu+$ or $\mu-$

Alternating solenoid focusing

- necessary for damping both transverse modes in ~ straight channel



Resonance dispersion generation:

resonant dependence of the periodic orbit on the tune, $x_{p.o.} \sim 1/\sin\pi Q_x$,
+ large chromaticity $\Rightarrow$

$$D_x = \frac{dx_{p.o.}}{d\delta_p} = -\pi Q'_x x_{p.o.} \cot(\pi Q_x)$$

Naturally $Q'_x < 0 \;\Rightarrow\; D_x \cdot x_{p.o.} > 0 \Rightarrow$ positive momentum compaction

Challenges

The initial proposal - planar snake - reported at PAC07. Further analysis revealed certain difficulties:

- Insufficient transverse dynamic aperture

$$\frac{\partial B_z}{\partial z} \Rightarrow \frac{\partial B_r}{\partial r}, \quad \frac{\partial^2 B_z}{\partial z^2} \Rightarrow \frac{\partial^2 B_z}{\partial r^2}, \quad \frac{\partial^3 B_z}{\partial z^3} \Rightarrow \frac{\partial^3 B_r}{\partial r^3} \quad etc. \qquad \Rightarrow \text{strong aberrations}$$

Cures:
- increase number of cells per period 4→6 (smaller phase advance per cell)
- increase solenoid radius (smoother *Bz*)

This helped, but brought about another problem:

- Unequal damping rates of transverse modes

Cure: mix the transverse modes better by
- adding a constant magnetic field component (R.Palmer)
- making the snake helical (the subject of this report)

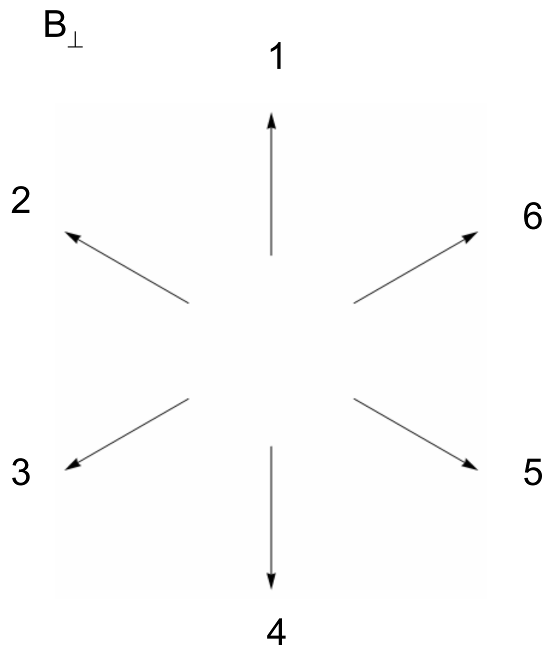
Basic Idea of Helical FOFO snake

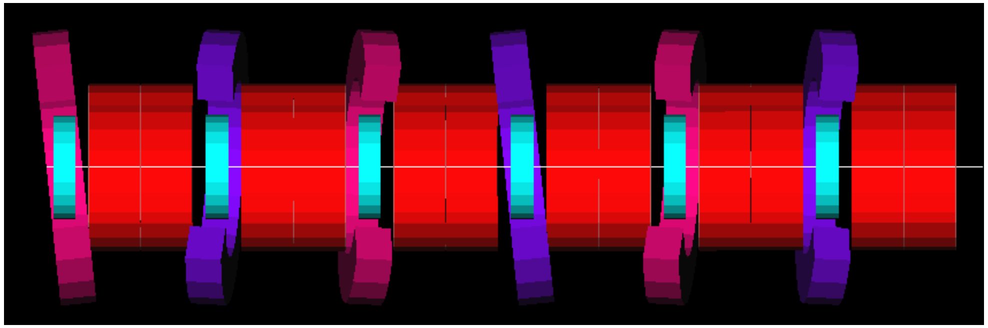
- Choose phase advance/period (6-cell period here) > $2\pi\times$integer to obtain $\alpha_p > 0$
- Create rotating $B_\perp$ field by tilting (or displacing) solenoids in rotating planes

$$x^*\cos(\phi_k)+y^*\sin(\phi_k)=0,\ k=1,2,\ldots$$

Example for 6-cell period:

Solenoid #	1	2	3	4	5	6
Polarity	+	-	+	-	+	-
Roll angle ϕ_k	0	$2\pi/3$	$4\pi/3$	0	$2\pi/3$	$4\pi/3$





Channel parameters:

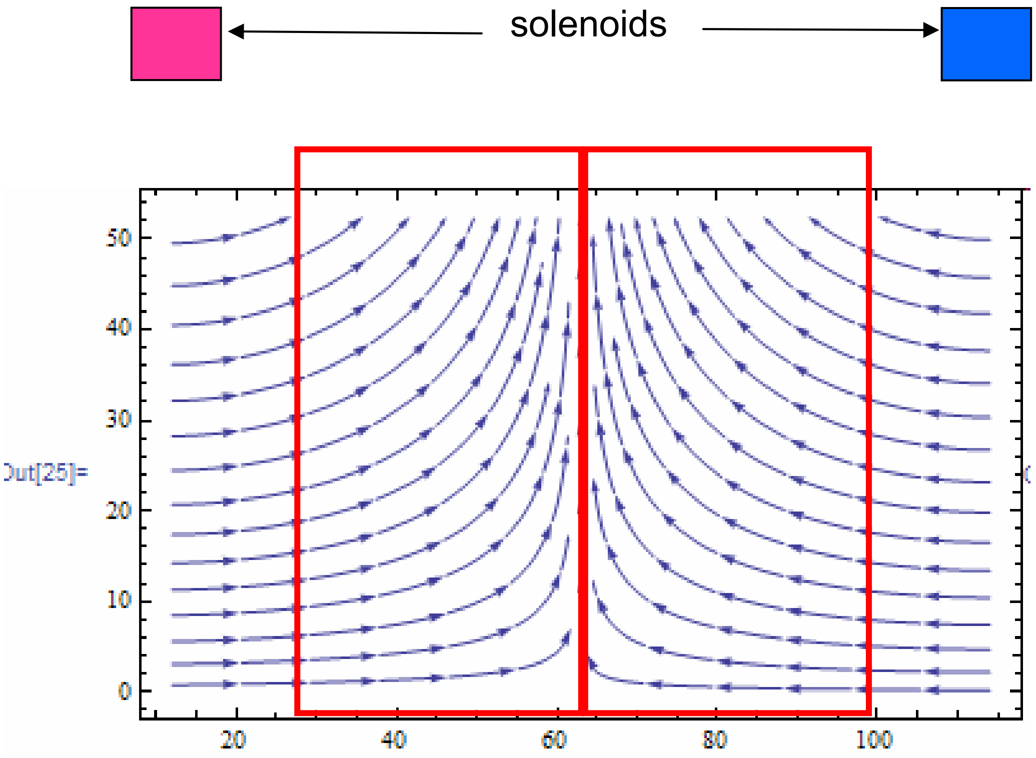
200 MHz pillbox RF 2x36cm, Emax=16MV/m

Solenoids: L=24cm, Rin=60cm, Rout=92cm,

Absorbers: LH2, total width (on-axis) 6x15cm,

Total length of 6-cell period 6.12m

Why double cavities?

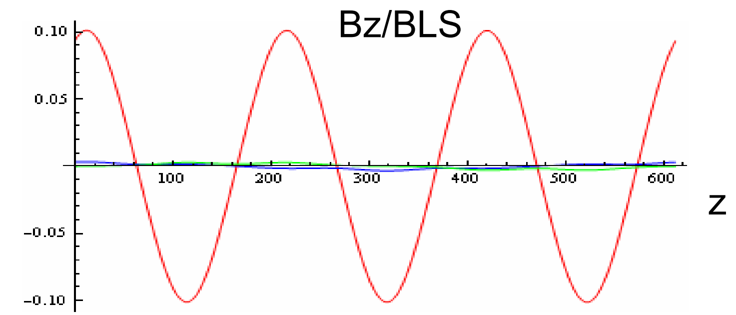


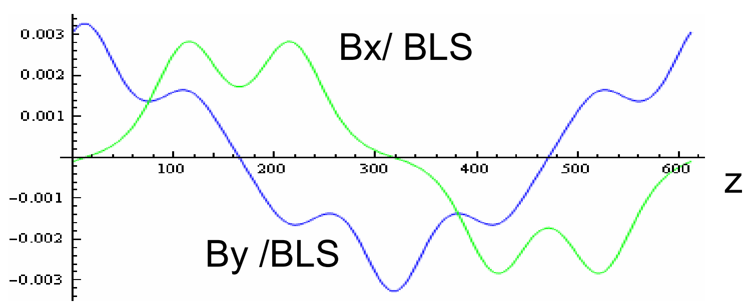
"Poor man" magnetic insulation for Ncavities ≤ 2

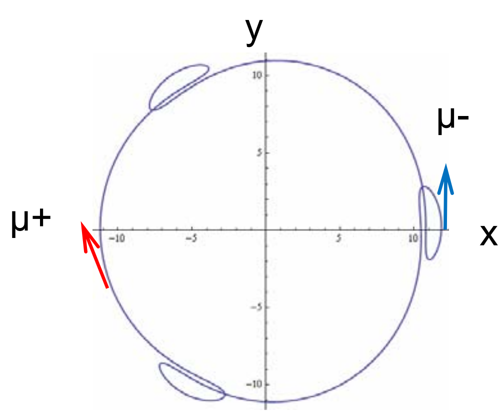
Length of 1 cavity is not enough for cooling rate

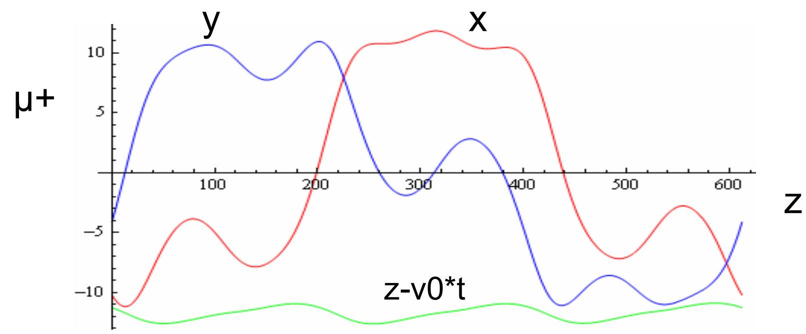
Periodic Helical Orbits

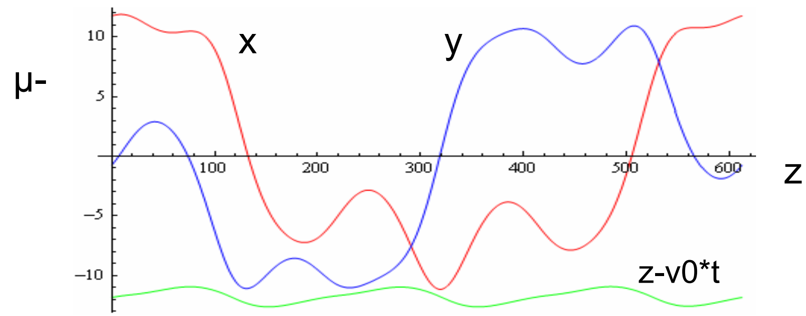
20mrad pitch angle, BLS=25.2 for p=200MeV/c

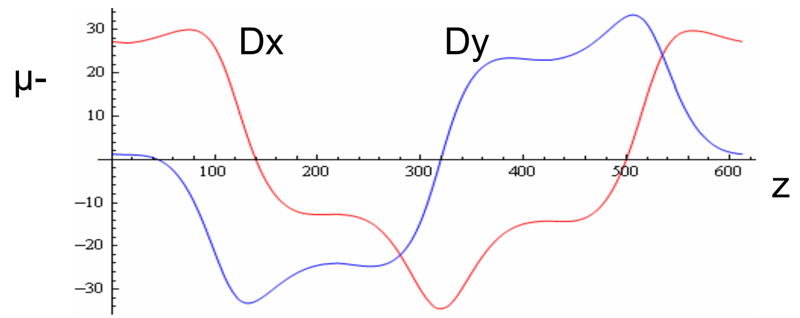






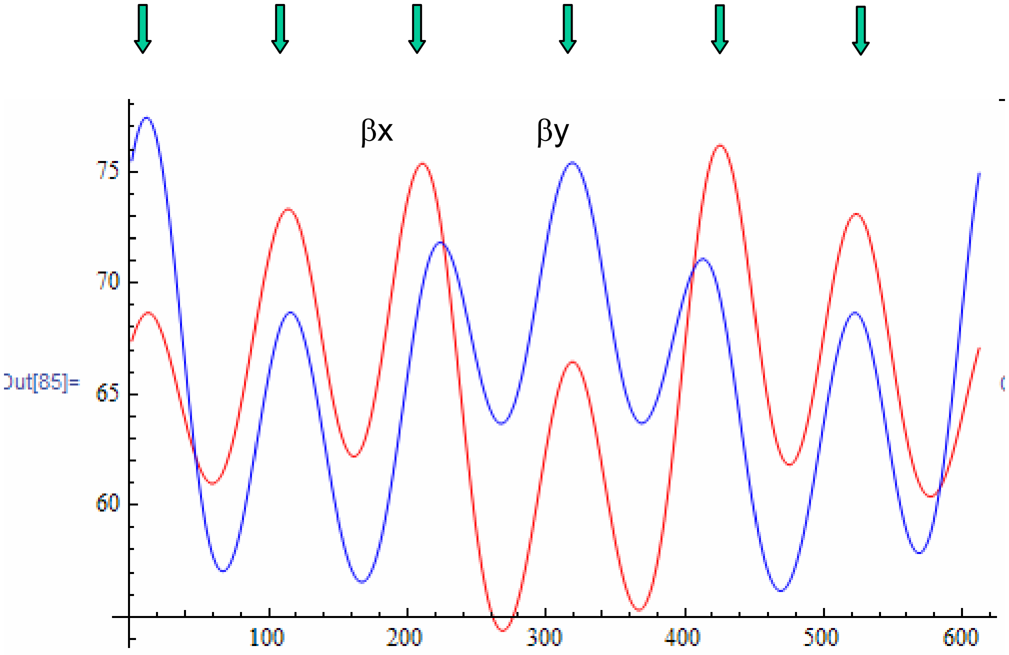






Helical FOFO snake – good for cooling both μ+ and μ-!

Beta-functions & tunes



at the absorber locations

< β > ~ 70cm

- compare with MICE's

< β > ~ 45cm

The best results with 7mrad pitch angle, no absorber wedge angle:

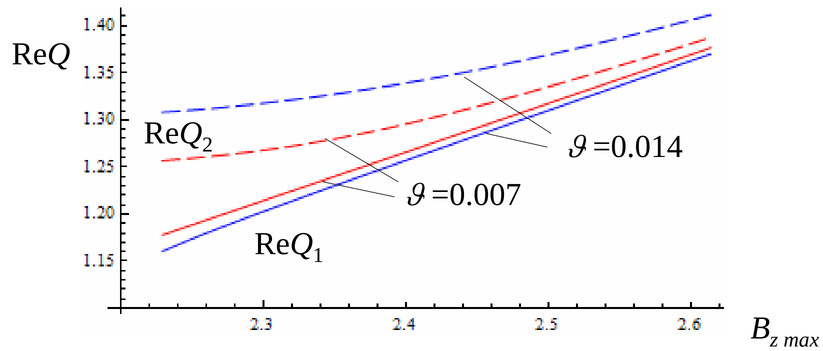
mode	I	II	III
tune	1.239+0.012i	1.279+0.007i	0.181+0.002i
ε_eq (mm)	3.2	4.5	6.9

ImQ=0.007 ⇒ cooling rate d log ε / dz = $2\times2\pi$/L ImQ = 1/70m

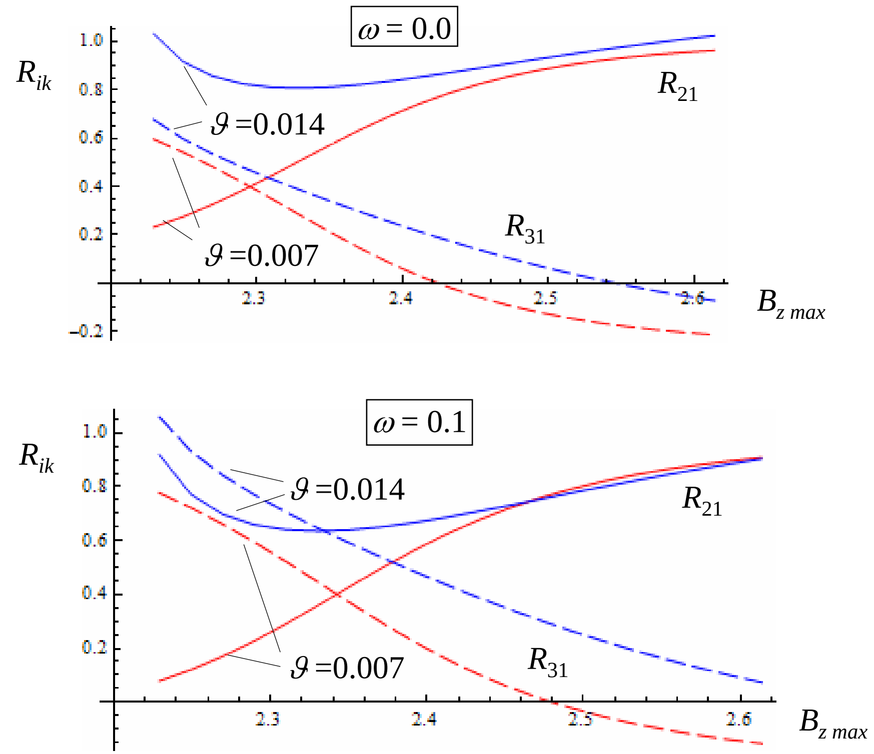
There is difficulty in equalization of damping rates of the transverse modes

Bmax=2.3T ⇒ j=58A/mm^2, Itot=4.4MA/solenoid

Parameter Scan



Transverse tunes vs $B_{z\ max}$ for pitch angle ϑ =7mrad (red) and ϑ =14mrad (blue).



Damping rate ratios R_{ik}=ImQ_i/ImQ_k for plane absorbers (top) and with wedge angle ω=0.1 (bottom)

Transmission

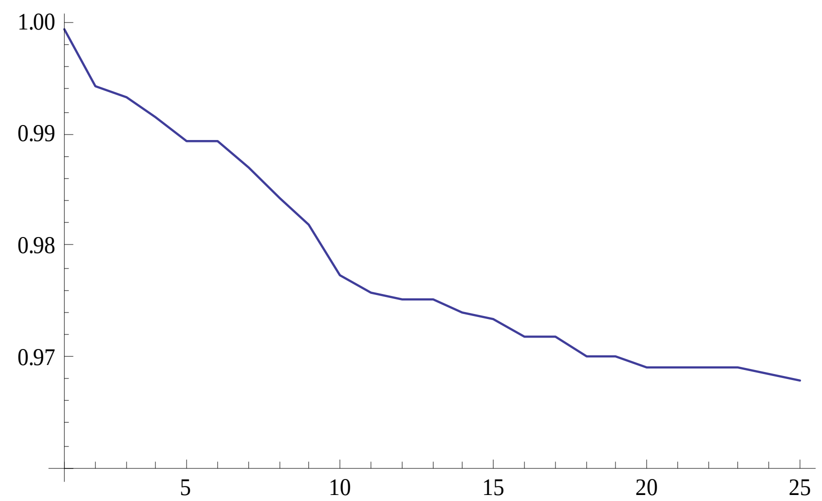
Tracking simulations:

- "True" action variables of 1771 particles evenly distributed in tetrahedron

  $(J_I + J_{II})/2.6 + J_{III}/4 < 1$ (cm)

- Phases chosen at random
- No decay nor stochastic processes
- Correlation between energy and oscillation amplitudes introduced into initial conditions to avoid immediate longitudinal blowup

$$\delta_p = \delta_{p0} + 0.042J_1 + 0.013J_2 + 0.039J_3$$



Survival after 25 periods (153m) 97%

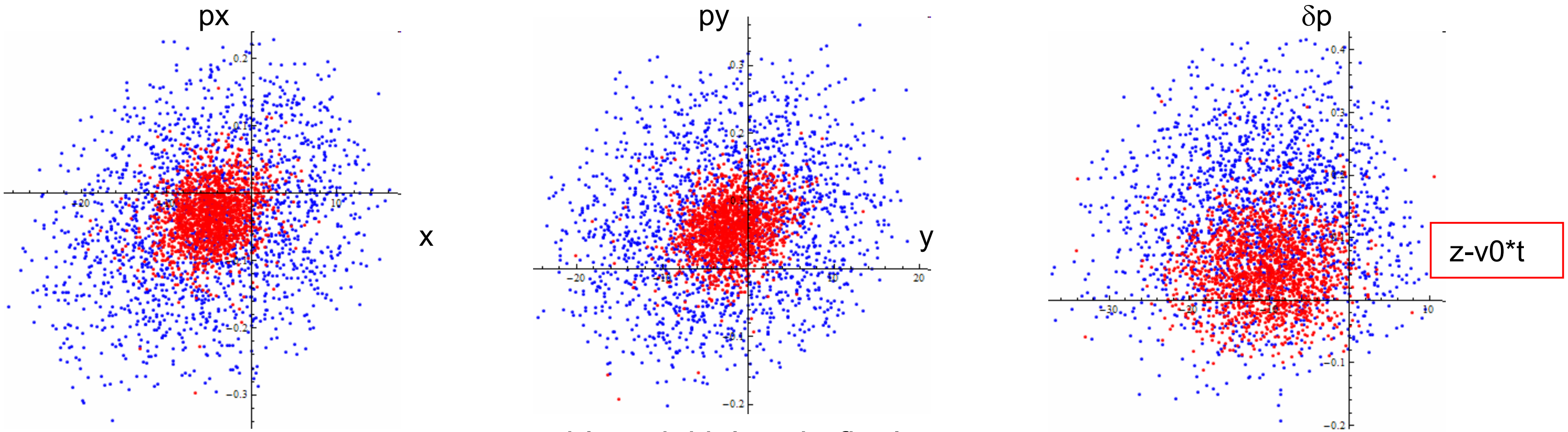
Note:

Courant-Snyder invariant =2J,

to compare with normalized emittance multiply by $\beta\gamma\approx2$:

$\beta\gamma\times$CSImax$\approx$10cm or 2.2σ for εN=2cm

Phase space distributions



blue - initial, red - final

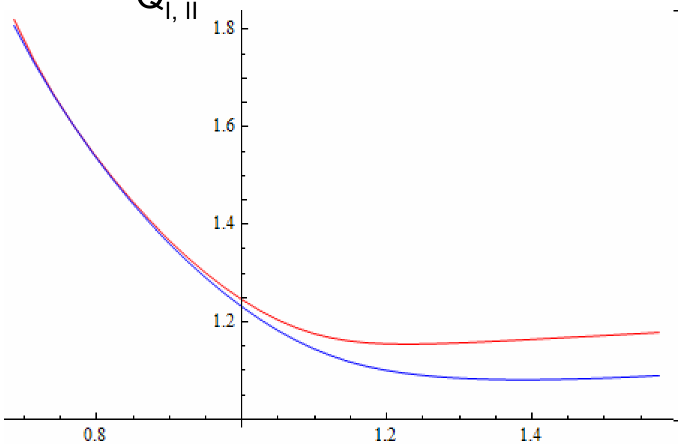
$$\delta_p = \frac{\gamma - \gamma_0}{\beta_0^2 \gamma_0},$$

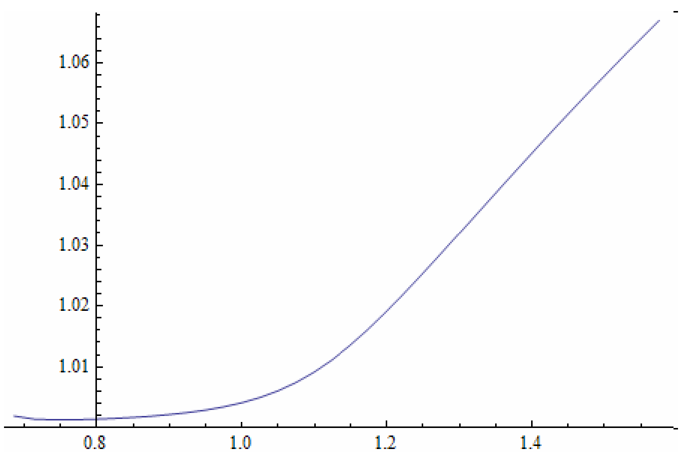
$$\frac{p}{p_0} = 1 + \delta_p - \frac{\delta_p^2}{2\gamma_0^2} + \ldots$$

“Emittances” (cm)	initial	final
6D	10.3	0.07
Trans. average	1.99	0.29
Longitudinal	3.75	1.46

Why momentum acceptance is so large (>60%) in the resonance case?

Tune "repulsion" from integer resonance





Nice surprise:

Large 2nd order chromaticity due to nonlinear field components keeps both tunes from crossing the integer !

Momentum compaction factor:

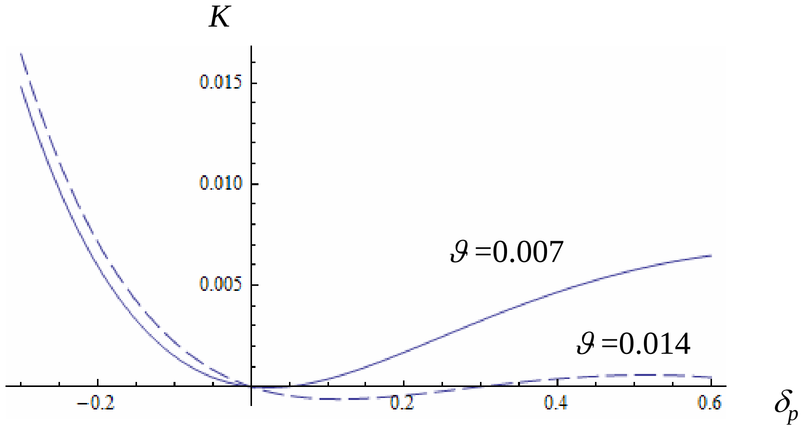
$$\alpha_p \approx 0.1 < 1/\gamma_0^2 \approx 0.22$$

- in contrast to classical HCC with homogeneous absorber where

$$\alpha_p > 2/\gamma_0^2$$

to ensure longitudinal damping.

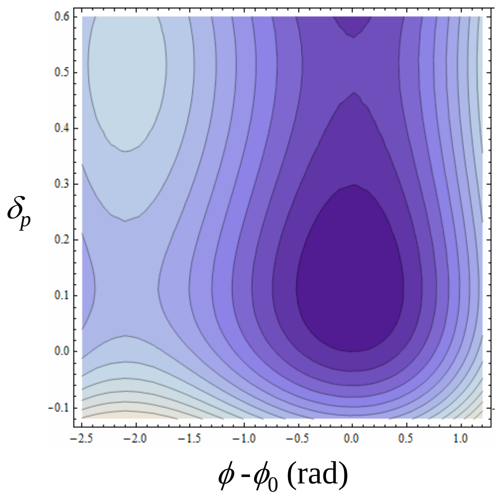
Longitudinal Hamiltonian



Kinetic energy in quasi-static approximation

$$K(\delta_p) = \int^{\delta_p} [1 - \lambda(\xi) \frac{\beta_0}{\beta(\xi)}]\, d\xi$$

where λ = orbit length/L_0



Contour plot of the longitudinal Hamiltonian for ϑ =0.014 - kinematic nonlinearity limits momentum acceptance, not insufficient RF bucket depth.

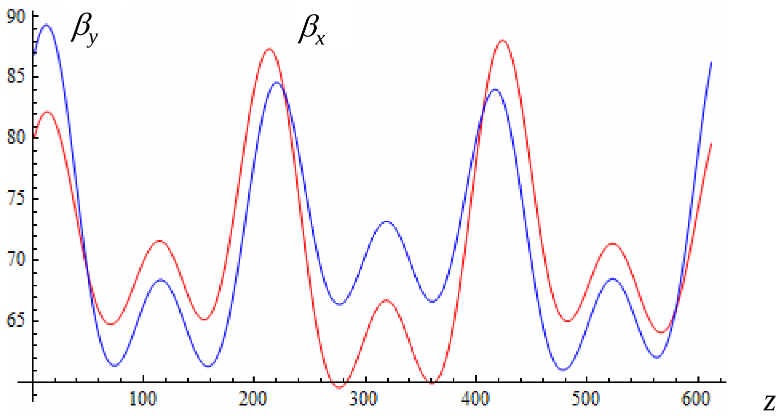
This makes problematic the achieving higher longitudinal damping rate with larger ϑ.

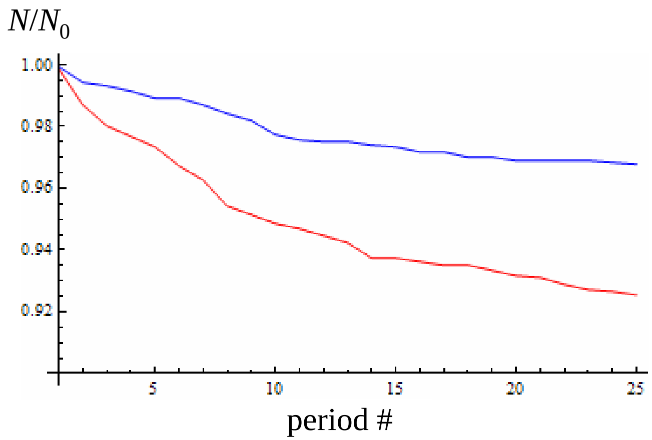
Equalization of transverse modes damping rates

R. Palmer proposed to add ~ constant solenoidal field to better mix the transverse modes and equalize their damping rates. This can be achieved by powering e.g. negative solenoids with slightly lower current. With just 1.6% difference in currents:

mode	I	II	III
tune	1.211+0.0100i	1.301+0.0108i	0.196+0.0003i
ε_eq (mm)	3.8	3.2	36.5

(fast transverse cooling w/o longitudinal blowup was the intent):





Unfortunately, there is no appreciable gain in equilibrium emittance due to large β-wave excited with unequal solenoid field

Transmission also suffers (red vs. blue)

Summary & Outlook

The proposed "helical FOFO snake:

- provides appreciable longitudinal cooling
- cools $\mu+$ and $\mu-$ simultaneously
- has sufficiently large acceptance / equilibrium emittance ratio ~20 (transmission with account of stochastic effects is yet to be simulated)
- 2nd stage with reduced overall dimensions (using 1-cell RF) will achieve equilibrium emittances to ~3mm for 200MHz channel.

Works is underway to:

- simulate the channel with "standard" codes like ICOOL and G4BL
- "taper" the channel to optimize cooling rate and transmission
- to combine it with D. Neuffer's buncher/rotator (see his report at this conference)