

Beam - RF Interactions in Twisted Cavities



PAC 2009, Vancouver

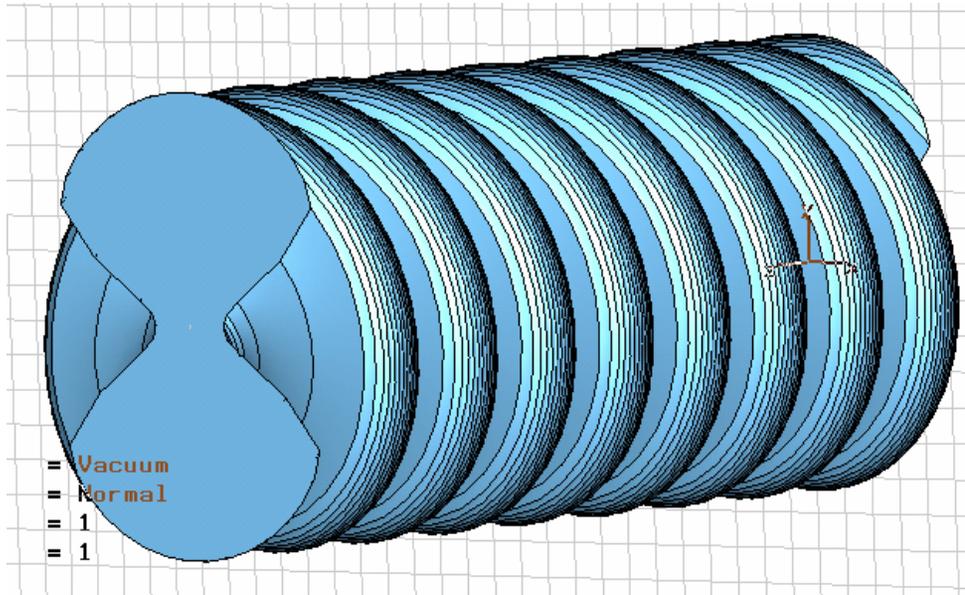
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J. Wilson**

Motivation

- 1) Twisted structures are promising candidates for accelerating cavities at velocities $\beta \leq 1$.**
- 2) Maxwell's equations were initially solved in twisted geometries using perturbation methods at small twist rates.**
- 3) They are now accurately solved for arbitrary twist rates by 2D and 3D finite difference methods.**
- 4) Bench models have been constructed and modes compared with numerical solutions (M. Awida, Y. Kang, et al. This conference). Prototype structures are being fabricated.**
- 5) We have initiated time-dependent 3D particle tracking studies to support the development of twisted accelerating cavities.**

Twisted Waveguide with Dumbbell Cross Section



$\nu = 2.74$ GHz

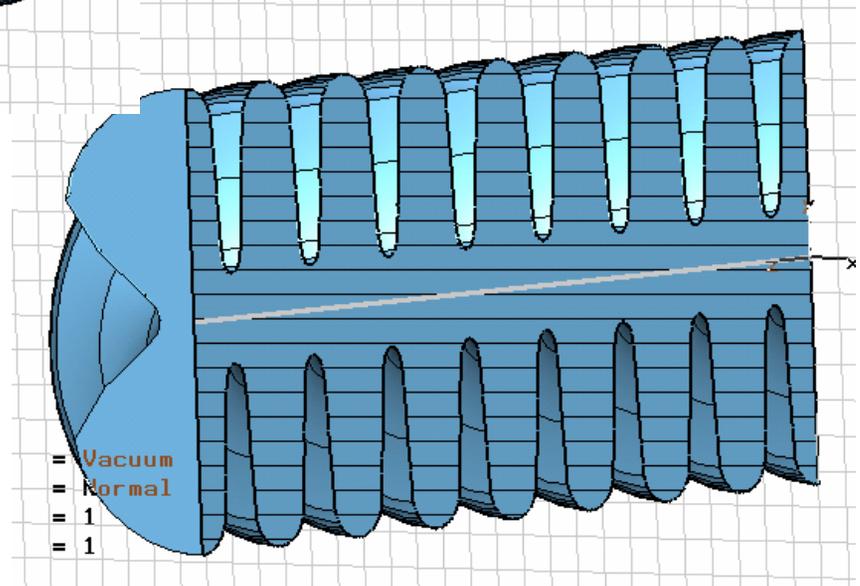
$L_{tw} = 6.6$ cm

$\beta = 0.6$

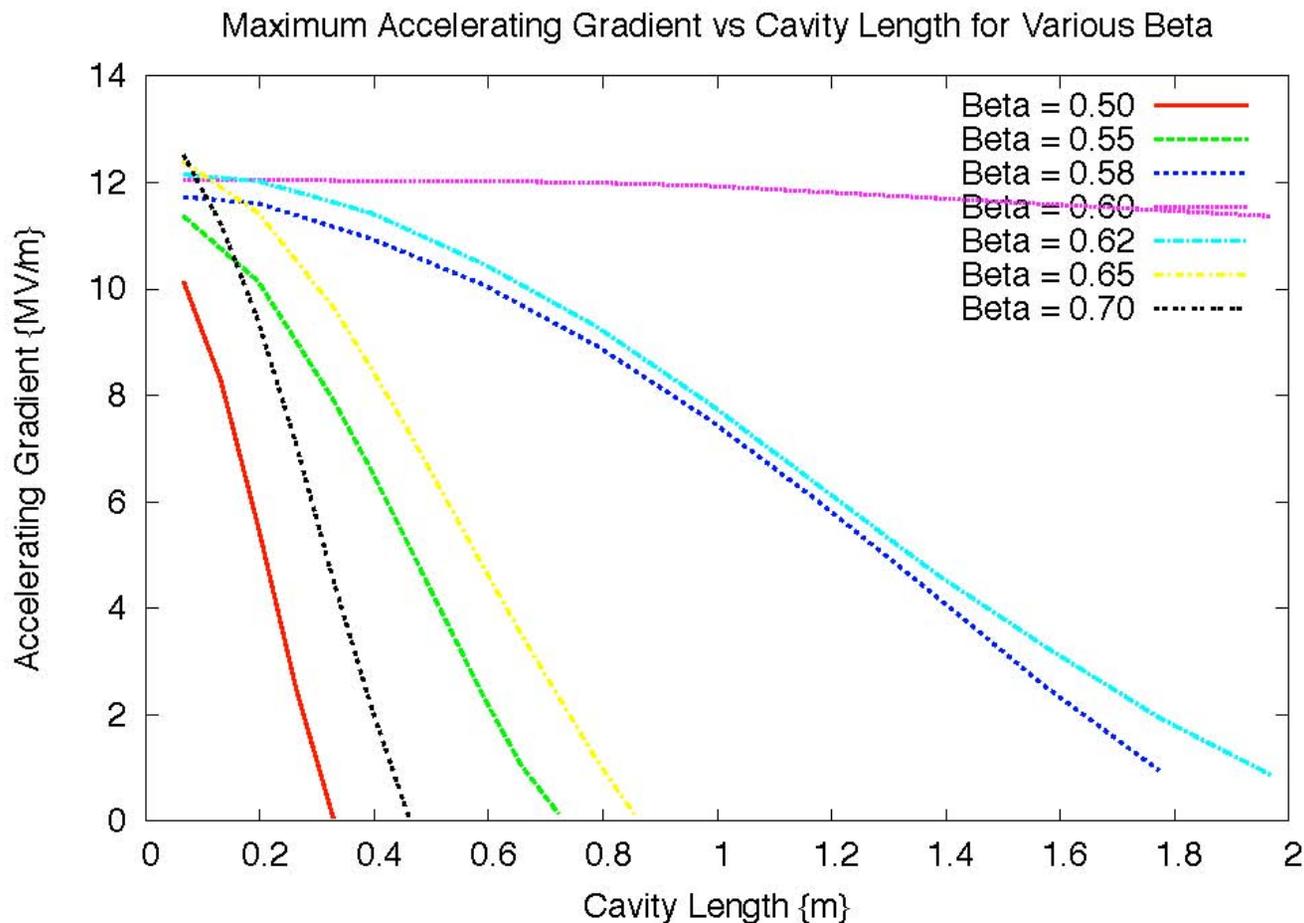
Set $E_{max} = 24$ MV/m

Tracking assumptions:

- * ORBIT code time-dependent Lorentz tracker
- * Ignore collective effects
- * Use time-dependent 3D fields from numerical cavity solution
- * Ignore end effects
- * Track protons



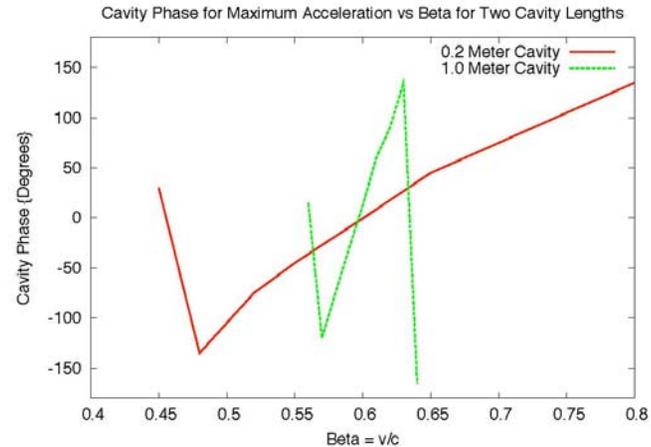
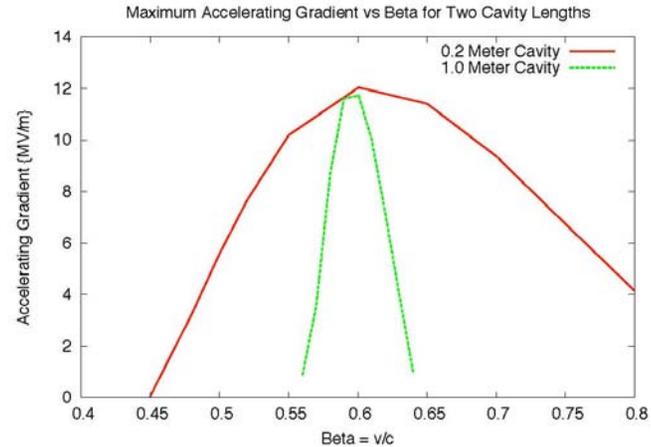
On-axis tracking: Accelerating Gradient Decreases with Increasing Number of Field Periods



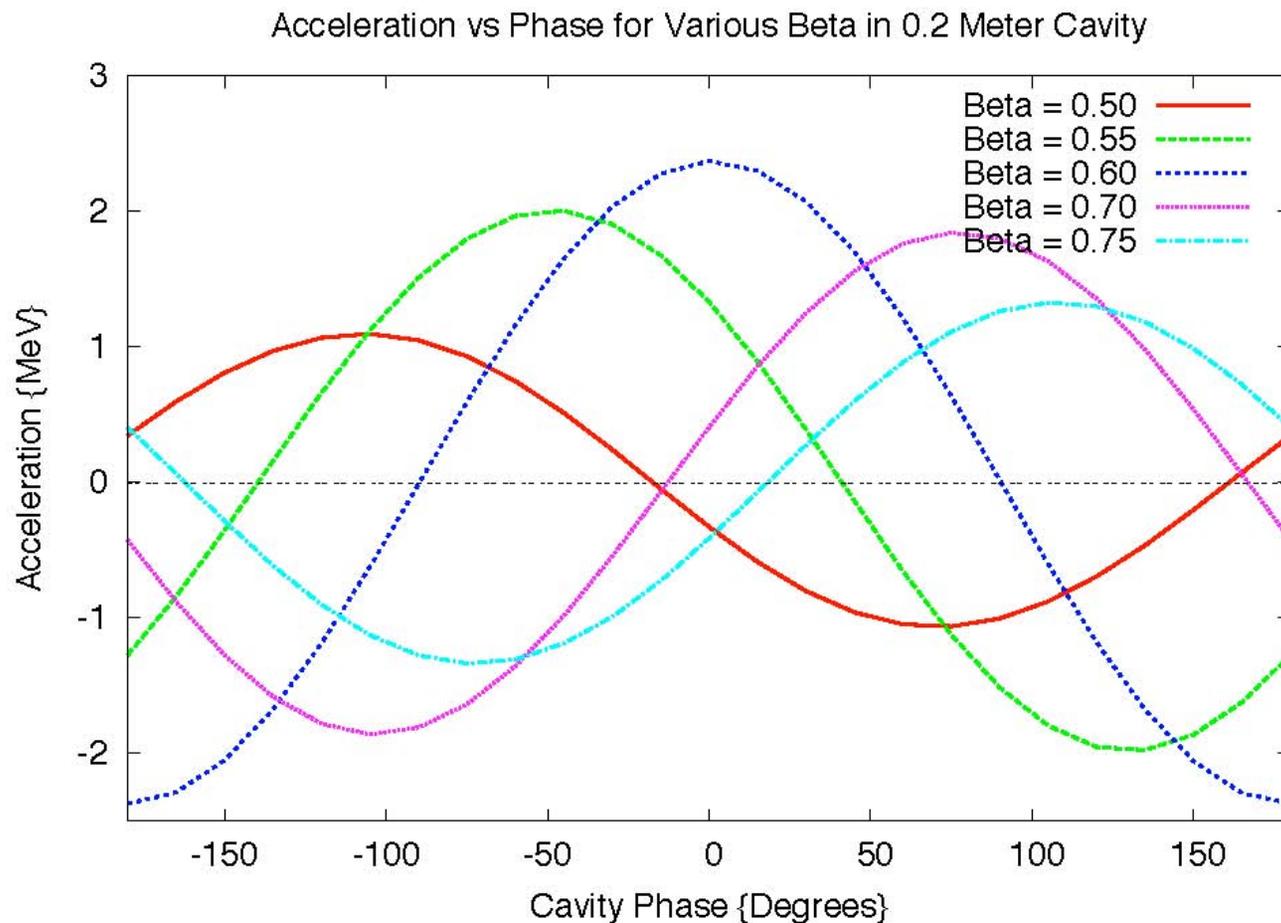
- * 1 m \approx 30 FP
- * Loss due to phase slip
- * Cavity lengths varied by replicating periodic solutions

Accelerating Gradient vs Velocity β

- For a 1 meter cavity (30 field periods), 1/3 the acceleration is lost with β deviations of ± 0.02 .
- For an 0.2 meter cavity (6 field periods), protons having $0.52 < \beta < 0.72$ get at least 2/3 the accelerating gradient.
- The relative phase between the protons and the cavity for maximum acceleration is a strong function of the particle velocity.

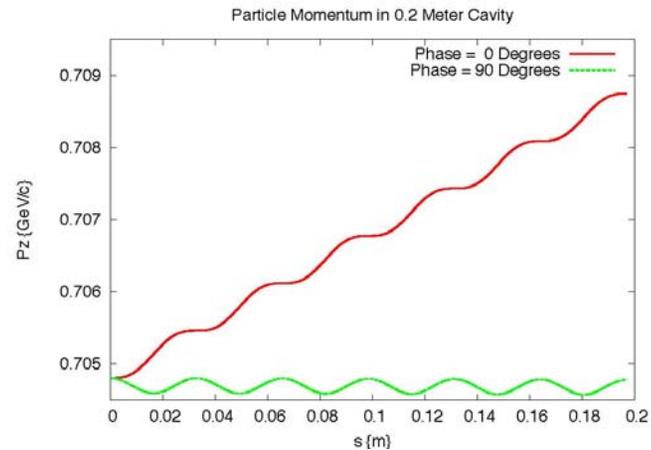
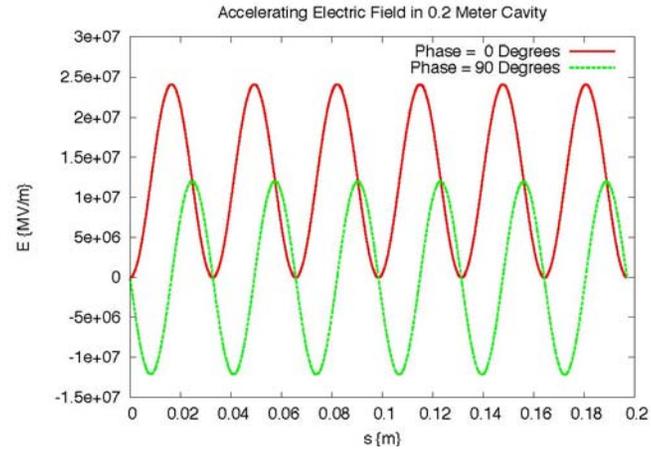


Acceleration vs Phase for Different Velocities



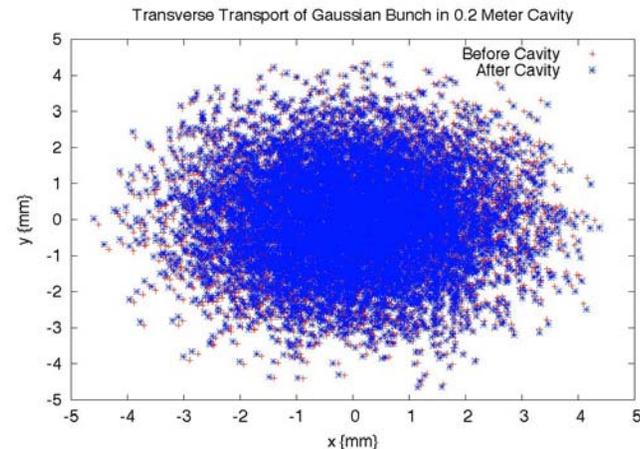
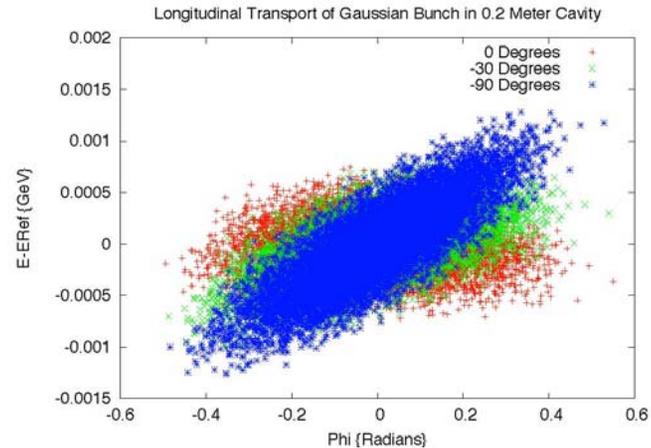
Accelerating Fields and Particle Momenta for Different Phases

- **Maximum acceleration decreases for velocities away from cavity β .**
- **Phase for maximum acceleration is a strong function of velocity.**
- **Depending on relative phase, cavity can accelerate or decelerate.**



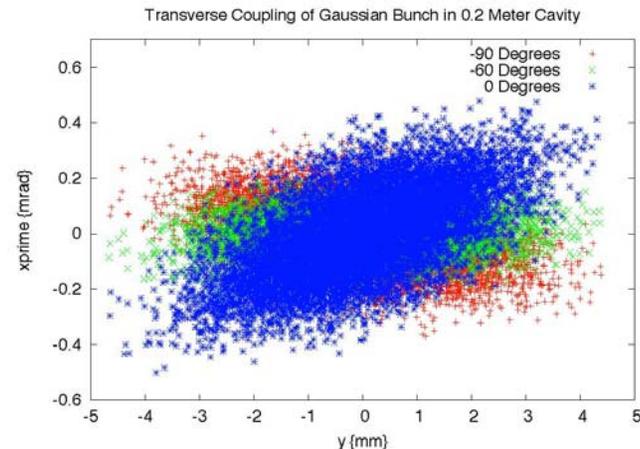
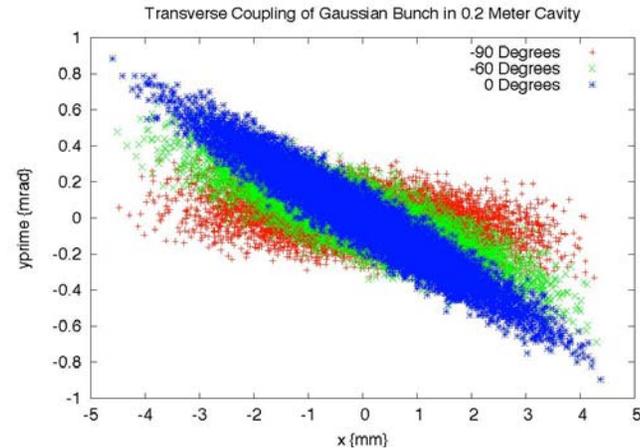
Acceleration of Bunches

- **Accelerate 6D Gaussian bunch with:**
 - $\sigma_x = \sigma_y = 1.4$ mm
 - $\sigma_{x'} = \sigma_{y'} = 0.14$ mradians
 - $\sigma_\varphi = 9$ degrees
 - $\sigma_E = 0.23$ MeV
- **Study bunch behavior with respect to reference particle.**
- **Longitudinal focusing depends on relative phase.**
 - At -30 degrees, particles get ~86% of maximum acceleration.
- **For transverse motion:**
 - Coordinates change little over length of cavity, but appear to rotate clockwise and expand slightly in x.
 - More can be learned by examining transverse momenta.

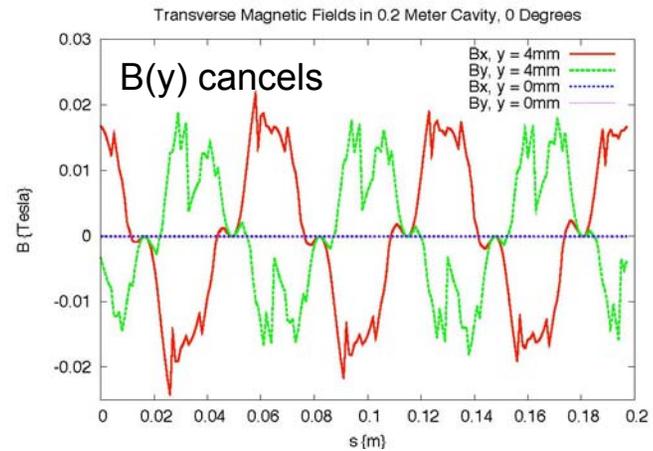
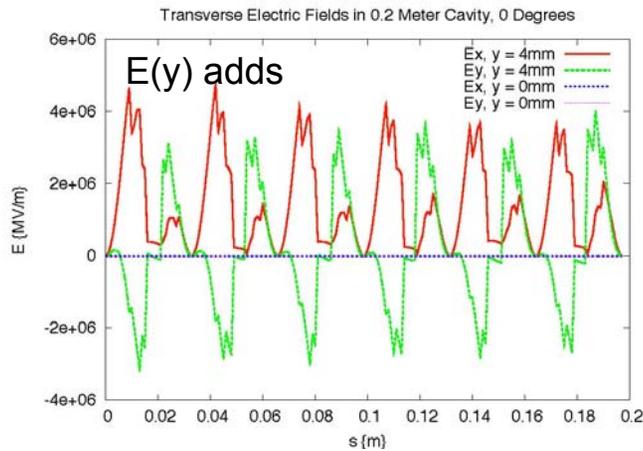
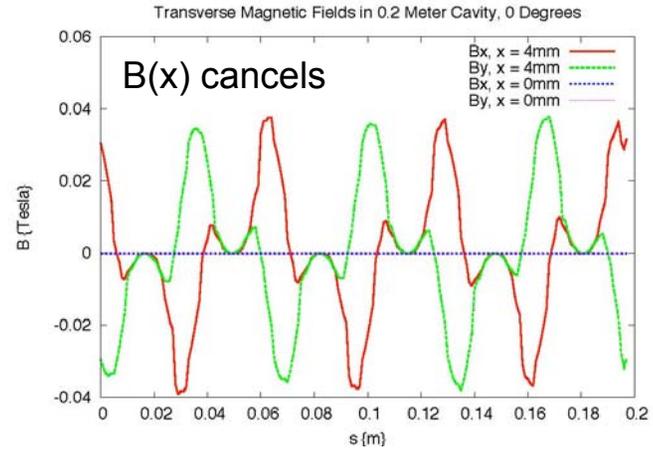
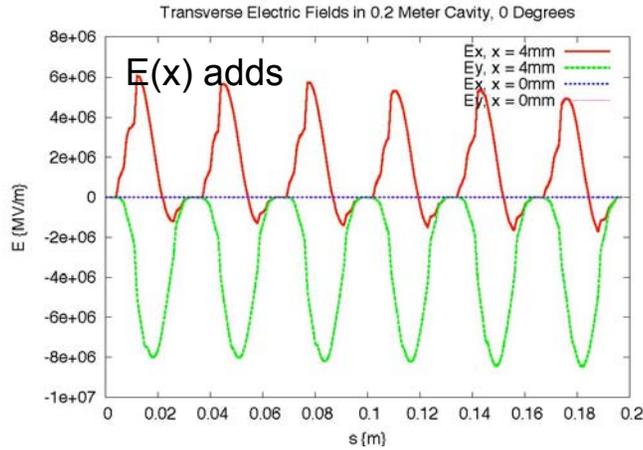


Effect of Transverse Fields

- The transverse fields couple the motion in the horizontal and vertical planes.
- The coupling strength is quite linear in the coordinate sizes.
- The effect is more than twice as pronounced in the horizontal plane than in the vertical plane.
- Although the effect weakens with increasing separation of particle phase and maximum accelerating phase, it is present at likely phases for acceleration.
- The observed coupling is independent of velocity over the range of accelerated particles.

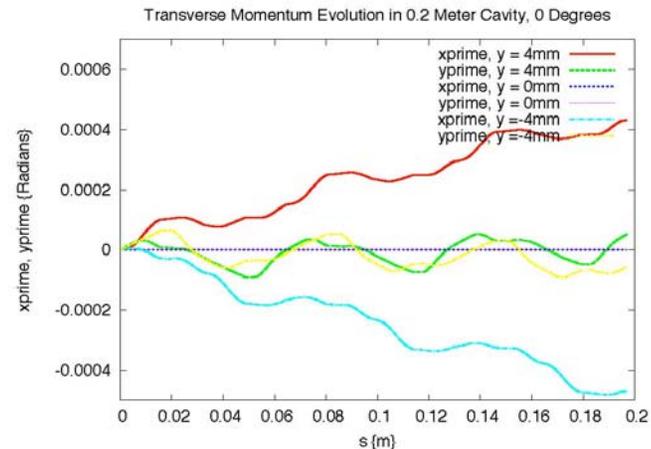
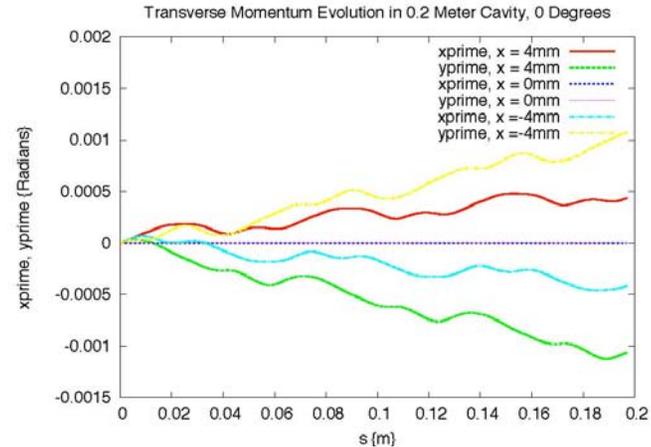


Source of the Coupling: Transverse E



Transverse Momentum Change

- **Horizontally displaced protons get vertical kicks.**
- **Vertically displaced protons get horizontal kicks.**
- **The kicks lead to clockwise rotation.**
- **The beam is defocused horizontally, but not vertically.**
- **Beam focusing issues can be addressed by including quadrupole magnets.**
- **For controlling coupling, skew quadrupoles are required.**



Next Steps

- **These results are just an initial exercise in tracking in twisted RF cavities. We learned that:**
 - **Twisted cavities can effectively accelerate protons over an acceptable range of velocities provided that the cavities have a limited number (~6) field periods.**
 - **Longitudinal dynamics (acceleration and focusing) of bunches appears to behave conventionally.**
 - **Transverse dynamics shows coupling between the x and y planes off axis due to transverse electric fields.**
- **We need to factor these results in to the iteration of the cavity design.**
- **We need to study other cavities: different β , different cross sections, $\beta=1$ for electrons.**
- **We need to generalize the model:**
 - **Consider end field effects.**
 - **Consider space charge and beam loading.**
 - **Incorporate into lattice design with tracking.**