Nonlinear Dynamics Study of Storage Ring with Super Periods

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Outline

- *Does higher super-periodicity of a storage ring help improving DA of a storage ring?*

- DA scaling with sextupole strength
- DA scaling with N_{SP}
- Examples
  - TBA: ALS
  - DBA: NSLS-II

Dynamic Aperture – Simple Analytic Approach

Dynamical system composed of sequence of linear and nonlinear elements:

$$\mathcal{M}_{\rm cell} = \mathcal{M}(q,p;s \to s+L)$$

$$= \mathrm{e}^{-:H_1:}\mathrm{e}^{-:H_2:}\cdots\mathrm{e}^{-:H_n:}$$

$$\mathcal{M}_{\rm SR} = (\mathcal{M}_{\rm cell})^{N_{\rm SP}}$$

In presence of sextupole:

$$\begin{aligned} H &= H_{\rm L} + H_{\rm NL} \\ &= \nu J + S J^{3/2} f(\phi) \end{aligned}$$

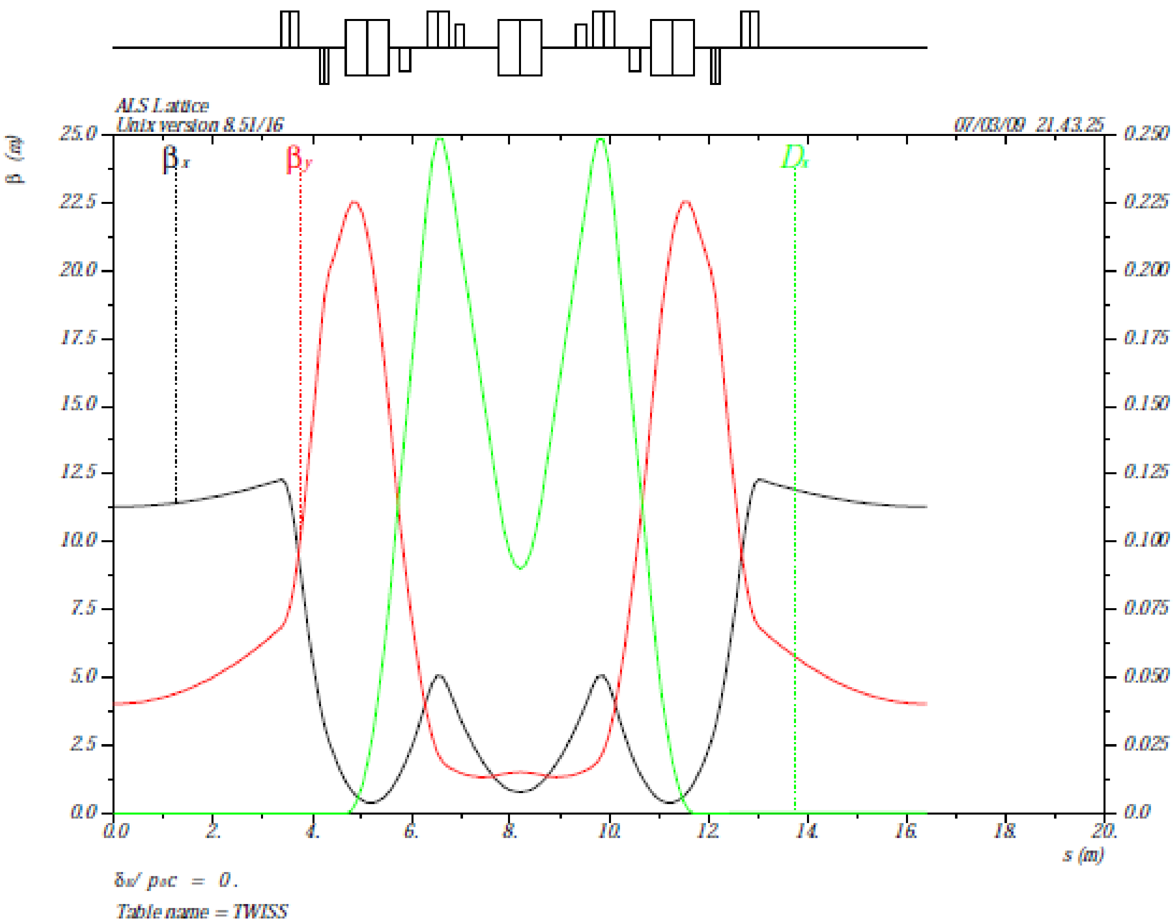
$$r = \left\langle \frac{H_{\rm NL}}{H_{\rm L}} \right\rangle_\phi \sim J^{1/2} S$$

- Assumption: at the DA boundary, $r_{\rm max}$ does not relate with *S*

$$J_{\rm max} \propto 1/S^2$$

E. Forest mentioned DA scales with sextupole strength (2000)

TBA - ALS Lattice



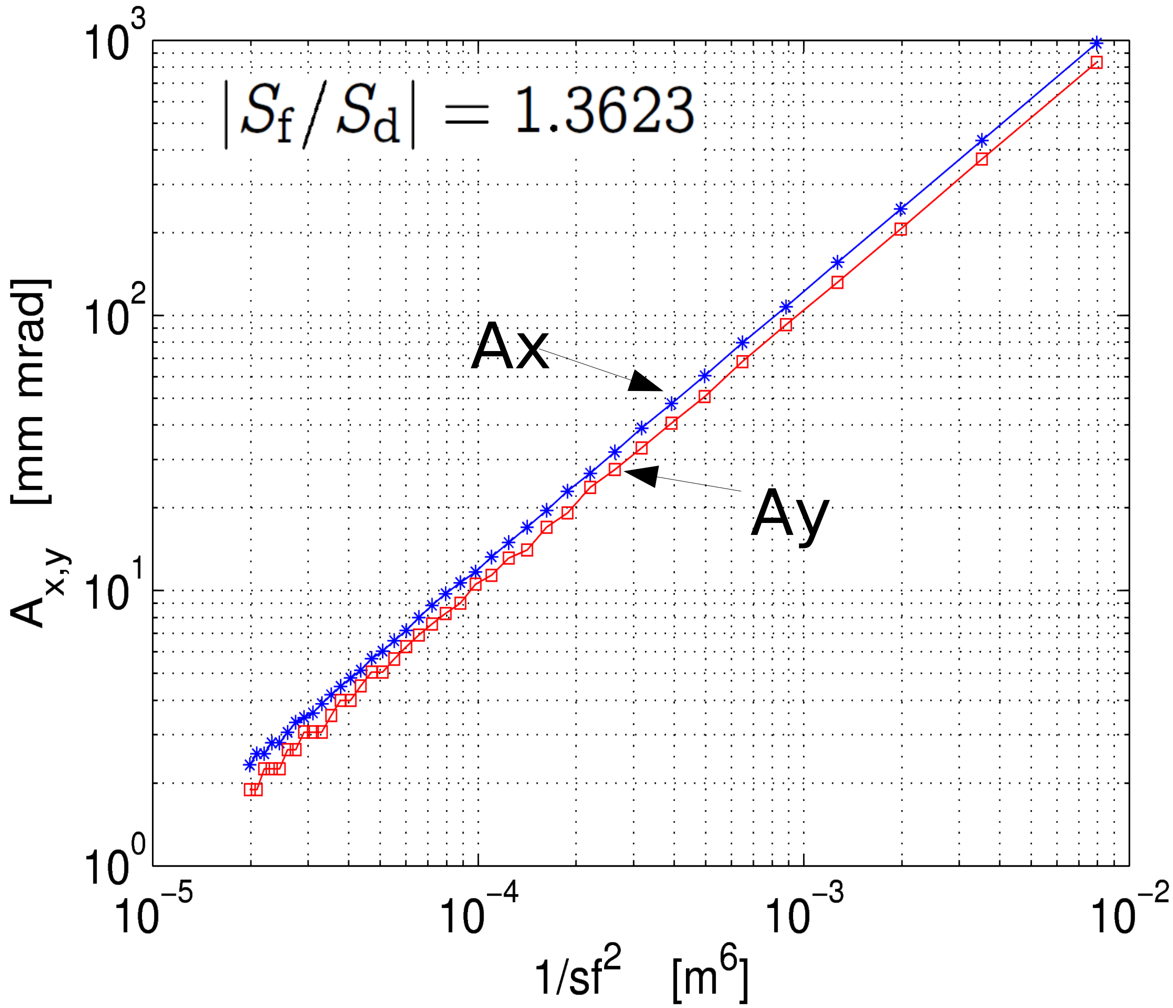
Courtesy of the ALS Staff, LBNL

- TBA
- Energy: 1.9 GeV
- Emit.: 5.5 nmrad
- C: 190 m
- N_{SP} =12
- 2 families of sextupoles
- $(\nu_x, \nu_y) = (14.25, 8.20)$

Dynamic Aperture $A \equiv \frac{x_{\max}^2}{\beta}$

ALS - Numerical Study

- $\vec{S}_0 = (S_\mathrm{f}, S_\mathrm{d})_{\xi=0}$, multiply $\vec{S}_0$ by factor of $\lambda \in [0.5, 5]$
- Calculate DA for different λ



$$A_{x,y} = \alpha_{x,y} / S_\mathrm{f}^2$$

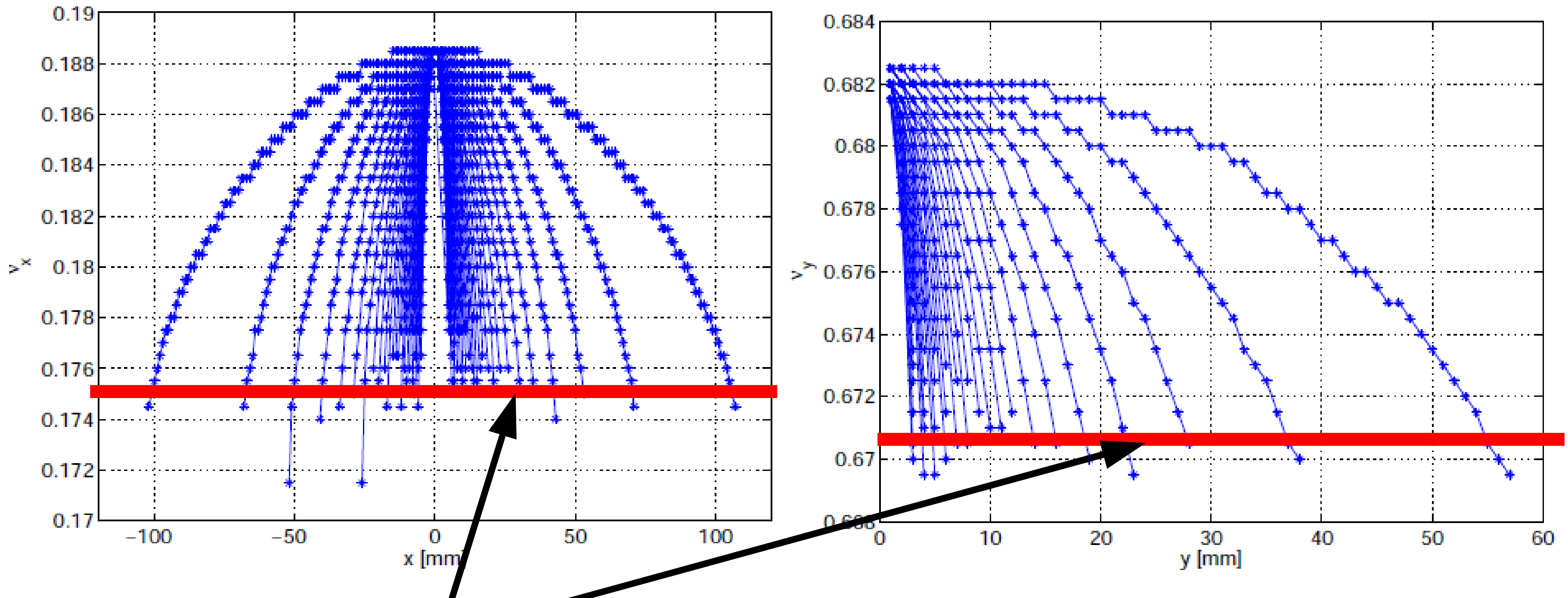
For ALS:

$\alpha_x = 0.1226$

$\alpha_y = 0.0453$

ALS - Numerical Study

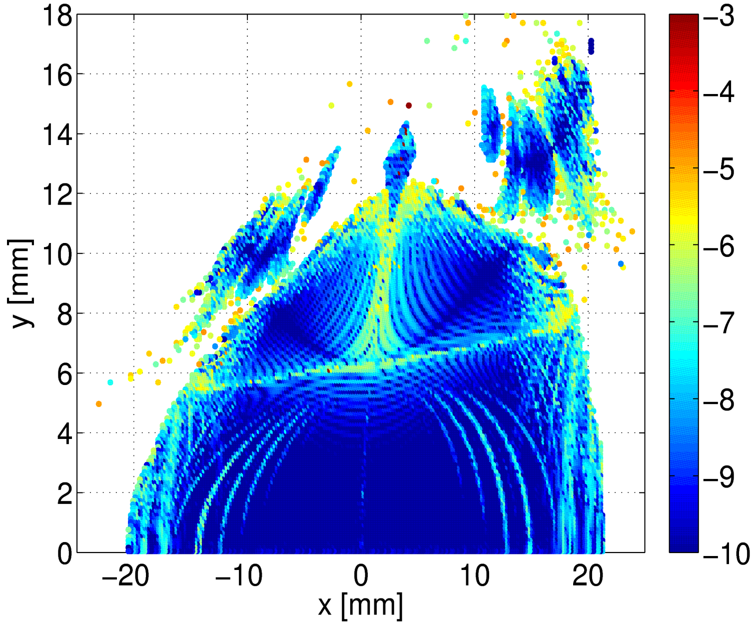
- $\vec{S}_0 = (S_{\rm f}, S_{\rm d})_{\xi=0}$ multiply $\vec{S}_0$ by a factor $\lambda \in [0.5, 5]$
- Observe the motion every SP

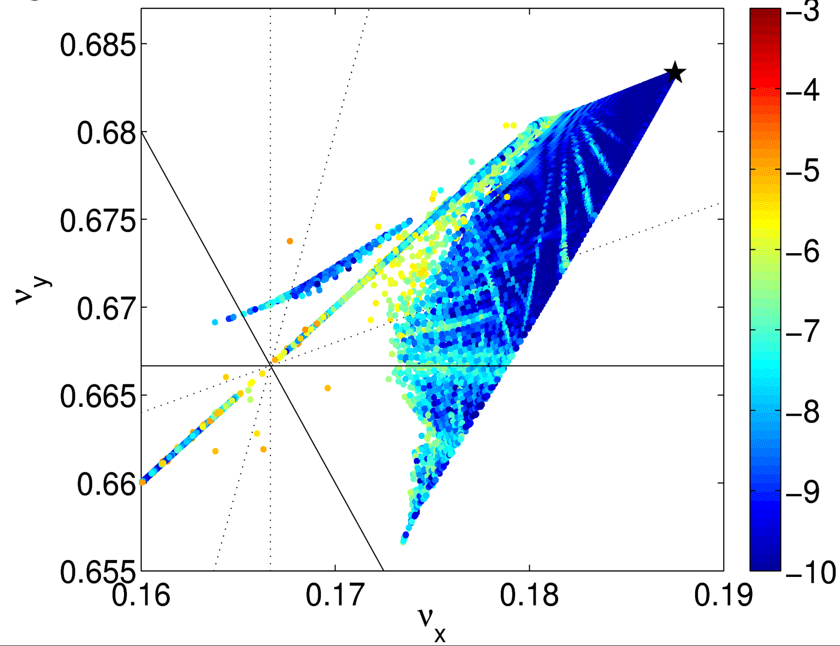


- There exists a "barrier" at which particles get lost
- Fixed range of nonlinear tune shift

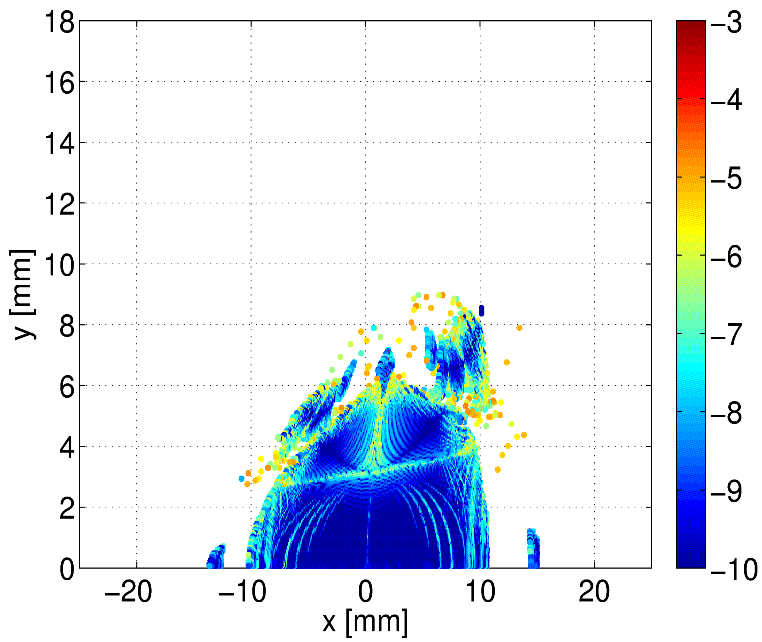
$$\frac{\mathrm{d}\nu}{\mathrm{d}J} \propto S^2 \quad \text{and} \quad |\Delta\nu| \quad \text{fixed} \quad \rightarrow \quad J_{\max} \propto 1/S^2$$

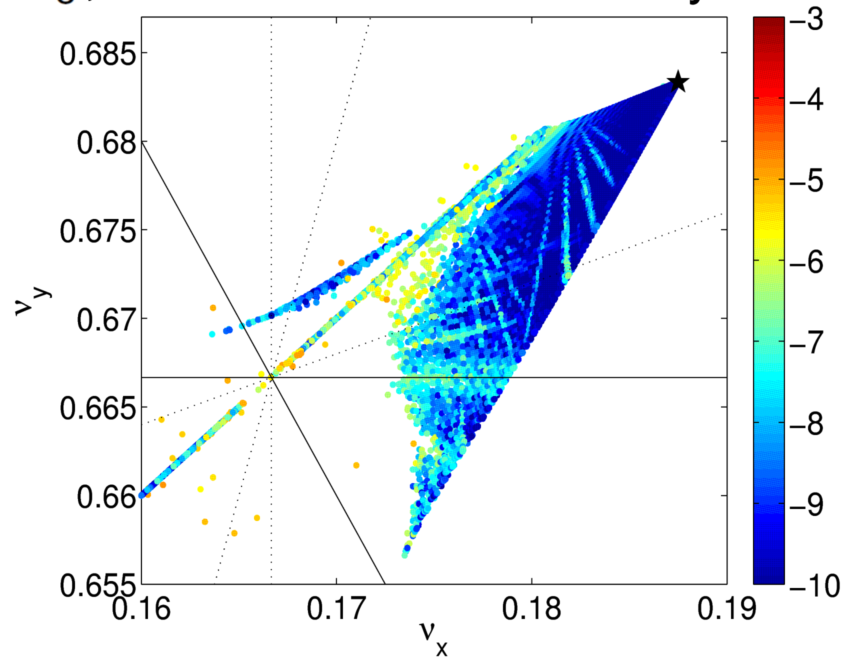
N_{SP}=12 tracking, $\vec{S}_0$, observe the motion every SP





N_{SP}=12 tracking, 2 $\vec{S}_0$, observe the motion every SP





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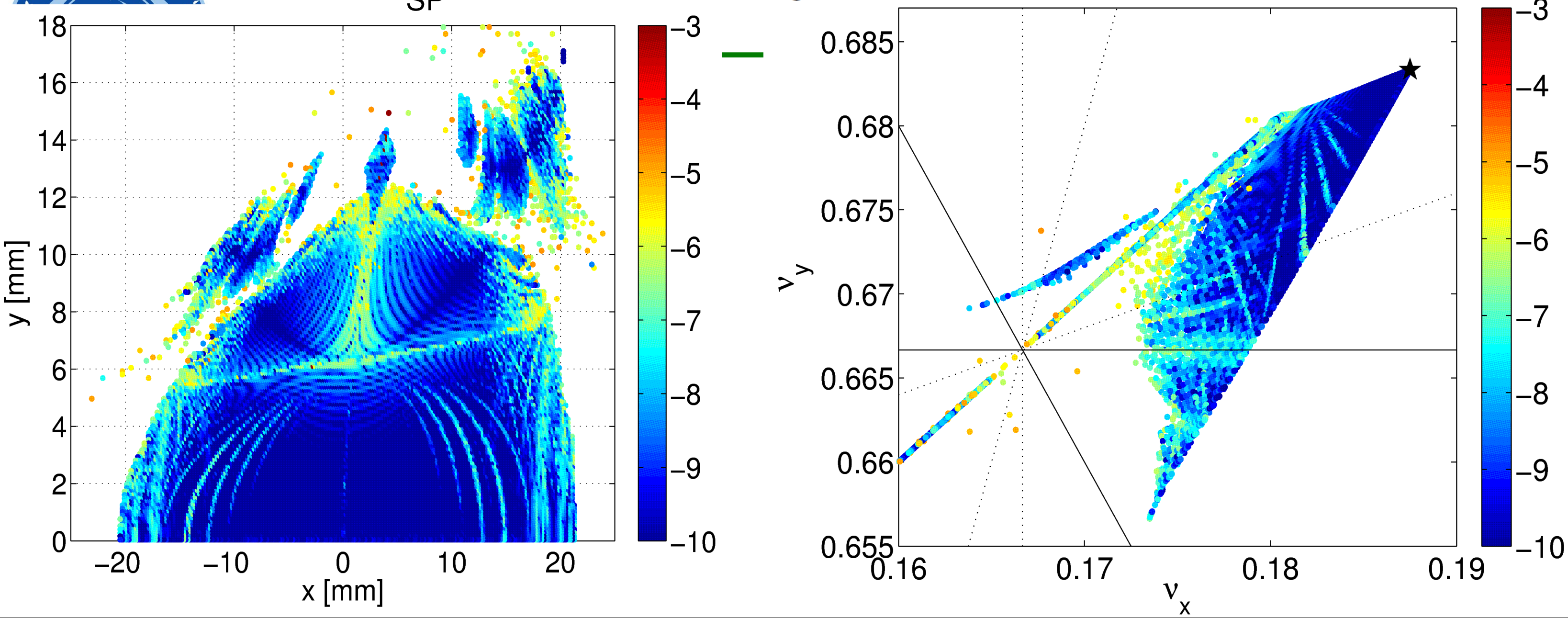

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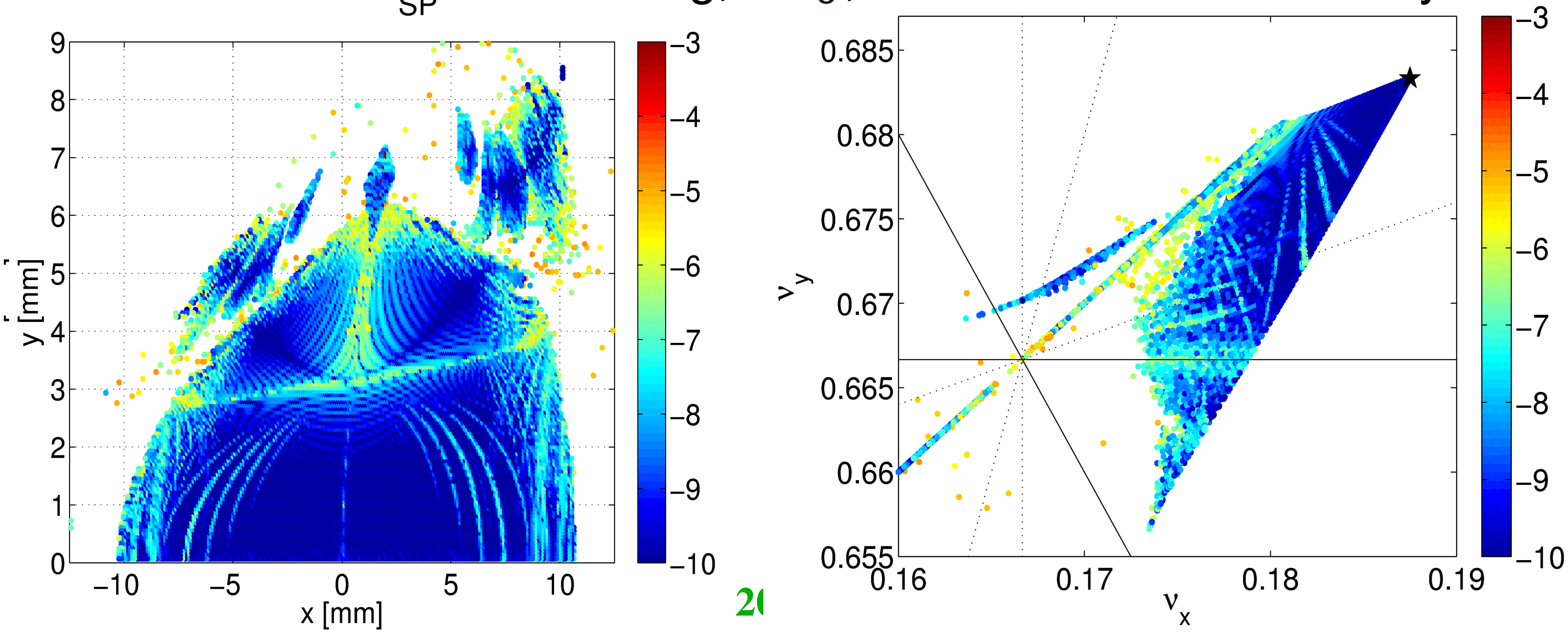


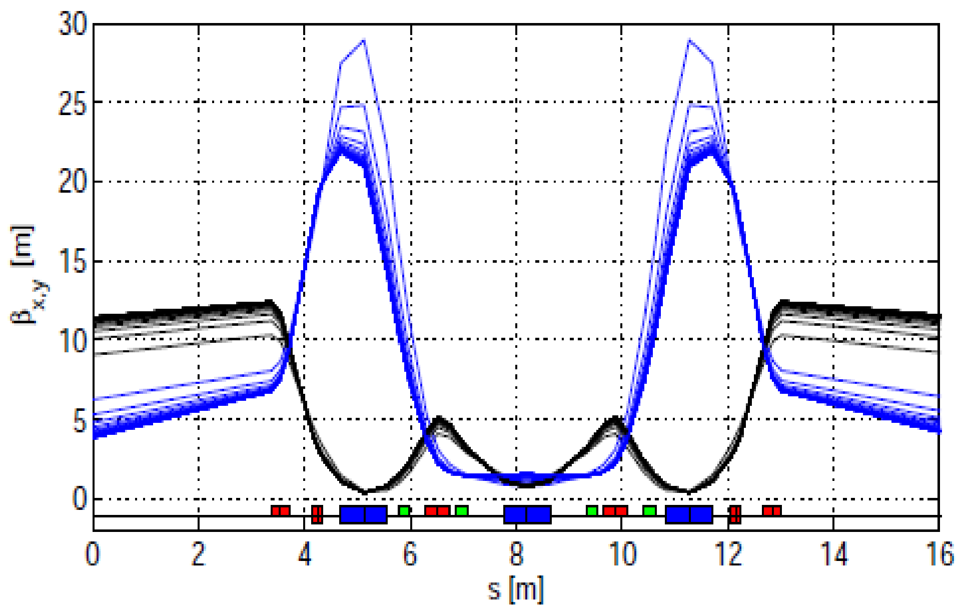
DA scales with S

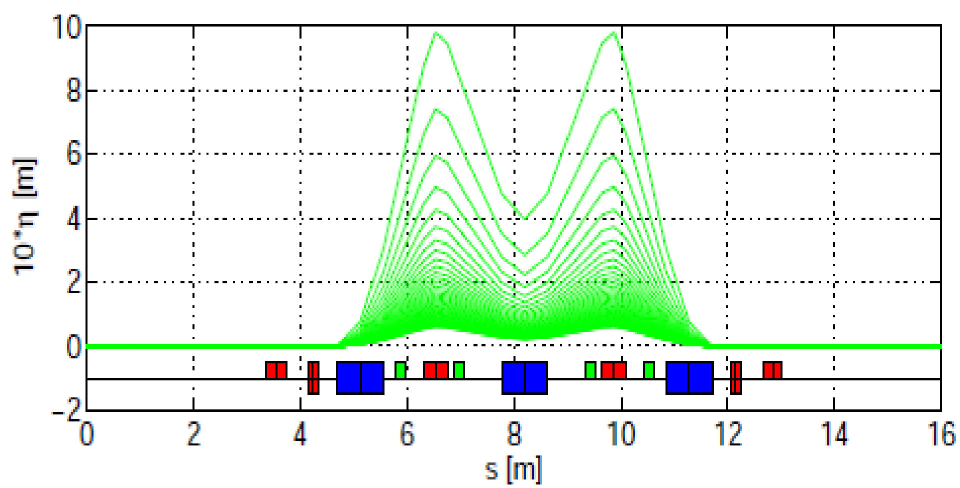
FMA does not

Vary the ALS Lattice

Increase of decrease N_{SP} from 12 to other numbers

- Vary N_{SP} of the ring
- Keep phase advance of one SP
- Minimize the β function change





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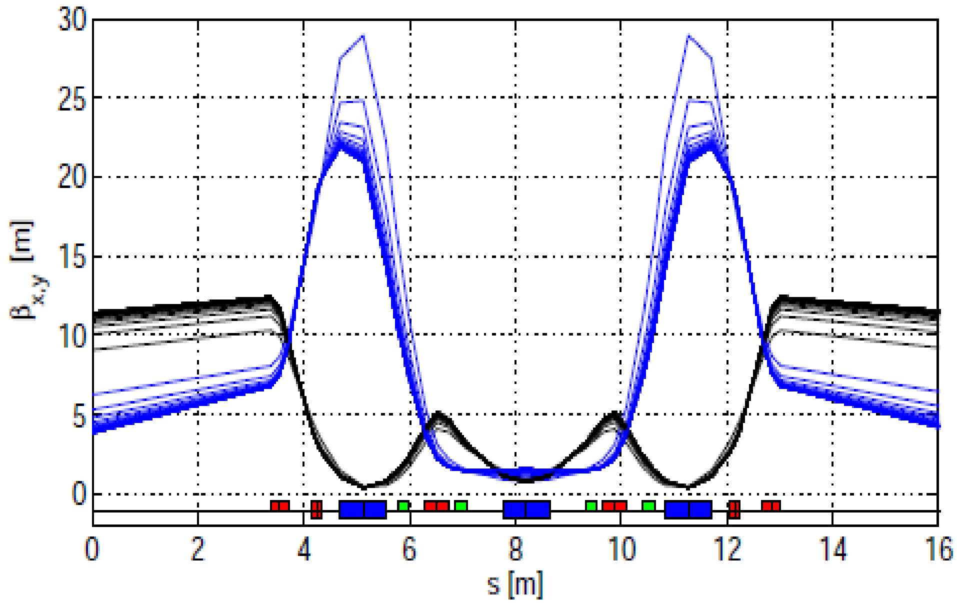
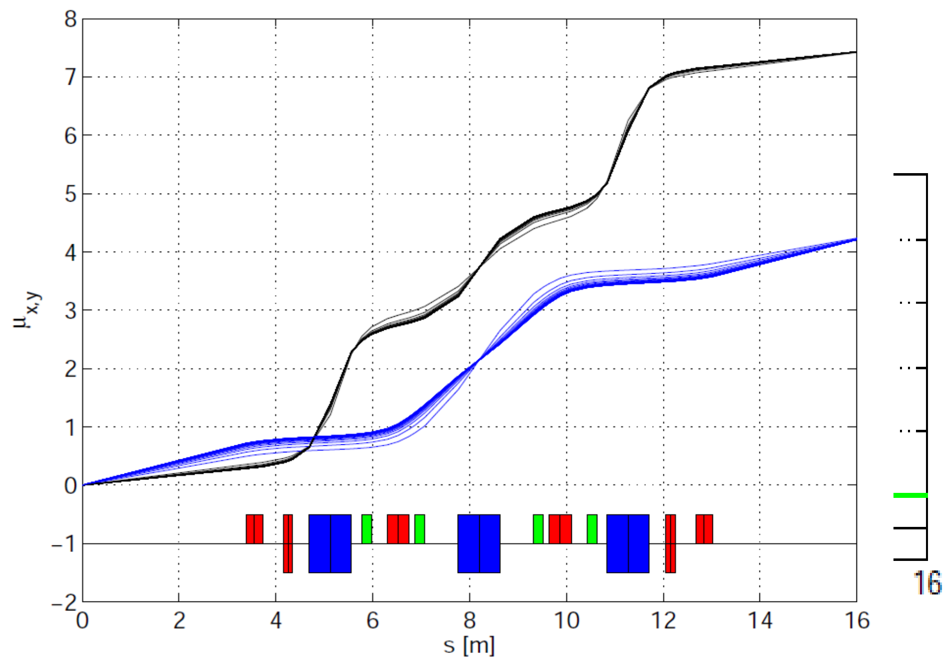
N_{SP} from 3 to 50+

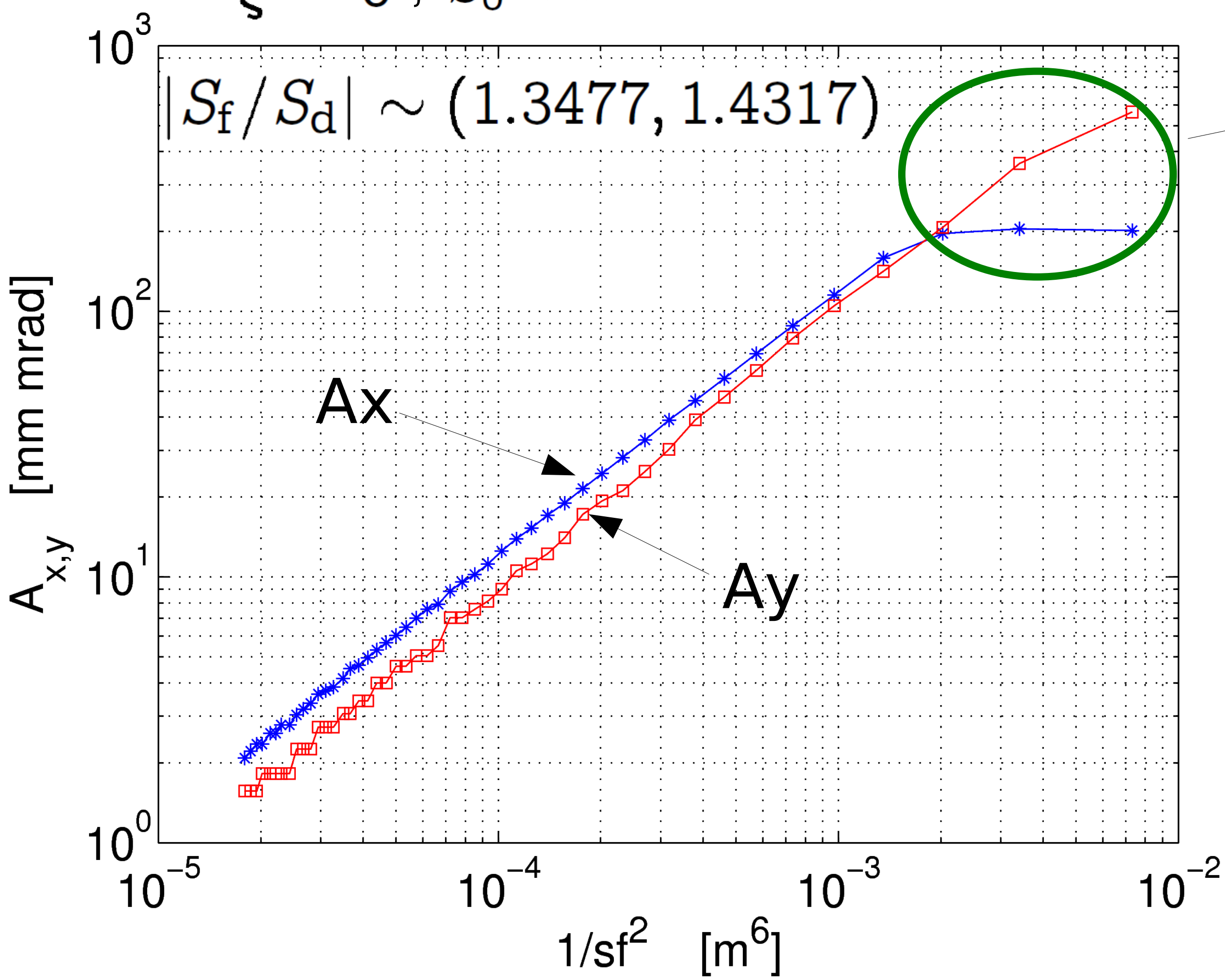




$$\epsilon \propto 1/N_{SP}^3 \qquad \eta \propto 1/N_{SP} \qquad \vec{S}_{\xi=0} \propto N_{SP}$$

Varied ALS Lattice – 1 SP Study

- Different N_{SP} lattices
- $\xi = 0$, $\vec{S}_0$ are different for different lattices



Linear lattice deformation for fewer SPs

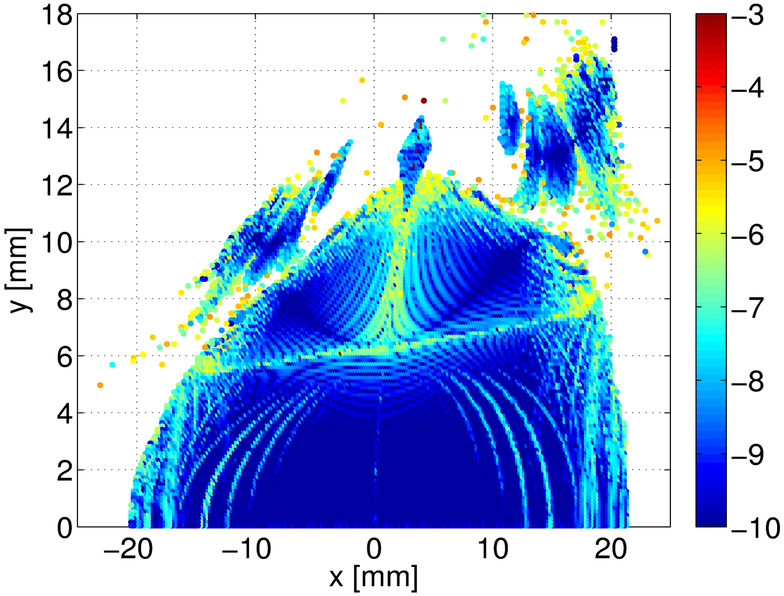
α_x=0.12
α_y=0.045

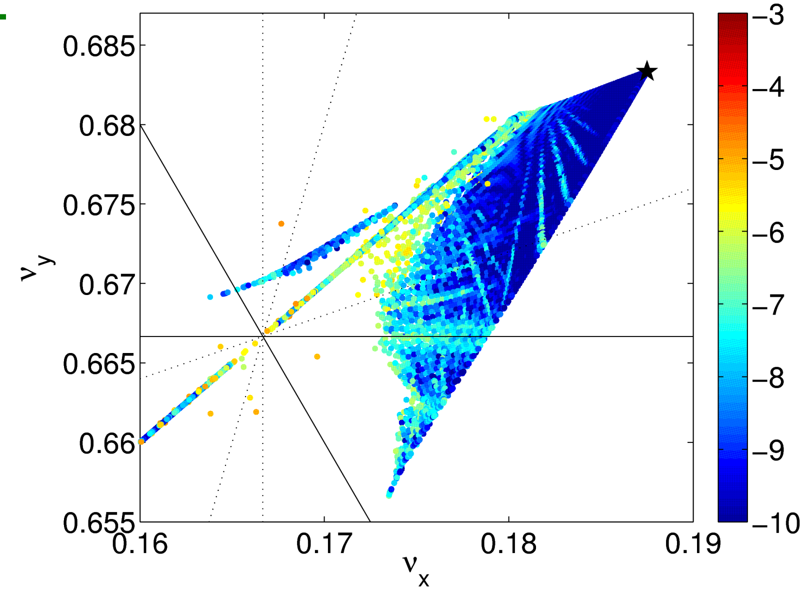
N_{SP} = 12, $\lambda \vec{S}_0$ result

α_x=0.12
α_y=0.045

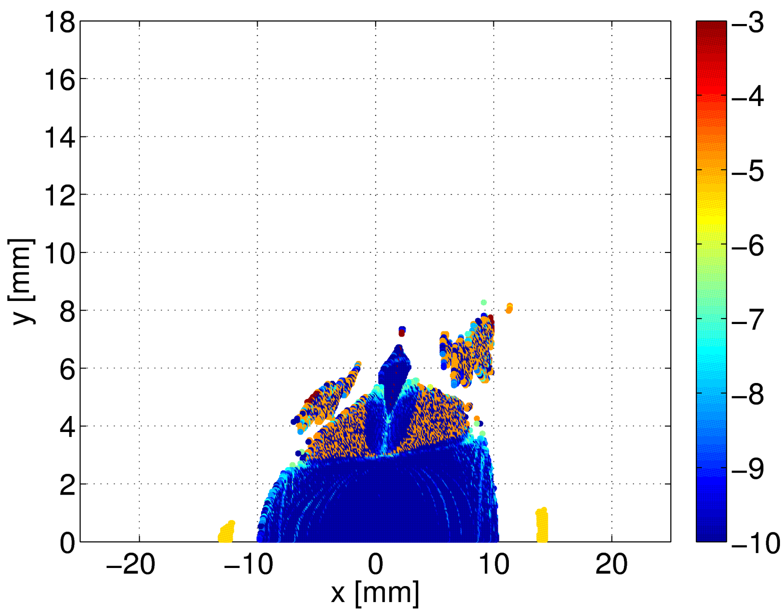
The $1/S^2 - A$ relation is the same as fixed lattice case

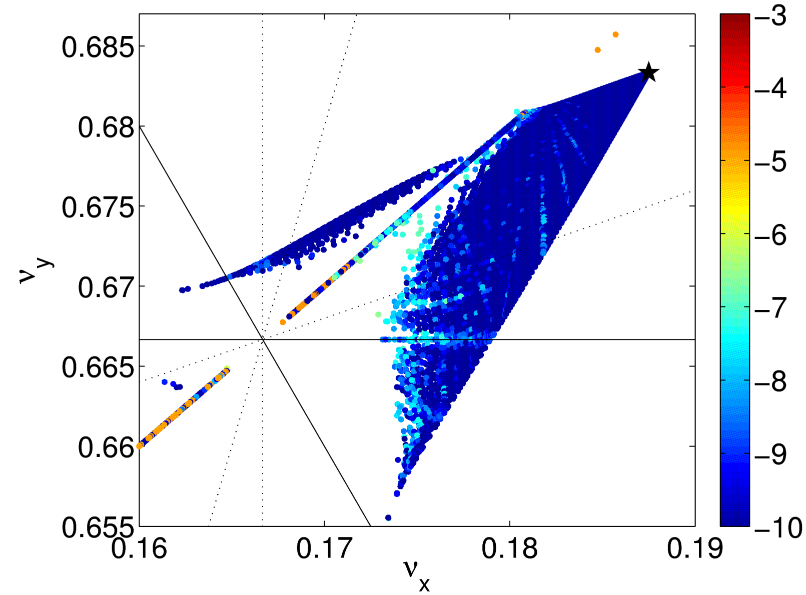
N_{SP}=12 tracking, 1 SP observation, $\xi = 0$





N_{SP}=24 tracking, 1 SP observation, $\xi = 0$





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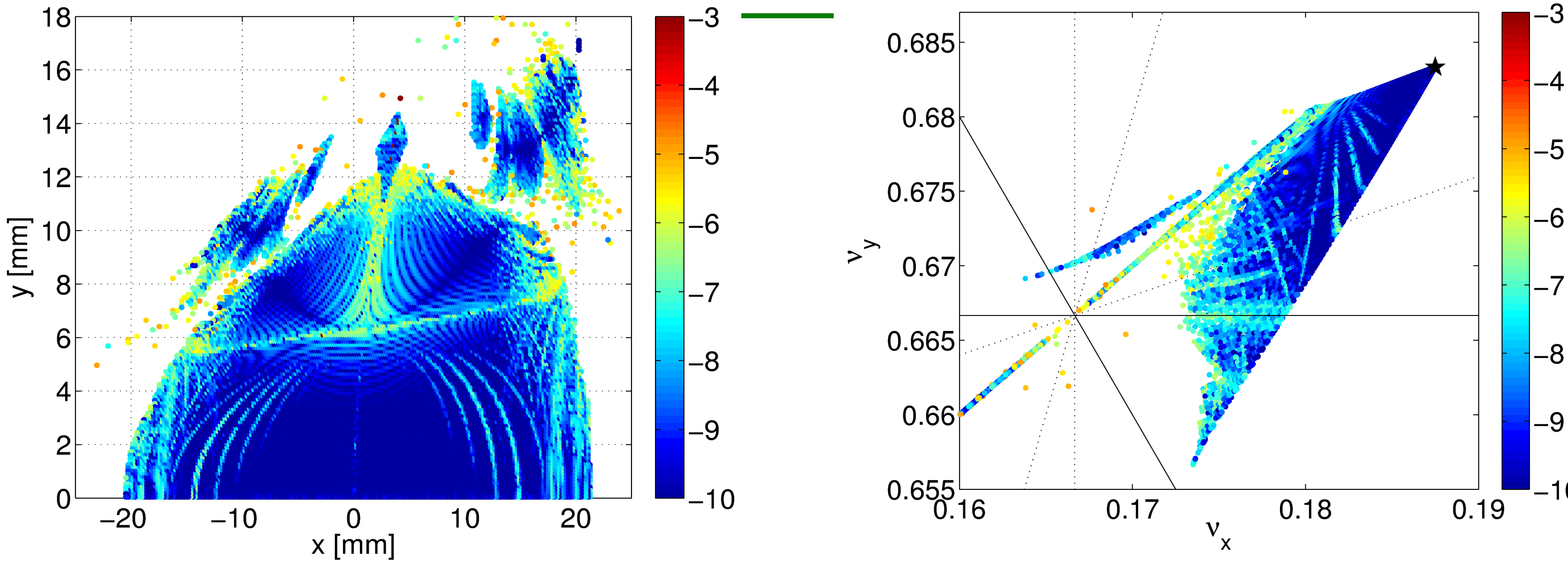

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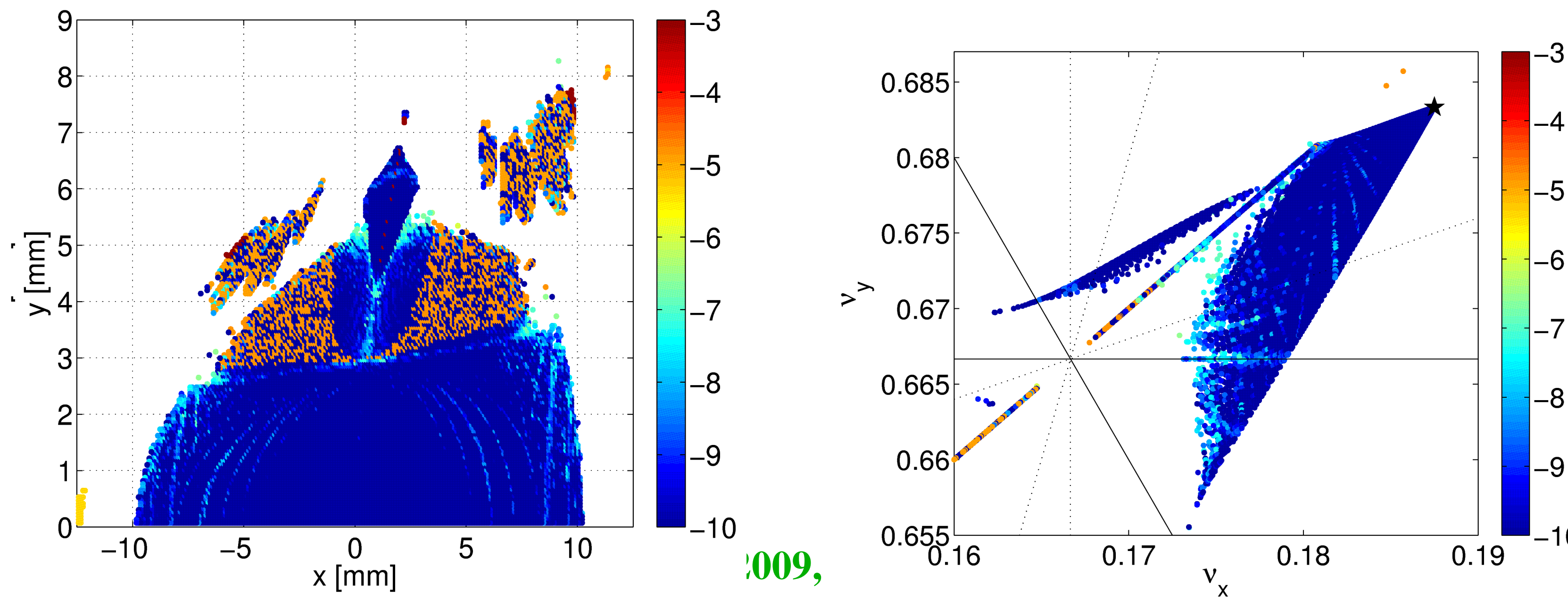


DA scales with N_{SP}

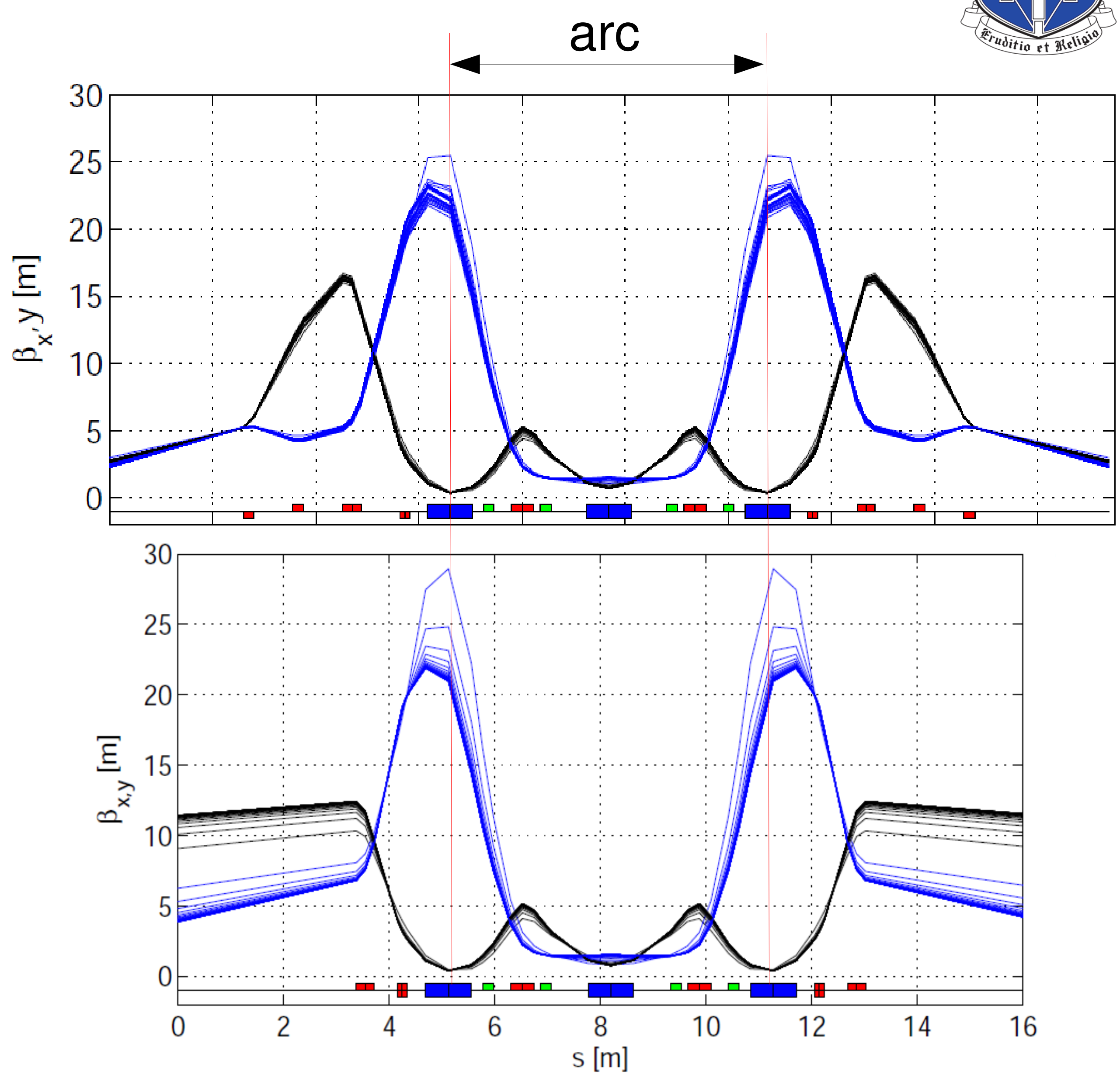
FMA does not

Modified ALS Lattice

- Modify the ALS lattice with arc part unchanged

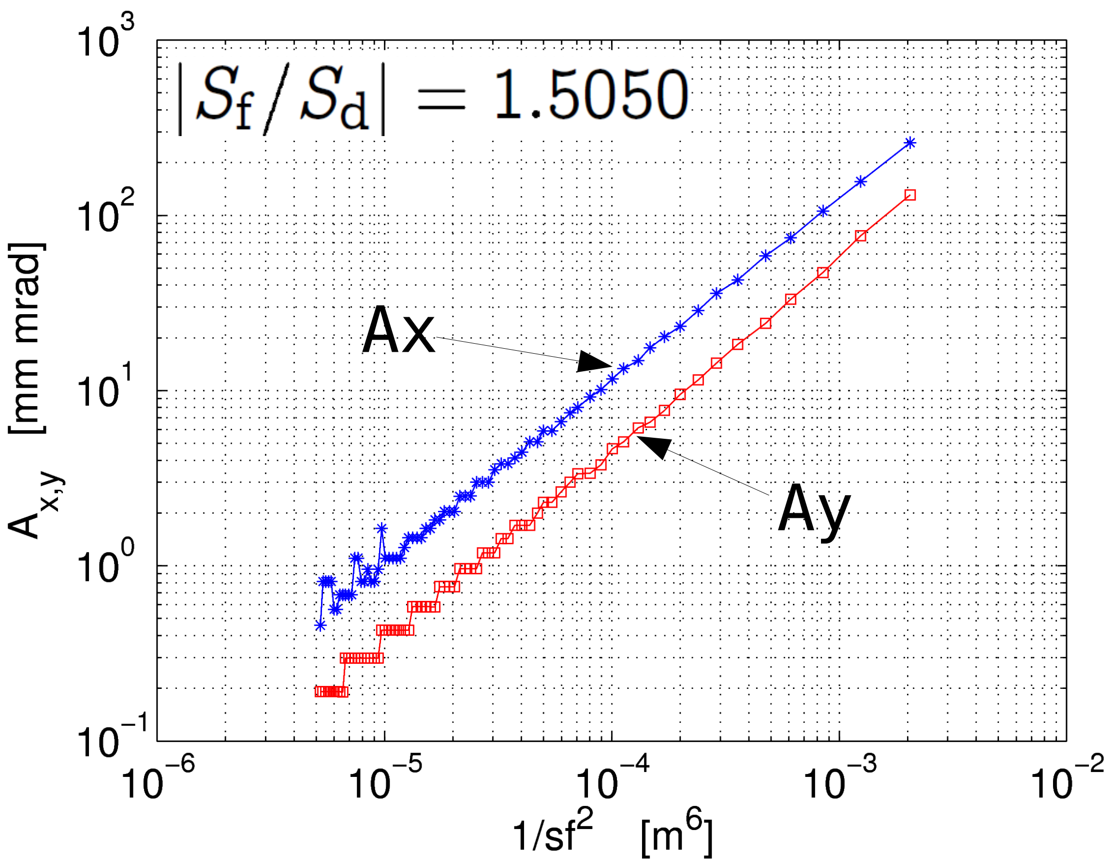
- $\beta_{x,0}$=11m→2.8m
- $\beta_{y,0}$=3m→2.8m
- ν_x=1.1875→1.4103
- ν_y=0.6833→0.8714

- N_{SP} from 4 to 50+



Modified ALS Lattice – 1 SP Study

- New ALS lattice, arc part unchanged, N_{SP} from 4 to 50
- $\xi = 0$



α_x=0.12

α_y=0.043

12 SPs, 0 chromaticity result:

α_x=0.12

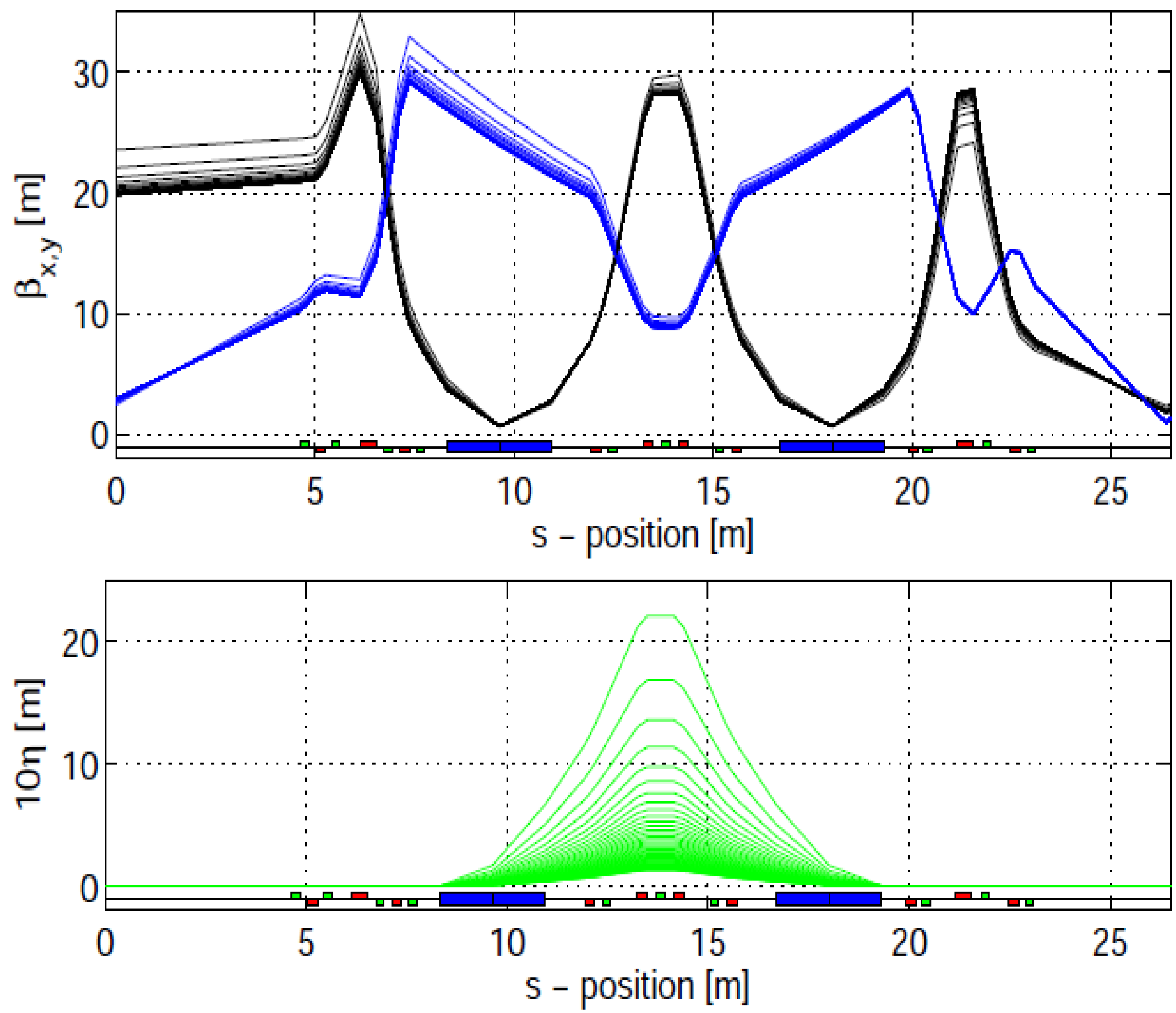
α_y=0.045

For ALS, the nonlinear beam dynamic is mainly determined by the design in arc part

DBA Example - NSLS-II

- DBA
- 3 GeV
- 52.8 m/SP
- N_{SP} = 15
- 2.02 nmrad
- 2 families of chromatic sextupoles
  (*sm1, sm2*)
- 8 families of harmonic sextupoles

- N_{SP} from 4 to 50+



Courtesy of NSLS-II Staff, BNL

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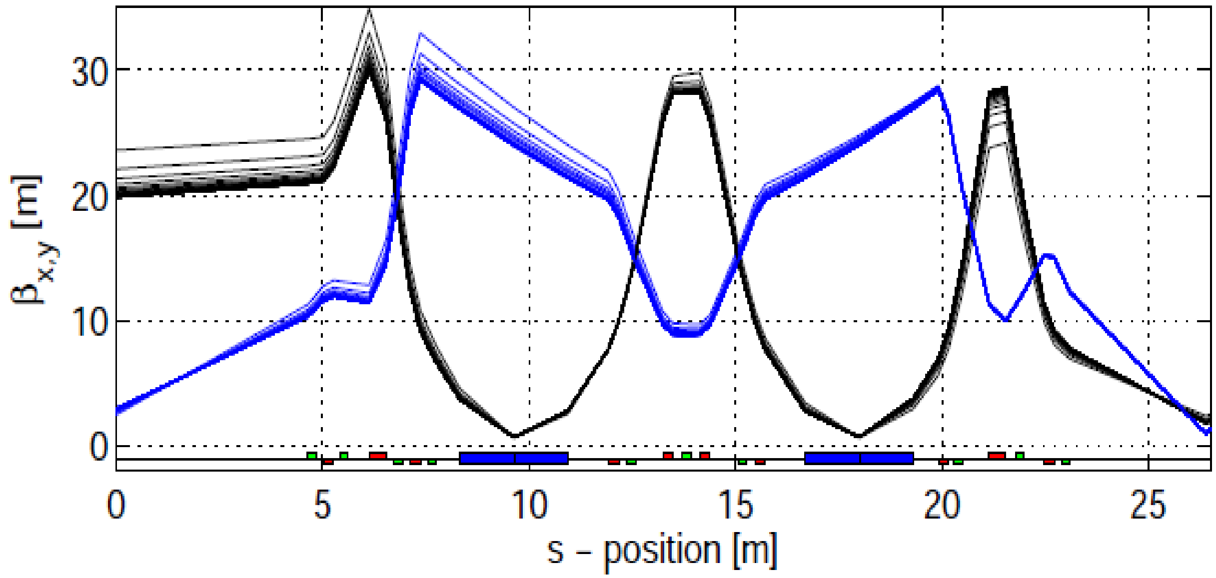

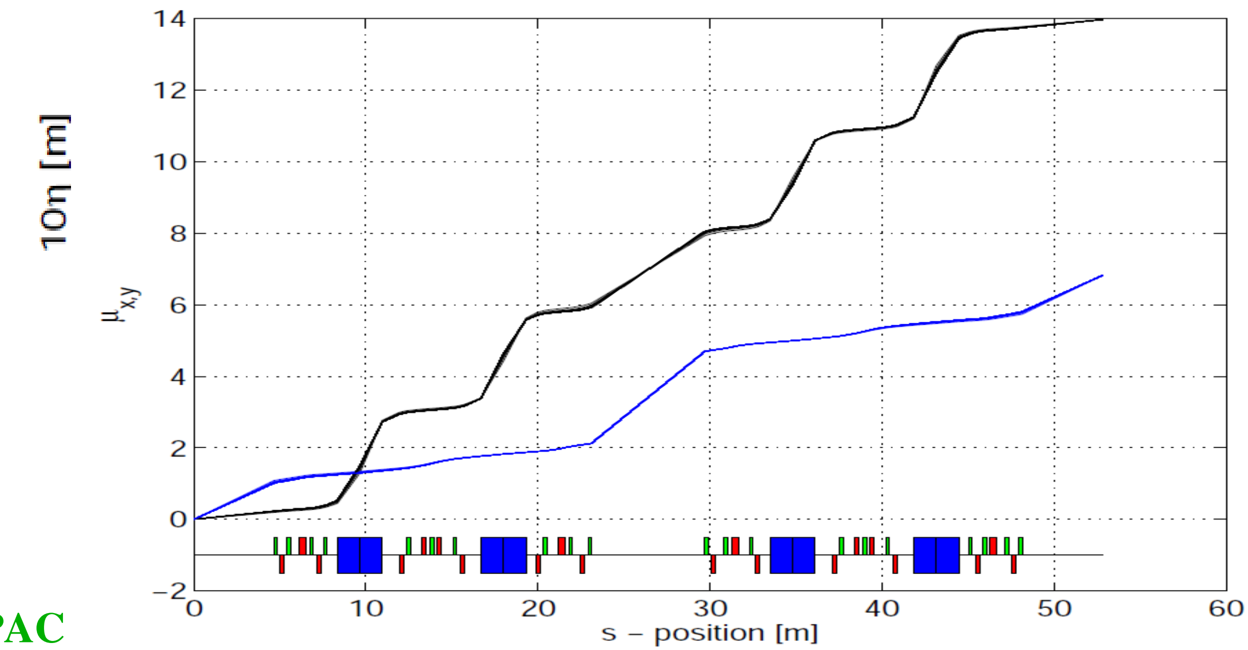

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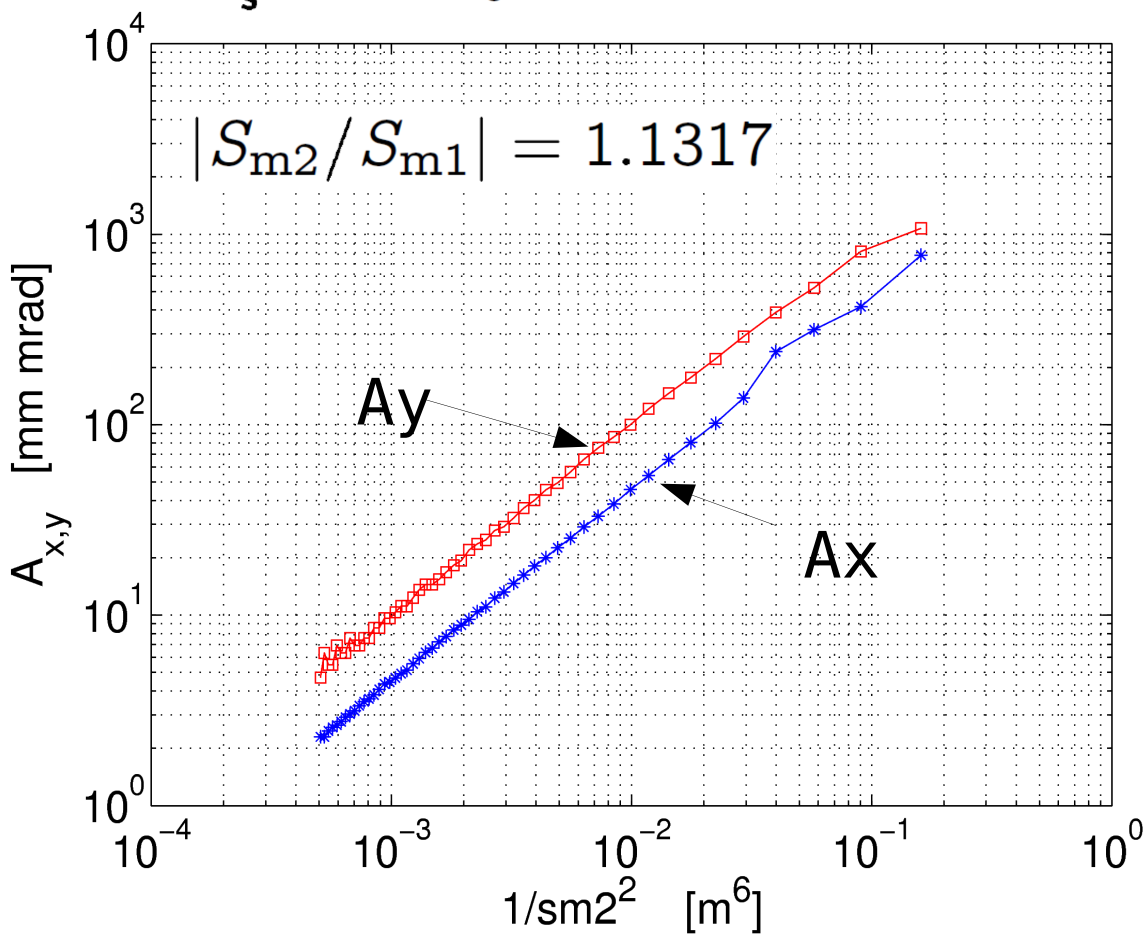


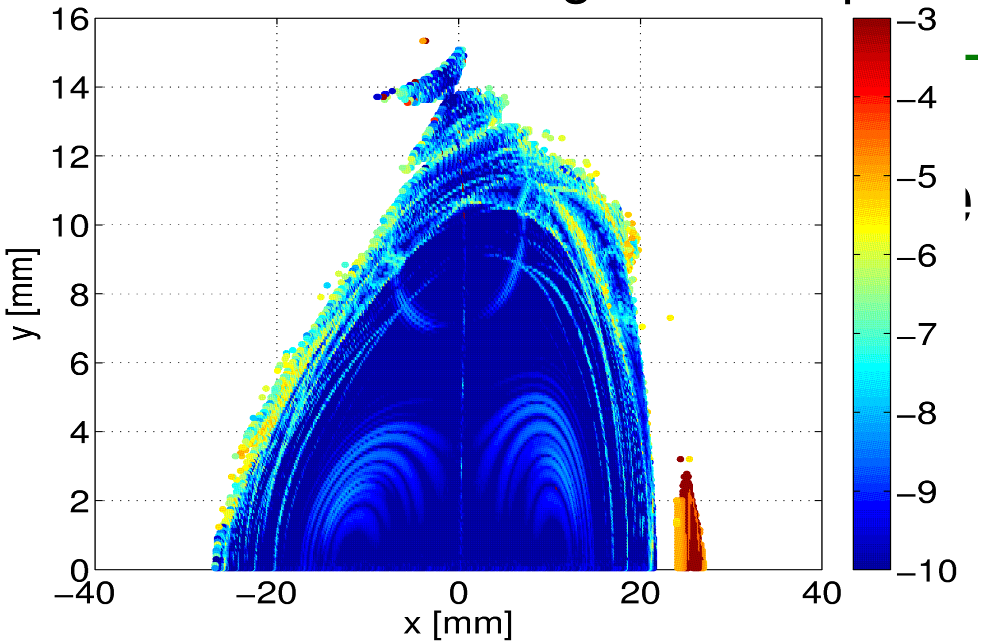
$$A_{x,y} = \alpha_{x,y}/S_{m2}^2$$

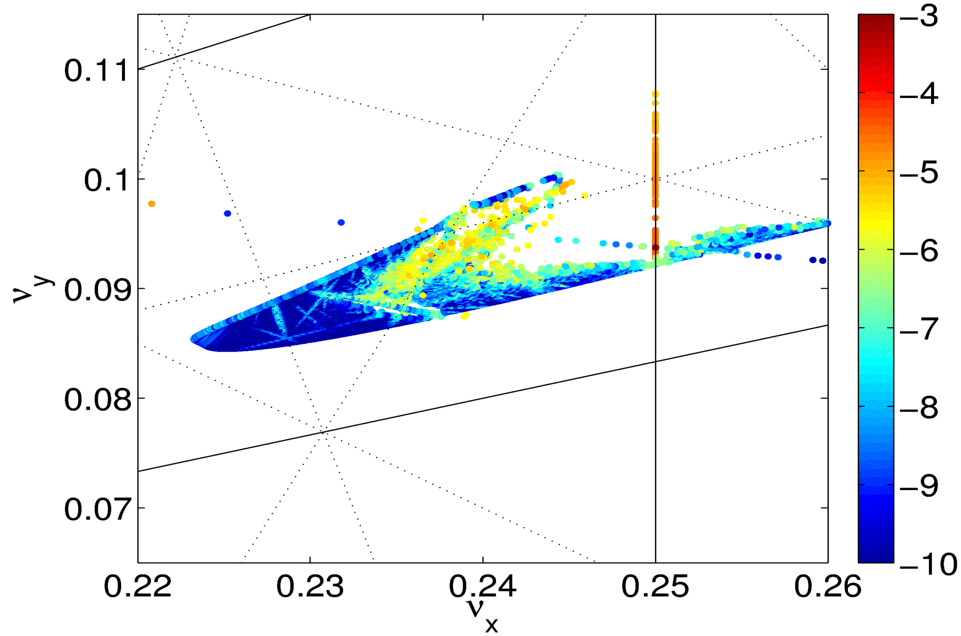
For NSLS-II

α_x=0.0057

α_y=0.0098

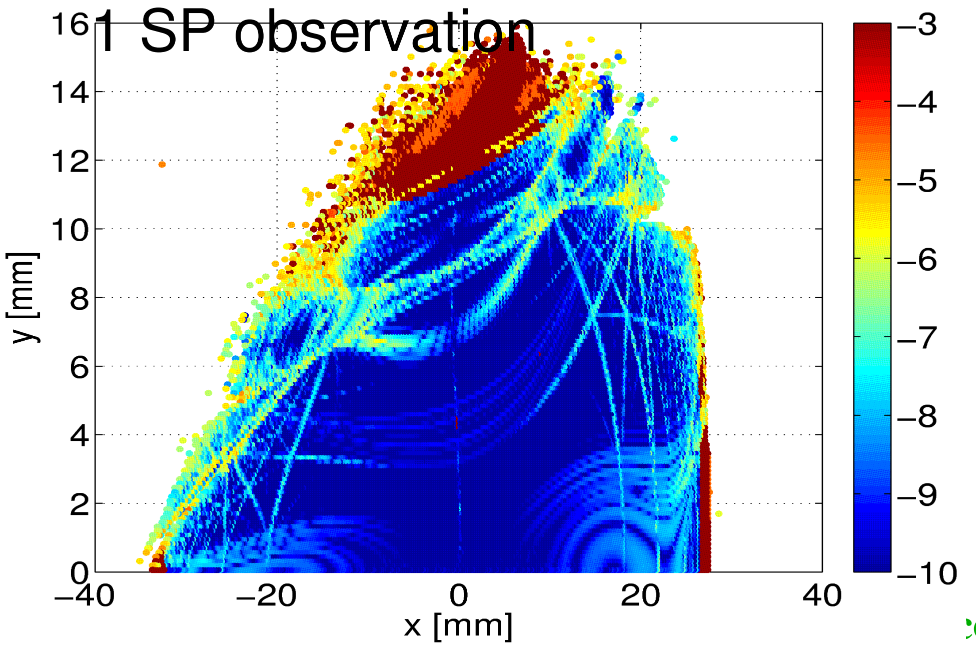
Original sextupoles setting, 1 SP observation

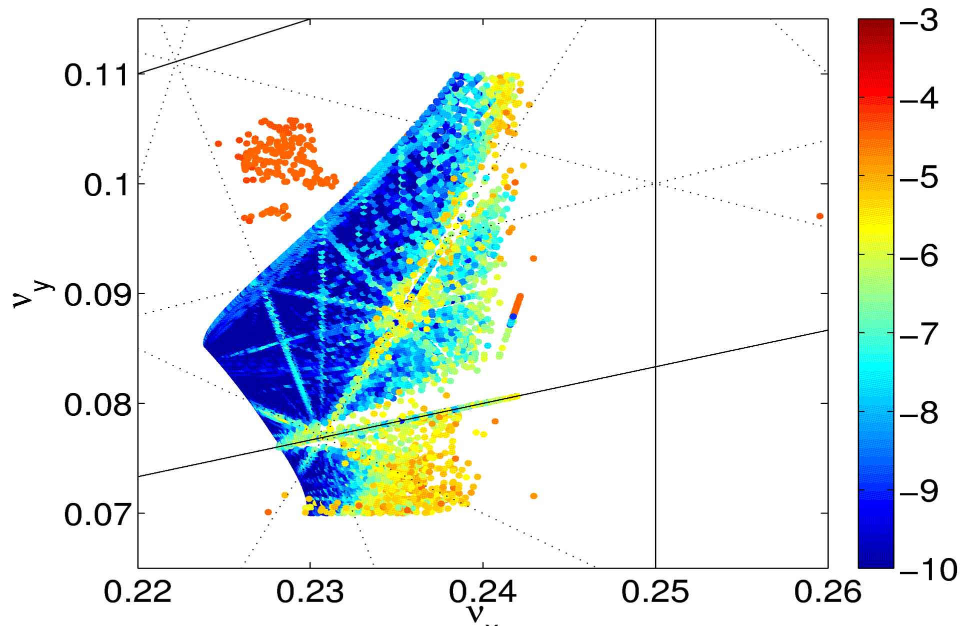




Modified sextupoles setting: harmonic sextupoles multiply factor of 0.8

1 SP observation





Summary

- Single cell of the *N*-cell storage ring $\rightarrow$ DA of the storage ring
- DA scaling with sextupole strengths:

define $\vec{\sigma} = (\sigma_1, \sigma_2, \cdots, \sigma_n)$ $\quad |\vec{\sigma}| = 1$

let $\vec{S} = \lambda\vec{\sigma}$

we have

$$\mathrm{DA} = \frac{\alpha(\vec{\sigma})}{\lambda^2}$$

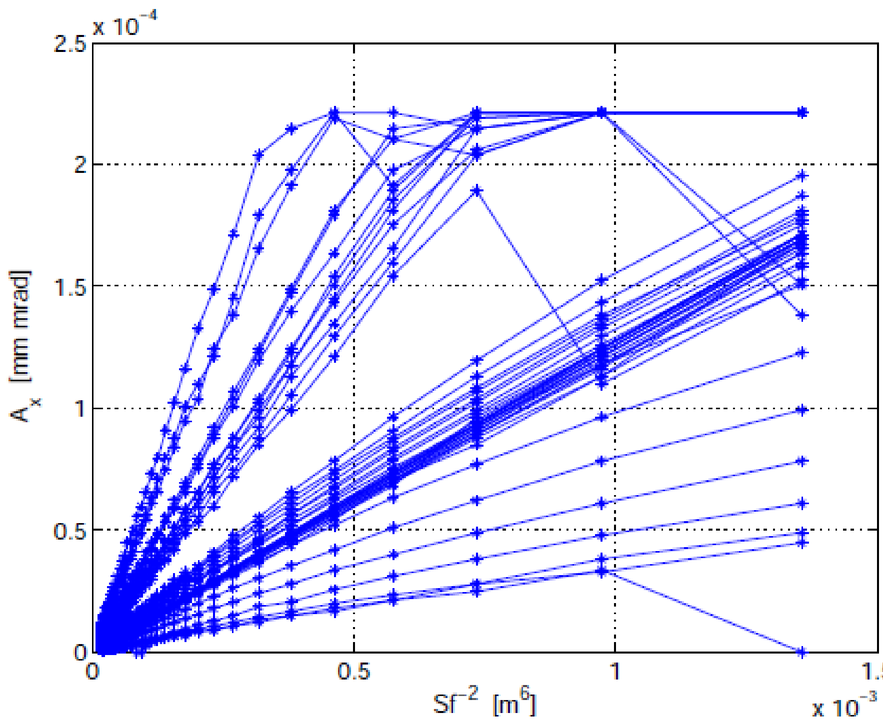
$$\mathrm{DA}_{\mathrm{frequency\ domain}} = g(\vec{\sigma}/|\vec{\sigma}|)$$

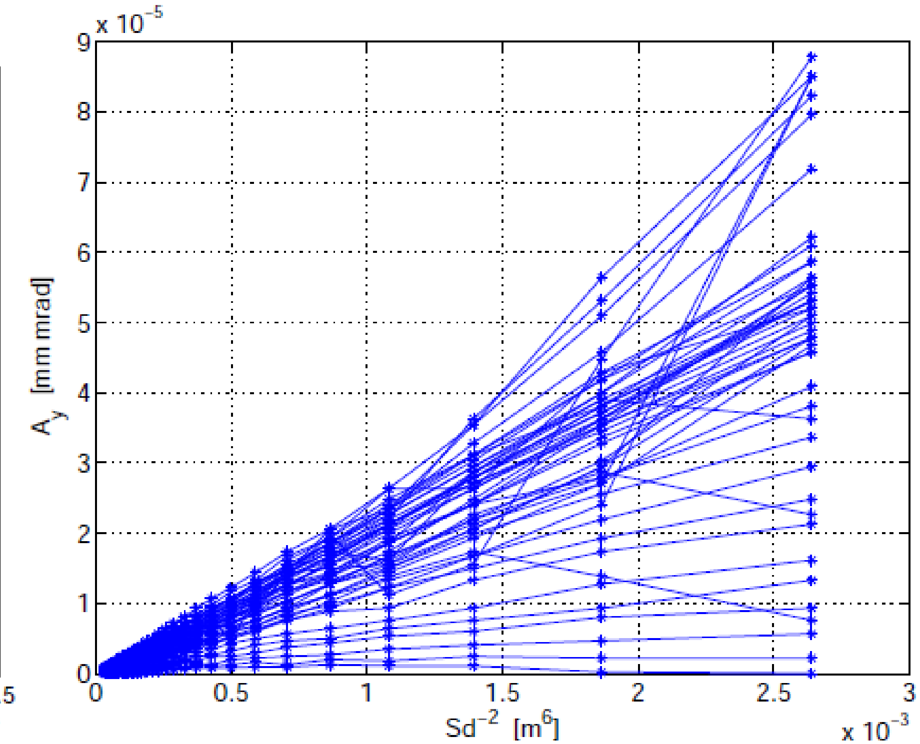
- N_{SP} could be changed without too much change in nonlinear dynamics
- For ALS, arc part $\rightarrow$ nonlinear dynamics
- *More work:*
  - *How about the relation with* δ
  - $\frac{\partial(\mathrm{DA})}{\partial\vec{\sigma}} = ?$

Thanks!

ALS – Off-momentum Case

- Different N_{SP}
- $\xi = 0$
- Different energy deviation from -15% to 15%

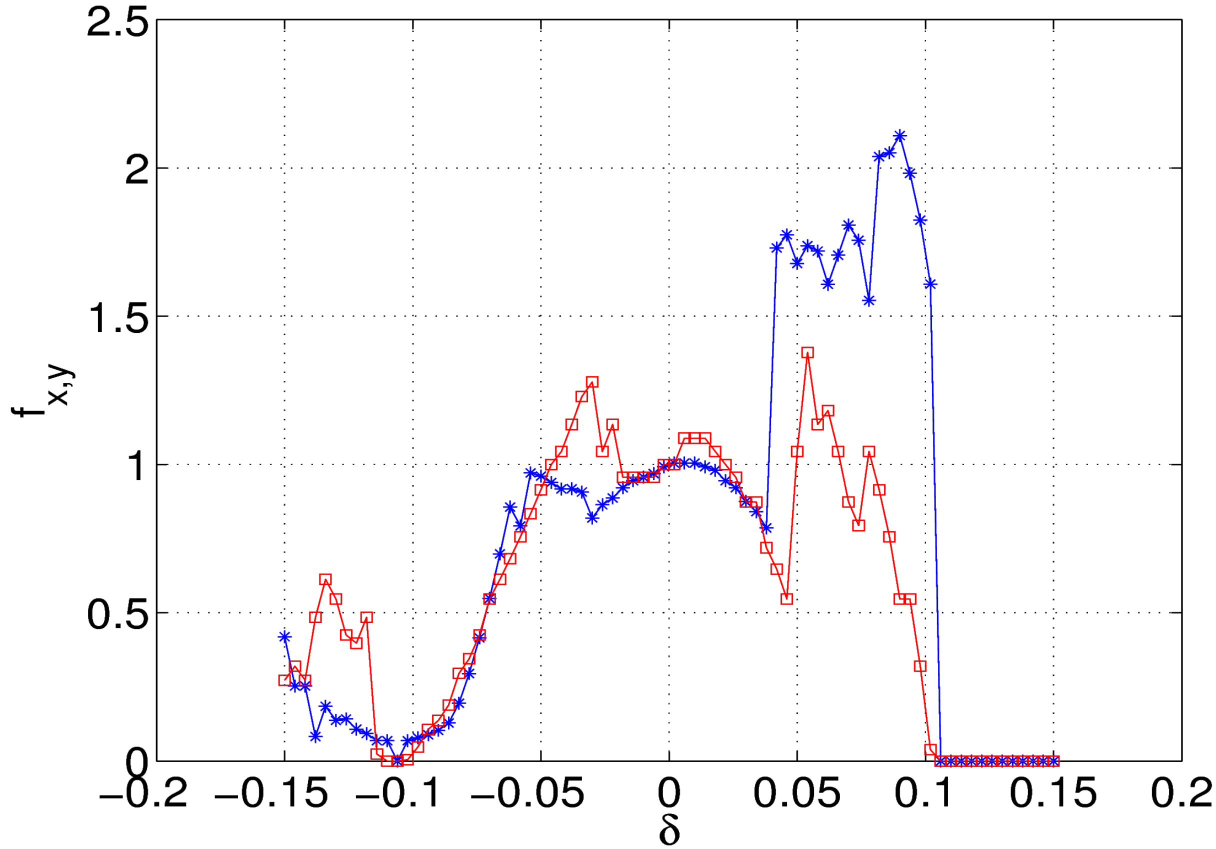




- For different energy deviation, slopes are different
- Different slope relates with the $f(\delta)$

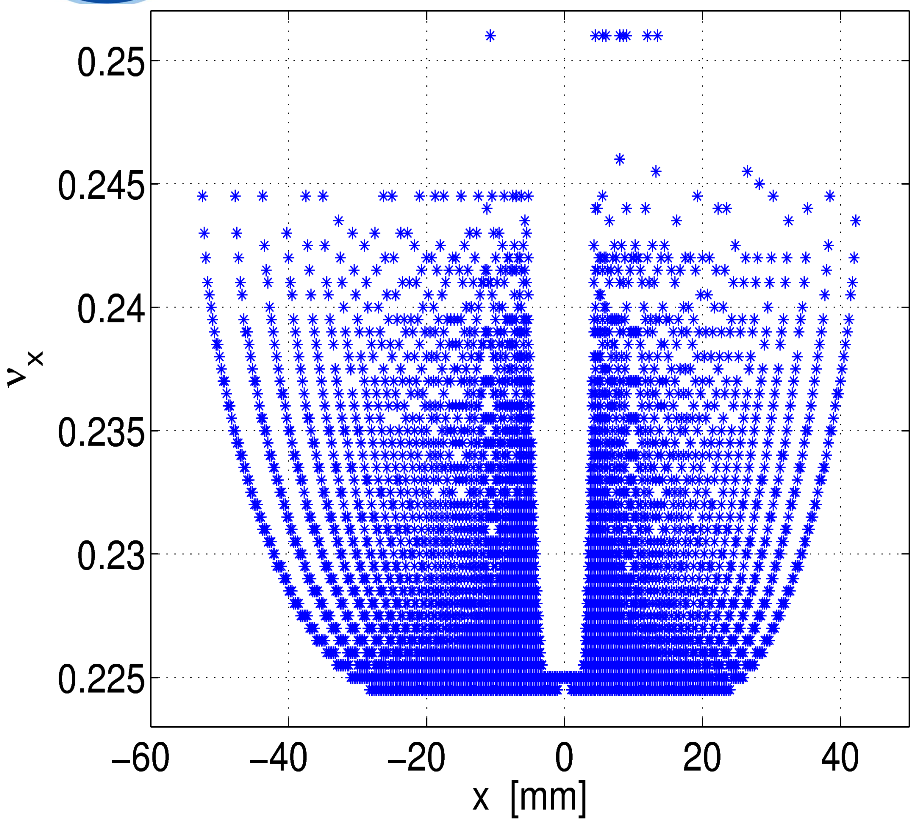
ALS – Off-momentum Case

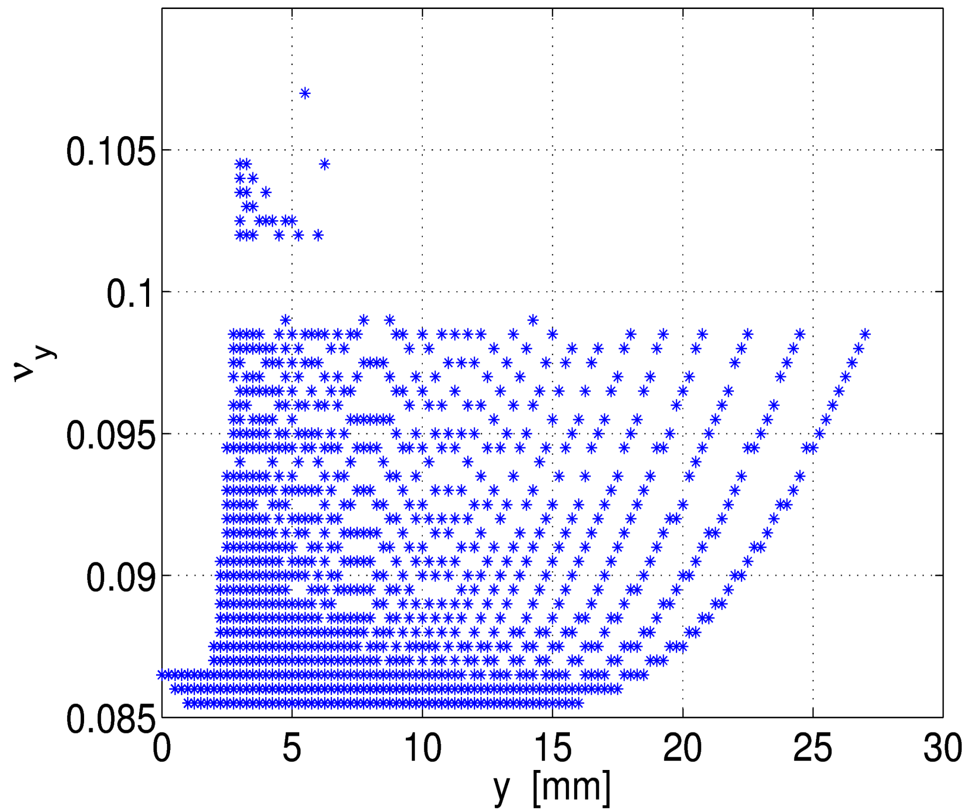
Normalized $\delta - A$ relation for ALS lattice



$$A_{x,y} = f(\delta)\alpha_{x,y}/S_f^2 \quad \text{with} \quad f(0) = 1$$

NSLS-II





NSLS-II, change sextupole strength

x,y versus tune