

Head-Tail Modes for Strong Space Charge

A. Burov

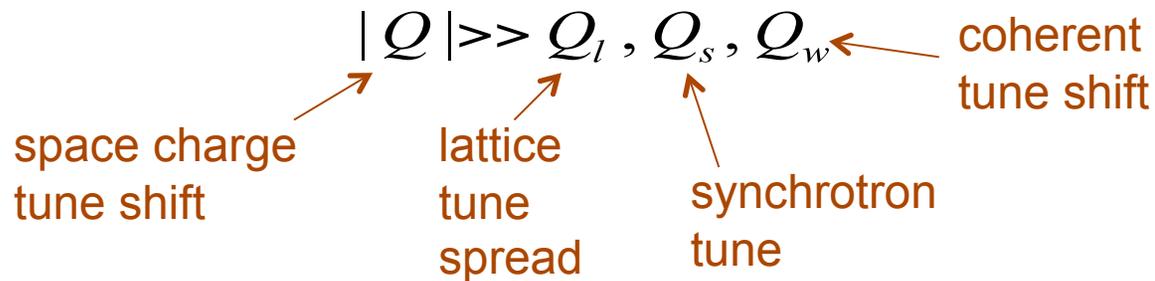
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Historical Remarks

- Head-tail modes = transverse coherent modes of a bunched beam.
- Without space charge, theory of head-tail stability was essentially shaped by Ernest Courant & Andrew Sessler, Claudio Pellegrini & Matthew Sands, and Frank Sacherer 40-30 years ago.
- With strong space charge, the problem was treated by Mike Blaskiewicz (1998). He solved it analytically for square potential well, air-bag (hollow beam) longitudinal distribution, transverse KV distribution and no octupoles. There is no Landau damping in his model. For short wakes, he found an analytical expression for the coherent spectrum.
- For no-wake case and square potential well, M. Blaskiewicz found a dispersion equation for arbitrary energy distribution, and transverse KV distribution.

Head-Tail Modes for Strong Space Charge

- In this talk, I will present my solution for arbitrary potential well, arbitrary 3D distribution function and possible lattice nonlinearity. Structure of the head-tail modes and their Landau damping for strong space charge will be described. Specifics of the transverse mode coupling instability is shown.
- The only crucial assumption is the dominance of the space charge tune shift:



- The paper is accepted by Phys. Rev. ST-AB; it can be found at <http://arxiv.org/ftp/arxiv/papers/0812/0812.3914.pdf>

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Coasting beam with space charge

- Space charge separates coherent and incoherent frequencies. Incoherent frequencies are shifted down by:

$$Q_{sc}(0) = -\frac{\rho r_0 C}{4\pi\beta^2\gamma^3\epsilon_{\text{rms}}}.$$

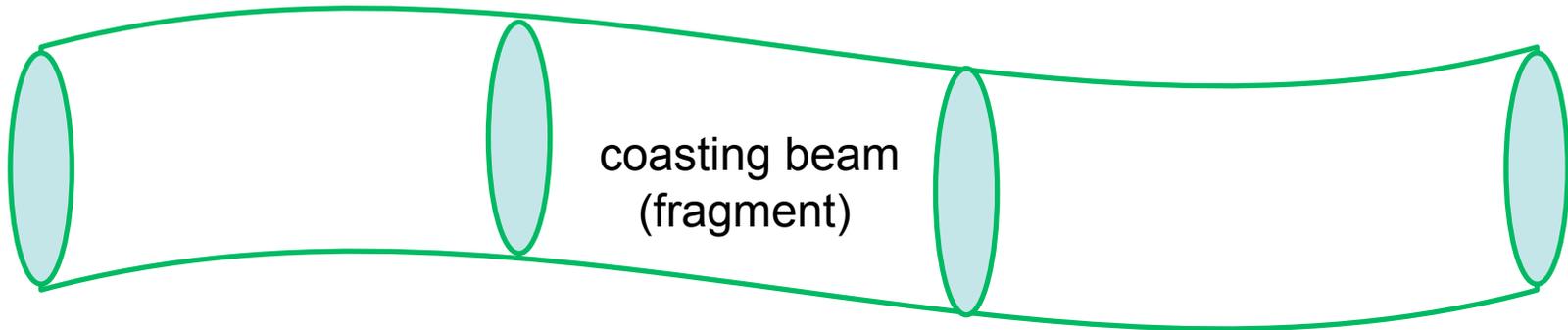
- In 1974, D. Möhl and H. Schönauer suggested to use the **rigid beam approximation** (rigid slice, frozen space charge):

$$\frac{d^2 x_i}{dt^2} + \underbrace{\Omega_i^2 Q_i^2}_{\text{lattice}} x_i + 2\Omega_0^2 Q_0 \left[\underbrace{Q_c}_{\text{wake}} \bar{x} + \underbrace{Q_{sc}}_{\text{space charge}} (x_i - \bar{x}) \right] = 0.$$

Condition for rigid-beam approximation

$$\frac{d^2 x_i}{dt^2} + \underbrace{\Omega_i^2 Q_i^2}_{\text{lattice}} x_i + 2\Omega_0^2 Q_0 \left[\underbrace{Q_c}_{\text{wake}} \bar{x} + \underbrace{Q_{sc}}_{\text{space charge}} (x_i - \bar{x}) \right] = 0.$$

- The assumption for the rigid beam approximation is that a core of any beam slice moves as a whole, there is no inner motion in it.



- This is a good approximation, when the space charge is strong enough:

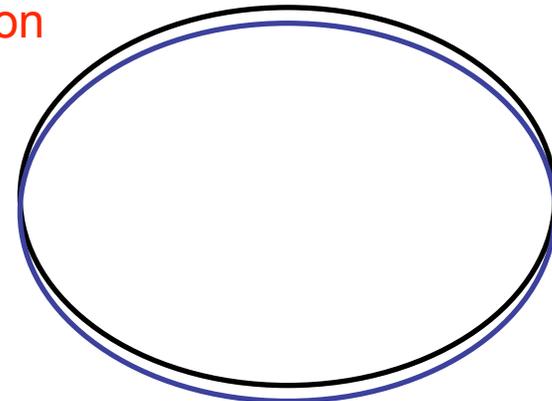
$$|Q_{sc}| \gg \Delta Q_l \equiv \sigma(\Omega_i Q_i) / \Omega_0.$$

Comments to Möhl-Schönauer Equation (MSE)

- Why non-linear space charge forces are presented by a linear term in the Möhl-Schönauer Equation? Does it mean that it is valid only for K-V (constant density) distribution?

$$\frac{d^2 x_i}{dt^2} + \underbrace{\Omega_i^2 Q_i^2}_{\text{lattice}} x_i + 2\Omega_0^2 Q_0 \left[\underbrace{Q_c}_{\text{wake}} \bar{x} + \underbrace{Q_{sc}(\mathbf{J}_\perp)}_{\text{space charge}} (x_i - \bar{x}) \right] = 0.$$

- No, it is valid for any transverse profile. The single-particle motion consists of 2 parts: free oscillations (beam size amplitude) and driven by the coherent offset oscillations (much smaller than the beam size). The equation describes the driven oscillations only, so it results from linearization of the original non-linear space charge term over infinitesimally small coherent motion, and averaging over the betatron phases. **This equation is a single possibility for a linear equation with constant coefficients, consistent with the given tunes and tune shifts.**
- The dispersion equation following from MSE is identical to what is obtained from Vlasov formalism (Pestrikov, NIM A, **562** (2006), p. 65)



Coasting beam: Landau damping (Burov, Lebedev, 2008):

$$\Lambda = -\pi \langle \Delta Q_{sep} \rangle \int \Delta Q_{sep} f_x J_x \delta(\Delta Q_l + Q_{sc} - \text{Re} v_c) d\Gamma, \quad f_x \equiv \partial f / \partial J_x ;$$

$$\Delta Q_{sep} \equiv \text{Re} \Delta Q_c - Q_{sc}(J_x, J_y), \quad \langle \Delta Q_{sep} \rangle \equiv - \left(\int \frac{f_x J_x d\Gamma}{\Delta Q_{sep}} \right)^{-1} ;$$

$$\text{Re} v_c = \text{Re} \Delta Q_c + \delta Q^{(1)} + \delta Q^{(2)},$$

$$\text{Im} v_c = \text{Im} \Delta Q_c - \Lambda$$

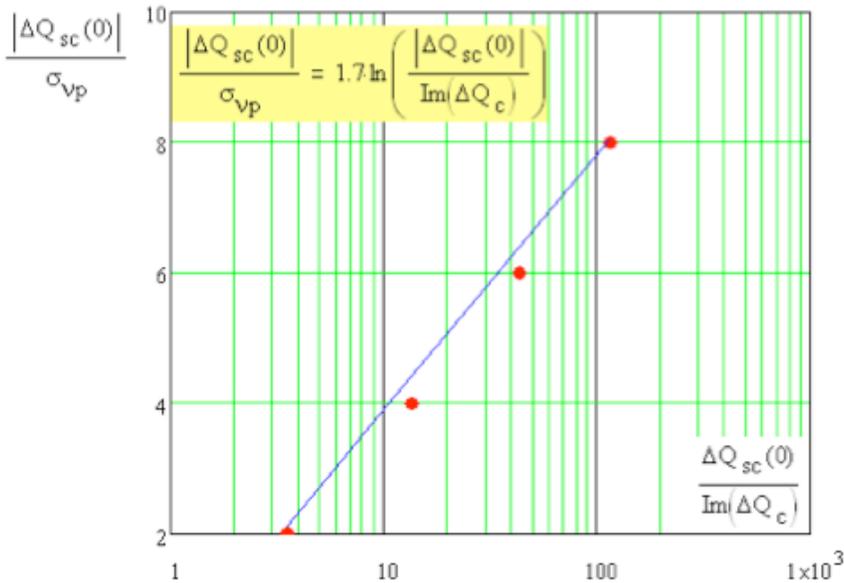
$$\delta Q^{(1)} = - \langle \Delta Q_{sep} \rangle \int \frac{\Delta Q_l f_x J_x d\Gamma}{\Delta Q_{sep}},$$

$$\delta Q^{(2)} = - \langle \Delta Q_{sep} \rangle \int \frac{\Delta Q_l^2 f_x J_x d\Gamma}{\Delta Q_{sep}^2},$$

$$\Delta Q_c = -i \frac{\rho r_0}{\gamma} \frac{\beta_x Z_x}{Z_0} .$$

Coasting Beam Thresholds (Burov, Lebedev, 2008)

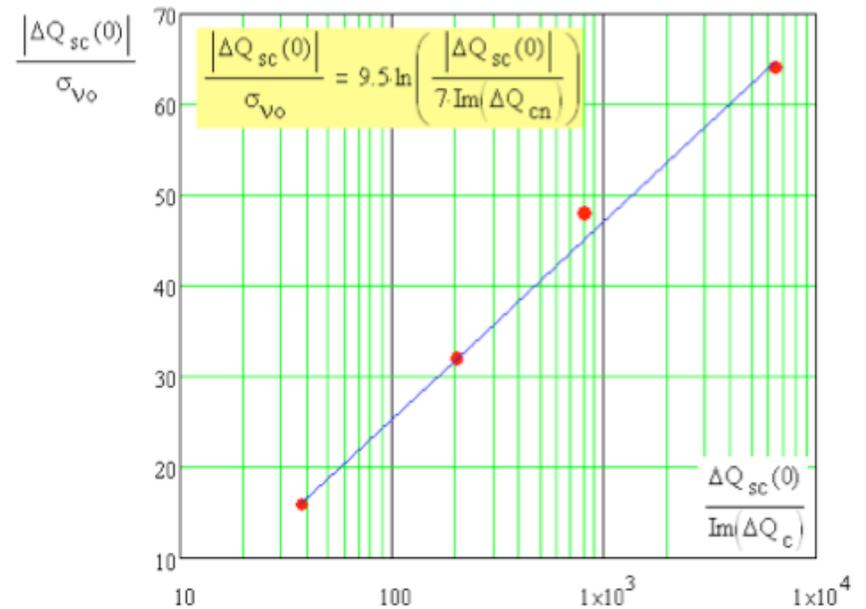
- Thresholds are determined by $\Lambda = \text{Im} \Delta Q_c$. For a round Gaussian beam:



Chromatic threshold

$$\sigma_{vp} \equiv \left| \xi - n\eta \right| \left(\frac{\delta p}{p} \right)_{\text{rms}}$$

$$\Delta Q_l = (\xi - n\eta) \hat{p} \equiv \sigma_{vp} \hat{p} / \sigma_p$$



Octupole threshold

$$\Delta Q_l = \Delta \sigma_{vo} (J_x + J_y) / 2\varepsilon > 0;$$

$$\langle J_x \rangle = \langle J_y \rangle = \varepsilon$$

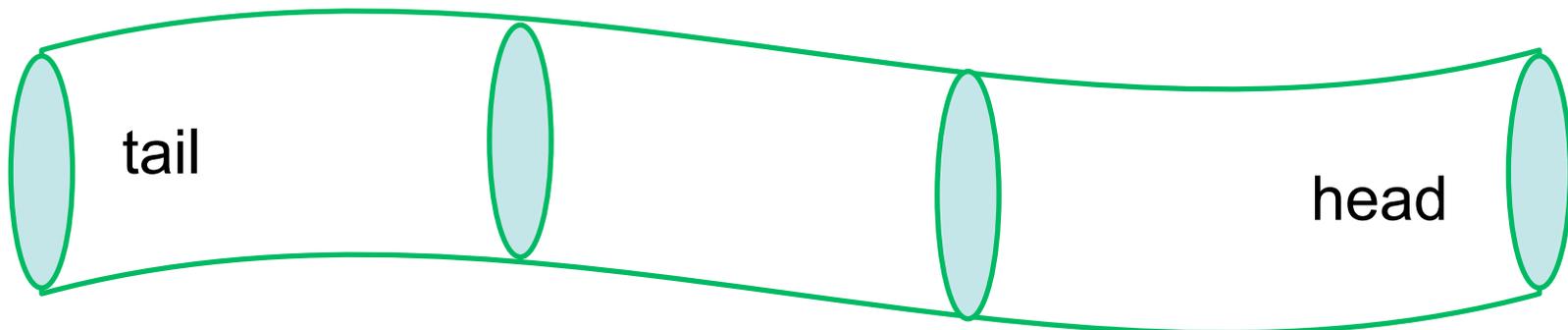
Head-tail with space charge

- The space charge can be considered as strong when

$$|Q| \gg Q_l, Q_s$$

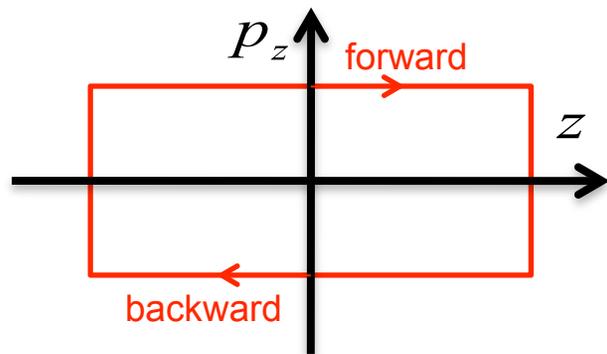
space charge tune shift lattice tune spread synchrotron tune

- In this case, all the particles of the local slice respond to the coherent field almost identically, similar to the coasting beam case.
- So, for the strong space charge, the rigid-beam approximation is justified for the bunched beam as well.



Square Well Model (M. Blaskiewicz)

- For a square potential well and KV transverse distribution, the head-tail modes with space charge were described by Mike Blaskiewicz.
- In 1998, he found an analytical solution for the air-bag longitudinal distribution and a short-range wake.
- In 2003, he generalized his square well result for arbitrary longitudinal distribution and zero wake.
- For the air-bag distribution, there are two particle fluxes in the synchrotron phase space:



Their betatron phases can be same or opposite.

Two sorts of modes

- For strong space charge, $Q \gg 2kQ_s$, the coherent tunes are shifted by the space charge:

$$\nu_{k+} = \frac{k^2 Q_s^2}{Q} \quad ; \quad k = 0, 1, 2, \dots \quad \text{in-phase modes}$$

$$\nu_{k-} = -Q - \frac{k^2 Q_s^2}{Q} \quad \text{out-of-phase modes}$$

- Only in-phase modes (+) are interesting for the beam stability, since they are much more separated from the incoherent spectrum, so they are much less Landau damped.
- Thus, out-of-phase modes, lost within the rigid-beam approximation, are not important for the beam stability.

Other form of MSE

Using slow betatron amplitudes $x_i(\theta)$,

$$X_i(\theta) = \exp(-iQ_b\theta)x_i(\theta)$$

the initial single-particle equation of motion writes as

$$\dot{x}_i(\theta) = iQ(\tau_i(\theta))[x_i(\theta) - \bar{x}(\theta, \tau_i(\theta))] - i\zeta v_i(\theta)x_i(\theta) - i\kappa\hat{W}\bar{x}$$

↑
space charge
↑
chromaticity
←
wake

$$\dot{x}_i = dx_i / d\theta, \quad \zeta = -\xi / \eta, \quad \eta = \gamma_i^{-2} - \gamma^{-2}, \quad \xi = dQ_b / d(\Delta p / p), \quad v_i(\theta) = \dot{\tau}_i(\theta)$$

Here θ and τ are time and distance along the bunch, both in angle units.

Solution of MSE for bunched beam

- After a substitution $x_i(\theta) = y_i(\theta) \exp(-i\xi\tau_i(\theta))$ with a new variable y , the chromatic term disappears from the equation, going instead into the wake term:

$$\dot{y}_i(\theta) = iQ(\tau_i(\theta)) [y_i(\theta) - \bar{y}(\theta, \tau_i(\theta))] - i\kappa \hat{\mathbf{W}} \bar{y}$$

$$\kappa = \frac{r_0 R}{4\pi\beta^2 \gamma Q_b} ; \quad \hat{\mathbf{W}} \bar{y} = \int_{\tau}^{\infty} W(\tau - s) \exp(i\xi(\tau - s)) \rho(s) \bar{y}(s) ds.$$

- Let for the beginning assume there is no wake, $W=0$. Then:

$$y_i(\theta) = -i \int_{-\infty}^{\theta} Q(\tau_i(\theta')) \bar{y}(\theta', \tau_i(\theta')) \exp(i\Psi(\theta) - i\Psi(\theta')) d\theta' ;$$

$$\Psi(\theta) \equiv \int_0^{\theta} Q(\tau_i(\theta')) d\theta' .$$

or

$$y_i(\Psi) = -i \int_{-\infty}^{\Psi} \bar{y}(\Psi') \exp(i\Psi - i\Psi') d\Psi' .$$

No-Wake Equation

- The phase Ψ runs fast compared with relatively slow dependence $\bar{y}(\Psi)$, so $\bar{y}(\Psi)$ under the integral can be expanded in the Taylor series:

$$\bar{y}(\Psi') \approx \bar{y}(\Psi) - (\Psi - \Psi') \frac{d\bar{y}}{d\Psi} + \frac{(\Psi - \Psi')^2}{2} \frac{d^2\bar{y}}{d\Psi^2} .$$

After that, the integral is easily taken:

$$y_i(\Psi) = \bar{y}(\Psi) - i \frac{d\bar{y}}{d\Psi} - \frac{d^2\bar{y}}{d\Psi^2} .$$

- Using that

$$\frac{d}{d\Psi} = \frac{v}{Q(\tau)} \frac{\partial}{\partial \tau} + \frac{1}{Q(\tau)} \frac{\partial}{\partial \theta} = \frac{1}{Q(\tau)} \left(v \frac{\partial}{\partial \tau} - i v_k \right),$$

eigenvalue

and averaging over all the particles inside the slice, equation for the **space charge modes** follows (no wake yet):

No-Wake Equation (2)

$$v_k \bar{y} + u(\tau) \frac{d}{d\tau} \left(\frac{u(\tau)}{Q(\tau)} \frac{d\bar{y}}{d\tau} \right) = 0;$$

$$u(\tau) \equiv \sqrt{\frac{\int_{-\infty}^{\infty} v^2 f(v, \tau) dv}{\int_{-\infty}^{\infty} f(v, \tau) dv}}.$$

- This equation is valid for **any longitudinal and transverse distribution functions**. The space charge tune shift $Q(\tau)$ stays here as transversely averaged:

$$Q(\tau) \rightarrow Q_{\text{eff}}(\tau) \equiv \left(q_{-2}^2 / q_{-1} \right) Q_{\text{max}}(\tau)$$

form-factors

$$\langle Q_i^p(\tau) \rangle_{\perp} = \int_0^{\infty} \int_0^{\infty} dJ_1 dJ_2 f_{\perp}(J_1, J_2) Q^p(J_1, J_2, \tau) \equiv q_p^p Q_{\text{max}}^p(\tau).$$

definition

General Equation for space charge modes

- With two wake terms (driving W and detuning D), the equation is modified as:

$$v \bar{y}(\tau) + u(\tau) \frac{d}{d\tau} \left(\frac{u(\tau)}{Q_{\text{eff}}(\tau)} \frac{d\bar{y}}{d\tau} \right) = \kappa (\hat{\mathbf{W}}\bar{y} + \hat{\mathbf{D}}\bar{y})$$

$$\hat{\mathbf{W}}\bar{y} \equiv \int_{\tau}^{\infty} W(\tau - s) \exp(i\zeta(\tau - s)) \rho(s) \bar{y}(s) ds ;$$

$$\hat{\mathbf{D}}\bar{y} \equiv \bar{y}(\tau) \int_{\tau}^{\infty} D(\tau - s) \rho(s) ds .$$

Boundary Conditions

- At far longitudinal tails, the space charge tune shift becomes so small, that local incoherent spectrum is covering the coherent line.
- This results in the decoherence of the collective motion beyond this point $\tau = \tau_*$, smearing out all the remaining gradients; thus:

$$\left. \frac{\partial \bar{y}}{\partial \tau} \right|_{\tau = \pm \tau_*} = 0$$

- The decoherence point τ_* can be found as $\left| \frac{d}{d\Psi} \bar{y}'(\tau_*) \right| \cong |\bar{y}'(\tau_*)|$ yielding $\tau_* = \sqrt{2 \ln(q/\tau_*)}$; $q \equiv Q_{\text{eff}}(0)/Q_s$ for a Gaussian bunch.
- With these boundary conditions, the obtained equation for the space charge modes has a **full orthonormal basis** of solutions:

Full orthonormal basis of no-wake problem

$$\int_{-\infty}^{\infty} \bar{y}_k(\tau) \bar{y}_m(\tau) u^{-1}(\tau) d\tau = \delta_{km}$$

$$\sum_{m=0}^{\infty} \bar{y}_m(\tau) \bar{y}_m(s) = u(\tau) \delta(s - \tau)$$

- At the bunch core, the k -th eigen-function $\bar{y}_k(\tau)$ behaves like $\sim \cos(k\tau / \sigma)$ or $\sim \sin(k\tau / \sigma)$; the eigenvalues are estimated as (zero wake)

$$v_k \cong k^2 \bar{Q}_s^2 / Q_{\text{eff}}(0) \ll \bar{Q}_s .$$

- If the wake is small enough, it can be accounted as a perturbation, giving the coherent tune shift as a diagonal matrix elements (similar to Schrödinger Equation in Quantum Mechanics):

$$Q_w = \kappa \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(\tau - s) \exp(i\zeta(\tau - s)) \rho(s) \bar{y}_k(s) \bar{y}_k(\tau) u^{-1}(\tau) ds d\tau ,$$

$$Q_d = \kappa \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(\tau - s) \rho(s) \bar{y}_k^2(\tau) u^{-1}(\tau) ds d\tau .$$

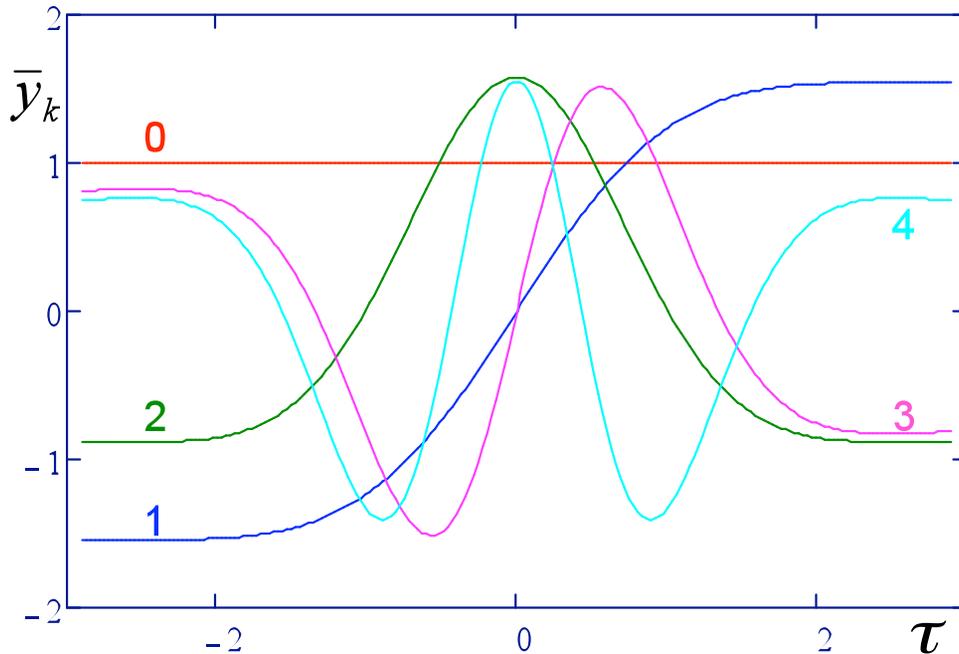
$$\text{Im} Q_d = 0 ; \quad \sum_{k=0}^{\infty} \text{Im} Q_w = 0$$

Modes for Gaussian bunch

- For the Gaussian distribution, $f(v, \tau) = \frac{N_b}{2\pi Q_s} \exp\left(-\frac{v^2}{2Q_s^2} - \frac{\tau^2}{2}\right)$, measuring

eigenvalues in $Q_s^2 / Q_{\text{eff}}(0)$, the no-wake equation reduces to (no parameters!):

$$v \bar{y} + \frac{d}{d\tau} \left(e^{\tau^2/2} \frac{d\bar{y}}{d\tau} \right) = 0; \quad \bar{y}'(\pm\tau_*) = 0.$$



Eigenfunctions for Gaussian bunch.

$$\tau_* = 2.5 \Leftrightarrow q = 60$$

Asymptotic:

$$\bar{y}'_k(\tau) \sim b_k \exp(-\tau^2/2) (\tau_* - \tau)$$

Coherent tune shifts

Coherent tune shifts (eigenvalues) for the Gaussian bunch, for $q=5, 15, 60$ (or $\tau_* = 1.5, 2.0, 2.5$) in the units of Q_s / q , $q \gg 2k$:

τ_* \ k	0	1	2	3	4	5	6	7	8	9
1.5	0	1.2								
2.0	0	0.78	4.0	9.2	17					
2.5	0	0.55	3.2	7.7	14	22	32	45	60	75

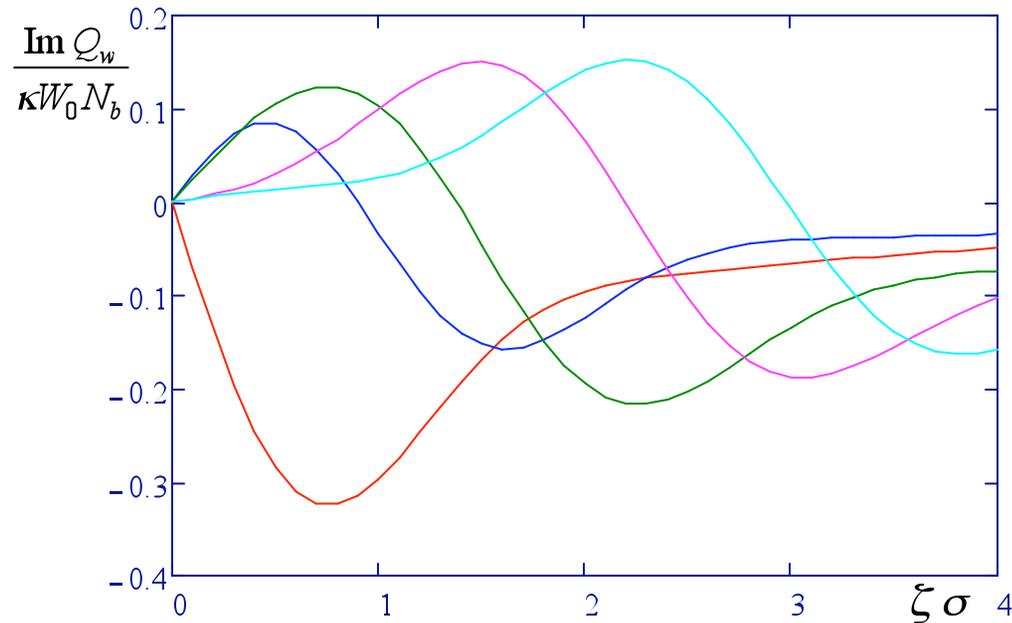
Each eigenvalue, except the first three, lies between two nearest squares of integers:

$$k^2 - k/2 \leq \nu_k \leq k^2 + k/2; \quad k = 0, 1, 2, \dots$$

Note the **structural difference** between this and no-space-charge coherent spectra. While the former is counted by squares of natural numbers, the latter is counted by integers.

Growth rates for Gaussian bunch and constant wake

- Coherent growth rates for the Gaussian bunch with the constant wake $W(\tau) = W_0$ as functions of the head-tail phase $\chi \equiv \zeta \sigma$, for the modes 0, 1, 2, 3, 4.



- The growth rates reach their maxima at $\chi \approx 0.7(k-1)$. After its maximum, the high order mode changes its sign at $\chi \approx 0.7k$

Intrinsic Landau damping

- Landau damping is dissipation of coherent motion due to transfer of its energy into incoherent motion. The coherent energy is transferred only to resonant particles - the particles whose individual frequencies are in a resonance with the coherent frequency.
- How the resonant particles can exist, when the space charge strongly separates coherent and incoherent motion?
- In the longitudinal tails, space charge tune shift ≈ 0 , so the Landau energy transfer is possible at the tails.
- For a given particle i , that happens at its own 'decoherence point', where

$$Q_i(\tau) \cong \left| v_i(\tau) \frac{\partial}{\partial \tau} \right|$$

$$\left| \frac{\partial}{\partial \tau} \right| = \left| \frac{1}{Q} \frac{\partial Q}{\partial \tau} \right| \cong \tau$$

For Gaussian bunch

Intrinsic Landau damping (2)

- After M times of passing its decoherence point, the individual amplitude is excited by

$$\Delta y_i(M) = \tilde{y} \sum_{m=0}^{M-1} e^{im\Psi} = \tilde{y} e^{iM\Psi/2} \frac{\sin(M\Psi/2)}{\sin(\Psi/2)}.$$

where Ψ is the space charge phase advance per synchrotron period:

$$\Psi_s(\tau_0) \equiv 4 \int_0^{\tau_s/4} Q(\tau_0 \sin(Q_s \theta)) d\theta.$$

- The entire Landau energy transfer from the mode after $M \gg 1$ turns can be expressed as

$$\Delta E(M) = 4 \int d\mathbf{J} f(\mathbf{J}) \tilde{y}^2 \frac{\sin^2(M\psi/2)}{\sin^2(\psi/2)}.$$

Intrinsic Landau damping (3)

- The power of the mode energy transfer is calculated as

$$\Delta \dot{E} = \frac{d\Delta E(M)}{T_s dM} = 4Q_s \int d\mathbf{J} f(\mathbf{J}) \tilde{y}^2 \delta_p(\Psi),$$

$$\delta_p(\Psi) \equiv \sum_n \delta(\Psi - 2\pi n). \leftarrow \text{Many resonant lines}$$

- Since the space charge phase advance $\Psi \gg 1$, the sum over many resonance lines n can be approximated as an integral over these resonances. After transverse average and longitudinal saddle-point integration, it yields the Landau damping rate:

$$\Lambda_k \equiv -\frac{\Delta \dot{E}_k}{2E_k} = 1.5b_k^2 Q_s / q^3 ; \quad q = \frac{Q_{\text{eff}}(0)}{Q_s}; \quad E_k \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\tau \exp(-\tau^2 / 2) \bar{y}_k^2$$

- Gaussian bunch asymptotic derivative factors b_k^2 numerically calculated for the space charge parameter $q = 5, 15, 60$.

$q \backslash k$	0	1	2	3	4	5	6	7	8	9
5	0	2.5								
15	0	1.3	15	64	160					
60	0	0.85	7.5	40	105	260	500	1060	1700	2300

Landau damping by lattice nonlinearity

- The nonlinearity modifies the single-particle equation of motion:

$$\dot{y}_i(\theta) = iQ(\tau_i(\theta))[y_i(\theta) - \bar{y}(\theta, \tau_i(\theta))] - \delta Q_i y_i(\theta)$$

- For $\delta Q_i > 0$, there is a point in the bunch $\tau = \tau_r$, where the nonlinear tune shift exactly compensates the local space charge tune shift:

$$\delta Q_i = Q(\tau_r)$$

- At this point, the particle actually crosses a resonance of its incoherent motion with the coherent one. Crossing the resonance excites the incoherent amplitude by

$$|\Delta y_i| = Q(\tau_r) \bar{y}(\tau_r) \sqrt{\frac{2\pi}{|\dot{Q}(\tau_r)|}}; \quad \dot{Q}(\tau_r) = \frac{dQ}{d\tau} \frac{d\tau}{d\theta} = \frac{dQ}{d\tau} v,$$

leading to a dissipation of the coherent energy.

Landau damping by lattice nonlinearity (2)

- This energy dissipation gives the Landau damping rate

$$\Lambda_k = -\frac{Q_s}{\pi E_k} \int d\mathbf{J}_\perp \int_{J_r}^{\infty} dJ_\parallel \frac{\partial f(\mathbf{J}_\perp, J_\parallel)}{\partial J_\parallel} J_\parallel |\Delta y_i|^2.$$

- For a round Gaussian bunch and symmetric octupole-driven nonlinear tune shift $\delta Q = \langle \delta Q \rangle (J_1 + J_2) / 2$,

$$\Lambda_k = A \frac{\bar{y}_k^2(\infty)}{\tau_*} \frac{\langle \delta Q \rangle^2}{Q_{\max}(0)}; \quad A \approx 1 \cdot 10^2$$

$$\bar{y}_k(\infty) = \begin{cases} 1, & \text{if } k = 0 \\ -b_k / \nu_k, & \text{otherwise} \end{cases}$$

- Total damping rate is a sum of the intrinsic rate and the nonlinearity-related rate.

General equation for space charge modes

- When both synchrotron tune and coherent tune shift are small compared to the space charge tune shift, then:

$$v \bar{y}(\tau) + u(\tau) \frac{d}{d\tau} \left(\frac{u(\tau)}{Q_{\text{eff}}(\tau)} \frac{d\bar{y}}{d\tau} \right) = \kappa (\hat{\mathbf{W}}\bar{y} + \hat{\mathbf{D}}\bar{y})$$

$$\hat{\mathbf{W}}\bar{y} \equiv \int_{\tau}^{\infty} W(\tau - s) \exp(i\zeta(\tau - s)) \rho(s) \bar{y}(s) ds ;$$

$$\hat{\mathbf{D}}\bar{y} \equiv \bar{y}(\tau) \int_{\tau}^{\infty} D(\tau - s) \rho(s) ds .$$

- This is valid for any ratio between the coherent tune shift and the synchrotron tune.
- The sought eigenfunction $\bar{y}(\tau)$ can be expanded over the full orthonormal basis of the no-wake modes $\bar{y}_{0k}(\tau)$,

$$\bar{y}(\tau) = \sum_{k=0}^{\infty} B_k \bar{y}_{0k}(\tau),$$

Method to solve the general equation

- This reduces the problem to a set of linear equations:

$$(\kappa \widehat{\mathbf{W}} + \kappa \widehat{\mathbf{D}} + \text{Diag}(\mathbf{v}_0)) \mathbf{B} = \nu \mathbf{B}$$

$$\widehat{\mathbf{W}}_{km} = \int_{-\infty}^{\infty} \int_{\tau}^{\infty} W(\tau - s) \exp(i\zeta(\tau - s)) \rho(s) \bar{y}_{0k}(\tau) \bar{y}_{0m}(s) u^{-1}(\tau) ds d\tau ,$$

$$\widehat{\mathbf{D}}_{km} = \int_{-\infty}^{\infty} \int_{\tau}^{\infty} D(\tau - s) \rho(s) \bar{y}_{0k}(\tau) \bar{y}_{0m}(\tau) u^{-1}(\tau) ds d\tau .$$

TMCI

For zero chromaticity, instabilities are possible due to

- Over-revolution (or couple-bunch) wake (not considered here...)
- Mode coupling with large enough coherent tune shift (TMCI = transverse mode coupling instability). This may be expected when the coherent tune shift is comparable with the distance between the unperturbed modes, $Q_w \cong Q_s^2 / Q_{\max}$.
- However, this expectation appears to be incorrect: the TMCI threshold is normally much higher than that.

Vanishing TMCI

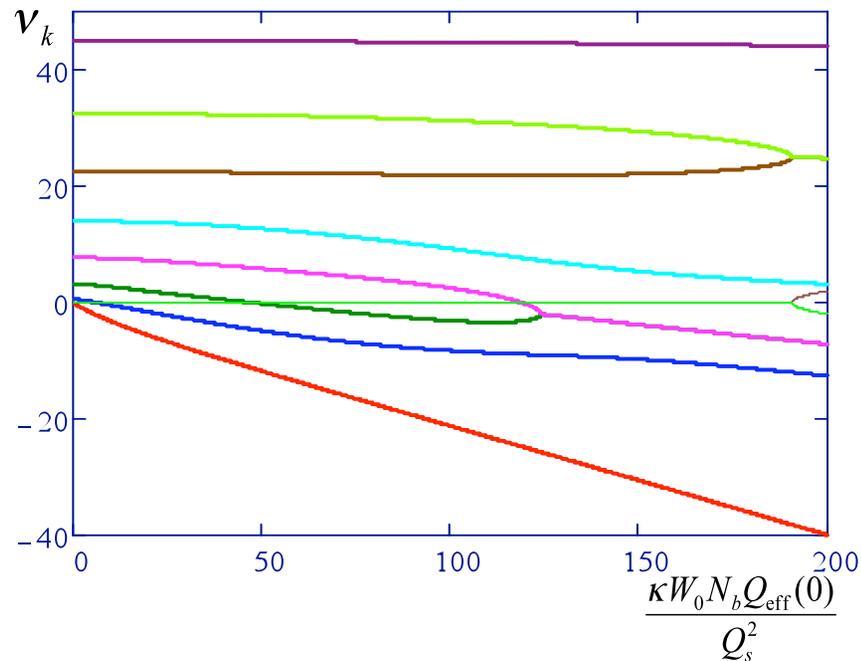
- While the conventional head-tail modes are numbered by integers,

$$\nu_k = kQ_s, k = 0, \pm 1, \pm 2, \dots$$

the space charge modes are numbered by natural numbers:

$$\nu_k \propto k^2.$$

- This is a structural difference, leading to significant increase of the transverse mode coupling instability: the most affected lowest mode has no neighbor from below.

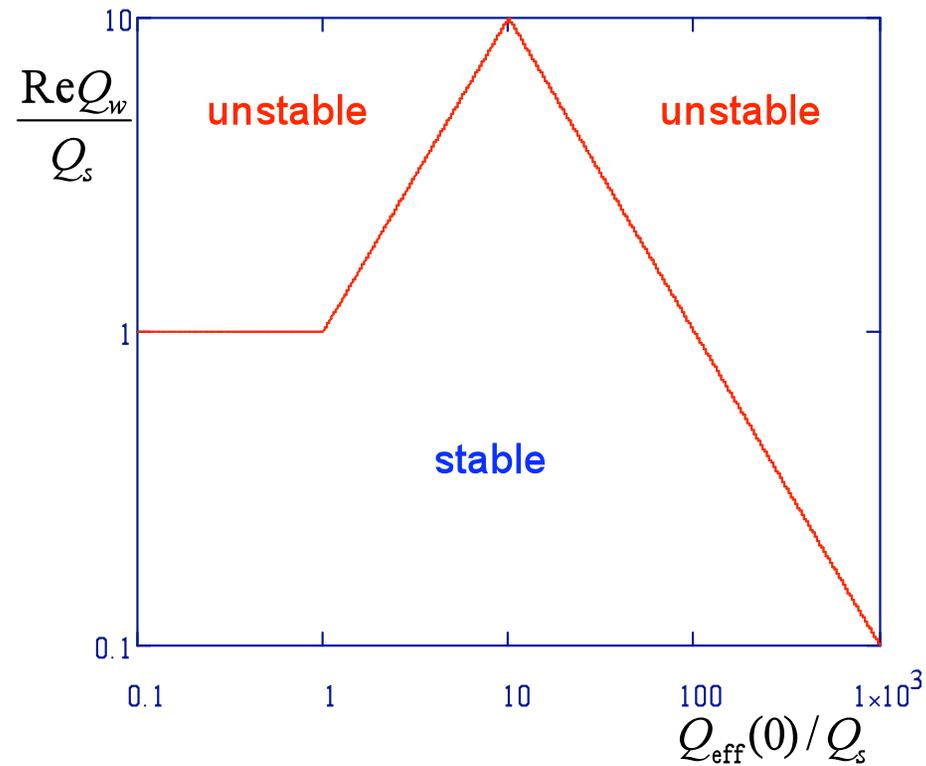


Coherent tunes of the Gaussian bunch for zero chromaticity and constant wake versus the wake amplitude.

Note high value of the TMCI threshold.

TMCI threshold for arbitrary space charge

A schematic behavior of the TMCI threshold for the coherent tune shift versus the space charge tune shift.



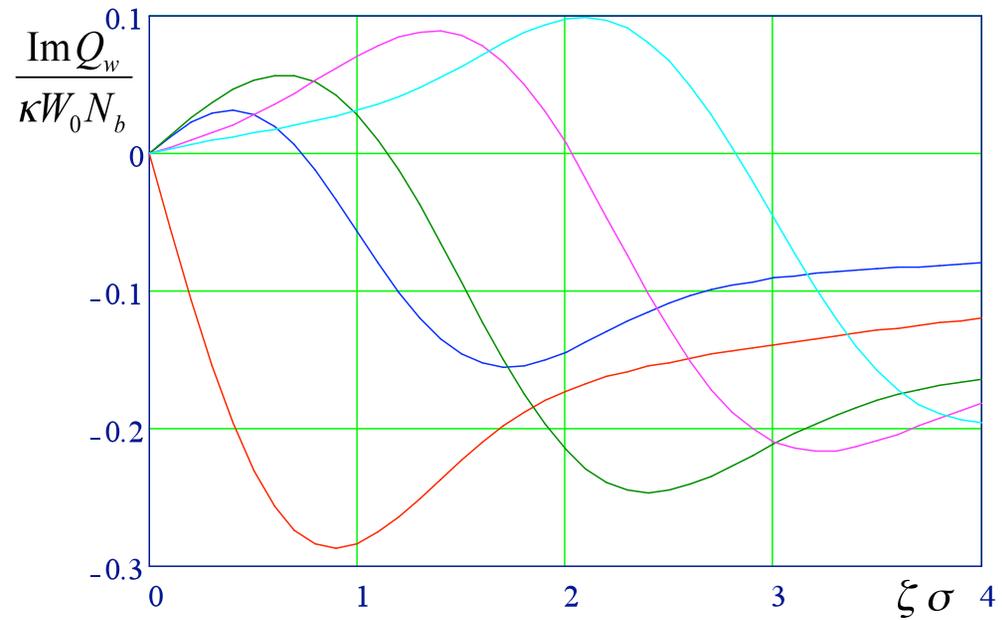
Multi-turn wake and coupled bunches

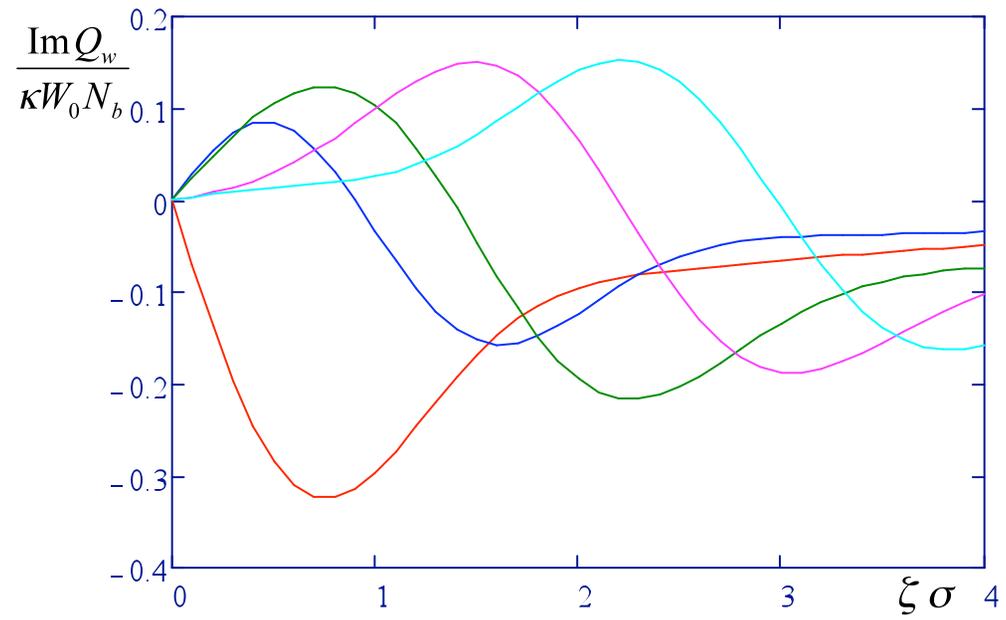
- Multi-turn (and bunch-to-bunch) wake can be taken into account for the space charge modes in the very same manner, how that is done in the conventional no-space-charge theory (see e. g. A. Chao, “Physics of Collective Beam Instabilities...”, p. 360).
- Similar to the conventional case, the dependence on the sign of chromaticity is less pronounced for coupled-bunch modes, compared with the single-bunch case. The higher is the chromaticity – the smaller is the growth rate.

Summary

- A theory of head-tail modes is presented for space charge tune shift significantly exceeding the synchrotron tune and the coherent tune shift.
- A general equation for the modes is derived, its spectrum is analyzed.
- Landau damping is calculated, both without and with lattice nonlinearities.
- TMCI is shown to have high threshold due to a specific structure of the coherent spectrum of the space charge modes.
- The theory needs to be checked with simulations (Schottky noise and stability) and measurements.
- The results can be applied for any ring with bunched beam and significant space charge (Project X, SIS100, CERN).

Coherent rates for Gaussian bunch and resistive wake





Same as previous slide, but for the constant wake.

TMCI Resistive wake, Gaussian bunch

