Conditions for SC Cavity Matching and Their Relations to the Robinson Stability Criterion

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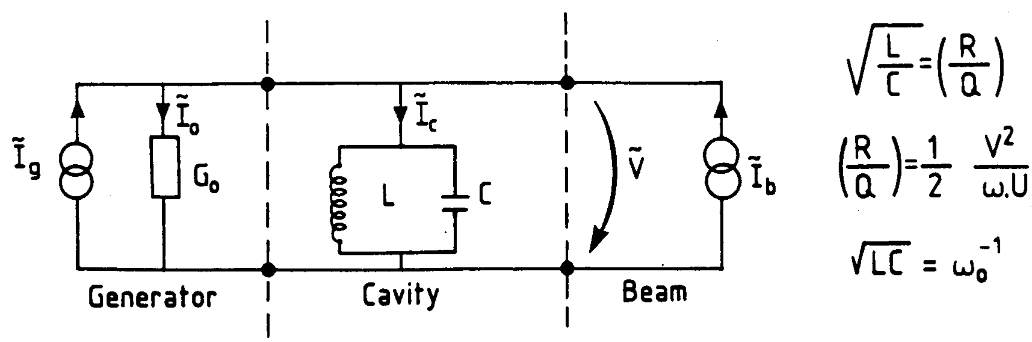
A superconducting cavity fed via an isolator from an rf generator with frequency ω and accelerating a bunched beam synchronous with ω can be described by the circuit diagram of Figure 1.



I_b is the rf beam current at ω and V the accelerating voltage. (Cavity losses are neglected.) Kirchoff's laws demand for the currents:

$$I_g + I_b = I_o + I_c \tag{1}$$

and defining

$$I_o + I_c = I_t \tag{2}$$

we can depict these two phasor equations by the two diagrams of Figs. 2a and 2b.

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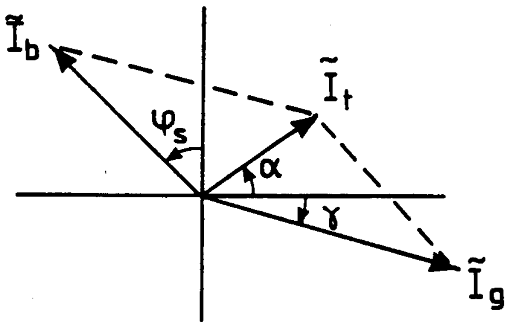

Fig. 2a

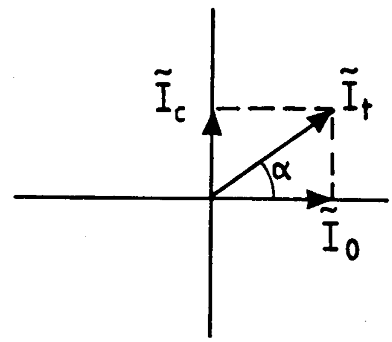

Fig. 2b

In drawing these diagrams we have chosen the time zero such that I_0 and hence $V = I_0 \times R_0$ become real quantities.

$$I_c = j\Omega\sqrt{\frac{C}{L}}\,V = j\,\omega\,V/(R/Q),\quad \Omega = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \qquad (3)$$

is then imaginary and I_b and I_g are complex currents.

These phasor-diagrams are specified if two amplitudes and angles are given, for instance I_b with the synchronous phase angle φ_s and I_t with the tuning angle α. The generator current I_g and its angle γ can then be calculated [1] from:

$$\mathrm{tg}\gamma = \frac{I_b \cos\varphi_s - I_t \sin\alpha}{I_b \sin\varphi_s + I_t \cos\alpha} \qquad (4)$$

$$I_g = \frac{I_b \sin\varphi_s + I_t \cos\alpha}{\cos\gamma} \qquad (5)$$

If we now want to match cavity and beam to the generator then gamma must be made zero to bring generator current and voltage in phase. Evidently, this demands (see eqn. 4 and Fig. 2b) a reactive beam current of

$$I_b \cos\varphi_s = I_t \sin\alpha = I_c = \Omega V/(R/Q) \qquad (6)$$

We must detune the cavity by Ω such that I_c compensates the reactive component of the beam current. With this compensation done (eqn. 5) requires the resistive beam current

$$I_b \sin \varphi_s = I_g - I_t \cos \alpha = I_g - I_o$$

The generator now "sees" a real load but for a match this load must draw the current $I_g/2$ which implies $I_o = I_g/2$. It follows

$$I_b \sin \varphi_s = \frac{I_g}{2} = I_o = I_t \cos \alpha \qquad (7)$$

The phasor diagram which satisfies eqns. (6) and (7) i.e. represents the matched case, is remarkably symmetric. (One aspect of this symmetry is the simple relation between synchronous phase angle φ_s and tuning angle α: $\alpha = 90° - \varphi_s$

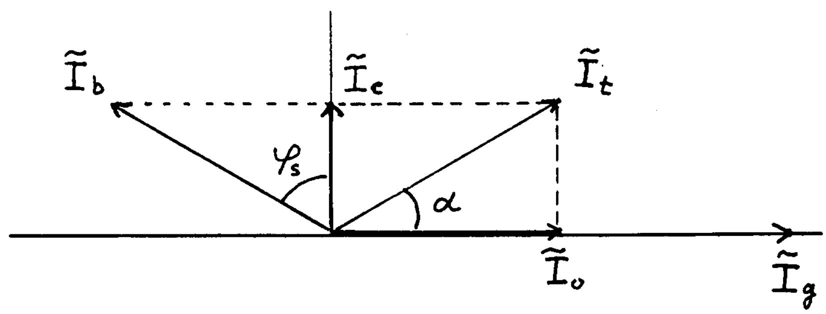

Figure 3: Phasor diagram for the matched case.

We will now combine (6) and (7) into a form which allows direct comparison with the two Robinson stability criteria [2]:

$$0 < \sin 2\alpha \qquad \text{(a)}$$

and

$$\sin 2\alpha < 2\,(I_o/I_b)\cos \varphi_s \qquad \text{(b)}$$

If an automatic loop is used to do the reactive compensation tuning then condition (a) is always satisfied. Whether this is also true for condition (b) if we want a match and choose a resistive beam current satisfying eqn. (7) shall now be discussed.

By combining eqns. (6) and (7), or by direct inspection of the diagram in

Fig. 3, we find:

$$\tan\alpha = \frac{I_c}{I_o} = \frac{I_b}{I_o}\cos\varphi_s \qquad (8)$$

and

$$I_o = I_b \sin\varphi_s \qquad (9)$$

squaring eqn. (9) $\qquad I_o^2 = I_b^2\,(1-\cos^2\varphi_s)$

$$\Rightarrow 1 = \frac{I_b^2}{I_o^2} - \frac{I_b^2}{I_o^2}\cos^2\varphi_s$$

$$\Rightarrow \frac{I_o}{I_b} = \frac{I_b/I_o}{1 + \frac{I_b^2}{I_o^2}\cos^2\varphi}$$

multiplying both sides by 2 cos φ_s and using eqn. (8) we obtain

$$2\,\frac{\tan\alpha}{1+\tan^2\alpha} = \sin 2\alpha = 2(I_o/I_b)\cos\varphi_s \qquad (10)$$

Comparing eqn. (10) with condition (b) we see that increasing the beam current (for a given cavity coupling [1]) to the value which produces the match brings us just to the limit of the stable region.

The physics behind this phenomenon is simple. For a fixed drive signal the generator delivers under matched conditions its available, i.e., maximum power which flows, if cavity losses are negligible, totally into the beam. Any deviation of the beam current's phase angle from its steady-state synchronous value can only diminish the beam power which thus cannot

1 The coupling Q_{ex} for cavity-generator matching is derived in the appendix.

change proportionally to φ_s as required for phase focussing and correction of the perturbation.

For a matched copper cavity-beam-generator system in contrast and with a substantial part of the available power spent as dissipation the beam power is free to change in the required way even though the total power diminishes.

One can also say that a matched superconducting cavity system works under heavy steady-state beam loading and active methods [3] to stabilize amplitude and phase of the cavity voltage become essential to ensure stability. Especially effective is rf feedback, the feasibility of which with a 4-cell superconducting LEP cavity has been recently demonstrated. [4,5]

References

[1] F. Pedersen, IEEE Trans. Nucl. Sci., NS-32, No. 5 (1985).
[2] K. W. Robinson, CEA, Report CEAL-1010 (1964).
[3] D. Boussard, IEEE Trans. Nucl. Sci., NS-32, No. 5 (1985).
[4] D. Boussard, A. P. Kindermann, V. Rossi, CERN/SPS 88-28 (ARF).
[5] Ph. Bernard, et al. Proceedings of the European Particle Accelerator Conference, EPAC, Rome, Italy, 7-11 (June 1982).

Appendix

Multiply eqn. (7) by V/2 and find

$$\frac{1}{2} V I_b \sin\varphi_s = \frac{1}{2} V I_g/2$$

At the left of this equation we have the beam power and at the right the available generator power. Both are shown to be equal as we expect. Now divide eqn. (7) by V and find

$$R_o^{-1} = I_o/V = \frac{I_b \sin\varphi_s}{V} = \frac{2 P_b}{V^2}$$

$$\Rightarrow Q_{ex} = \frac{R_o}{\sqrt{L/C}} = \frac{V^2}{2 P_b (R/Q)}$$