A DIFFERENT TUNING METHOD FOR ACCELERATING CAVITIES

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Abstract Superconducting structures ($\beta = 1$) are tuned by deformation of the individual cells. This procedure changes the cell frequency and the coupling factor between cells as well. Both parameters are evaluated in a lumped circuit element model to be used for tuning purpose. A description of the mathematical model and tuning experience are presented.

INTRODUCTION

The tuning procedure presented here has been developed for 4-cell 500 MHz superconducting accelerating structures for HERA. The frequency adjustment of each cell is done by inelastic cell deformation. One should reduce the number of tuning steps to avoid unwanted material stresses. We expect that at least 16 cavities have to be tuned in the foreseeable future and then a less time consuming adjustment procedure should be employed. For both reasons we have tried to develope a method which leads in very few steps to the required status of the cavity.

MATHEMATICAL METHOD

The method described in this paper is based on "reconstruction" of an uniquely determined matrix, the spectrum $\lambda_1 \dots \lambda_N$ and eigenvectors $A_1 \dots A_N$ of which are known. Each monopole passband of N-cell accelerating cavity consists of N resonant modes. Their resonant frequencies and field distributions (e.g. E_{acc} on axes) are related respectively to the eigenvalues and the eigenvectors of the uniquely determined square matrix M of order N. Elements m_{ij} of the matrix M depend on a current status of the cavity i.e. frequencies of the cells and coupling between them. If a lumped element replacement circuit is used for a description of the cavity passband, status of the cavity means: current values of all lumped elements. Functions f_{ij} relating m_{ij} to those elcments are defined for applied replacement circuit and then solution of the set of equations:

$$m_{ij} = f_{ij}\,(\text{model parameters}) \quad \text{for } i,j \leq N \tag{1}$$

should deliver their values. It is not a trivial problem to solve (1). If N*N < number of model parameters,usually there is no unique solution. Even for the opposite case but for large N, relevant coupling between next-neighbour cells or mixed capacitive-inductive coupling between the cells, the situation becomes quite complicated. Let us assume that we have a method to solve (1) and to find model parameters for each cavity status, as we will later show for N = 4 and only capacitive coupling between the cells. By the matrix M' we will denote the matrix which corresponds to the adjusted cavity. We can now write equations which have to be fulfilled by the matrix M'. These equations result from the eigenvalue problem of this matrix:

$$M'A_0 - \Lambda(F_0)A_0 = 0 \tag{2}$$

where $A_0 = (a_{01} .. a_{0N})$ is the required field distribution of an accelerating mode. F_0 is the required frequency of this mode. Function Λ is given explicitly together with the functions f_{ij} for chosen lumped element replacement circuit. We have N equations which must be satisfied by elements m_{ij} of the matrix :

$$\sum_{j=1}^{N} m'_{ij}\, a_{0j} - \Lambda(F_0)\, a_{0i} = 0 \qquad i = 1 \ldots N \tag{3}$$

If the adjustment of the cavity is accompanied with the change of N selected model parameters: $P_1 \ldots P_N$, we can rewrite (3) in the new form:

$$\sum_{j=1}^{N} f_{ij}(P'_1 \ldots P'_N, \text{other model param.}) \cdot a_{0i} - \Lambda(F_0)\, a_{0i} = 0 \tag{4}$$

where : $P'_1 \ldots P'_N$ are the values of selected parameters, satisfying equation (4). If we can solve (4), the differences between the initial value of P_i and the final P'_i for each of the chosen parameter gives the change of this parameter necessary for the adjustment of the cavity.

LUMPED ELEMENT REPLACEMENT CIRCUIT

The lumped element replacement circuit of 4-cell is shown in Fig.1

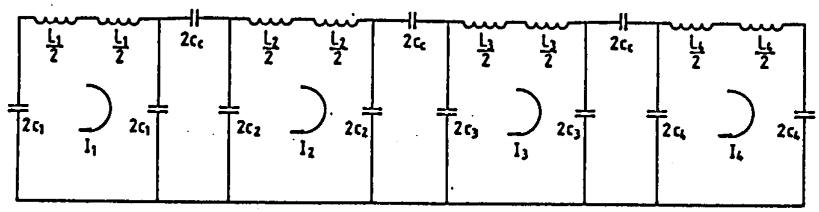

Fig.1 Lumped element replacement circuit of a 4-cell cavity (c_i, l_i - capacitance and inductance of cell no. "i", C_c - coupling capacitor)

The mesh current method applied to currents $I_1 \ldots I_4$ leads to the following eigenvalue problem:

$$\begin{vmatrix} f_{11} & f_{12} & 0 & 0 \\ f_{21} & f_{22} & f_{23} & 0 \\ 0 & f_{32} & f_{33} & f_{34} \\ 0 & 0 & f_{43} & f_{44} \end{vmatrix} A_S - 2\Omega_S^2 A_S = 0 \qquad (5)$$

where: $A_S = (a_{1S}, a_{2S}, a_{3S}, a_{4S})$ and Ω_S are respectively eigenvector and angular frequency of the resonant mode "s", and s = 1 ... 4. The functions f_{ij} are given by formulae:

$$f_{11} = \omega_1^2 \cdot \left(2 - \frac{K_1}{1 + K_1 + K_2} \right) \qquad (6.1)$$

$$f_{22} = \omega_2^2 \cdot \left(2 - \frac{K_2}{1 + K_1 + K_2} - \frac{K_2}{1 + K_2 + K_3} \right) \qquad (6.2)$$

$$f_{33} = \omega_3^2 \cdot \left(2 - \frac{K_3}{1 + K_2 + K_3} - \frac{K_3}{1 + K_3 + K_4} \right) \qquad (6.3)$$

$$f_{44} = \omega_4^2 \cdot \left(2 - \frac{K_4}{1 + K_3 + K_4} \right) \qquad (6.4)$$

$$f_{ij} = - \frac{\omega_i^2 K_j}{1 + K_i + K_j} \quad \text{for } |i-j| = 1 \qquad (6.5)$$

where: $K_j = C_c / C_j$ is the coupling factor, and $\omega_j = 2\pi f_j$ is an angular eigenfrequency of the cell no. "j". Function $\Lambda(F_S) = 8\pi^2 F_S^2$ gives the relationship between eigenvalues and resonant frequencies.

MEASUREMENTS OF A_i AND RECONSTRUCTION OF MATRIX M

Standard frequency and beadpull measurements give Ω_S and absolute values: $(|a_{1S}|, |a_{2S}|, |a_{3S}|, |a_{4S}|)$ of all resonant modes. To find matrix M the elements of which m_{ij} are current values of the functions f_{ij}' , one has to know also a phase relationship between elements a_{js} of each eigenvector A_s . For a strongly detuned cavity it may be necessary to measure also phases, but for typical machining imperfections phases are usually similar to those for the well tuned cavity. Phases of all 4 resonances for an adjusted cavity are shown in Table 1. In this Table we assumed the following sequence of resonant modes: $\Omega_1 < \Omega_2 < \Omega_3 < \Omega_4$

TABLE I Phase relationships of the cells at different modes

Mode \ Cell	1	2	3	4
Ω_1	+	+	+	+
Ω_2	-	-	+	+
Ω_3	-	+	+	-
Ω_4	+	-	+	-

Matrix M is uniquely defined by 4 sets of linear equations:

$$\begin{vmatrix} A_1^T \\ A_2^T \\ A_3^T \\ A_4^T \end{vmatrix} \times \begin{vmatrix} m_{i1} \\ m_{i2} \\ m_{i3} \\ m_{i4} \end{vmatrix} = \begin{vmatrix} 2\Omega_1^2 a_{i1} \\ 2\Omega_2^2 a_{i2} \\ 2\Omega_3^2 a_{i3} \\ 2\Omega_4^2 a_{i4} \end{vmatrix} \quad \text{for } i = 1 \ldots 4 \qquad (7)$$

Having all elements m_{ij} one can compute the current values of the model parameters.

THE PARAMETERS AND ACCURACY OF THE MODEL

We can now formulate equations, similary to (1), to find current model parameters. For this we will equate (6) and the corresponding elements of matrix M :

$$m_{ij} = f_{ij} \quad \text{for } |i-j| \le 1 \qquad (8)$$

This set of 10 equations with 8 unknowns: $\omega_1 \ldots \omega_4 , K_1 \ldots K_4$ one can solve in the following way. Six equations from (8) for $|i-j| = 1$ give expressions relating $\omega_1 \ldots \omega_4$ to the $K_1 \ldots K_4$

$$\frac{K_j}{1 + K_i + K_j} = - \frac{m_{ij}}{\omega_j^2} \qquad (9)$$

where: $|i-j| = 1$ and $i,j \le 4$. We can re-write the remaining four equations substituting all fractions $\frac{K_j}{1+K_i+K_j}$ for right sides of (9). This leads to 4 equations only with 4 unknowns: $\omega_1, \omega_2, \omega_3, \omega_4$ which can be solved easily:

$$\omega_2^2 = \frac{m_{23}\cdot m_{32} - s_1\cdot s_2}{z_2\cdot m_{32} - s_2\cdot z_1} \qquad \omega_3^2 = \frac{m_{32}}{z_1 - \frac{s_1}{\omega_2^2}}$$

$$\omega_4^2 = \frac{m_{44}}{2 + \frac{m_{34}}{\omega_3^2}} \qquad \omega_1^2 = \frac{m_{11}}{2 + \frac{m_{21}}{\omega_2^2}} \tag{10}$$

$$\text{where: } s_1 = \frac{m_{12}\cdot m_{21}}{m_{11}} - m_{22} \quad ; \quad s_2 = \frac{m_{43}\cdot m_{34}}{m_{44}} - m_{33}$$

$$z_1 = -2\cdot\left(1 + \frac{m_{12}}{m_{11}}\right) \quad ; \quad z_2 = -2\cdot\left(1 + \frac{m_{43}}{m_{44}}\right)$$

To find $K_1 \ldots K_4$ from 6 equations, we will use the least square method which seems to be better than the analytic one. Using the analytic method one should select 4 equations from (9). Because of unavoidable measurement errors and finite accuracy of applied lumped element approximation it is better to use a method in which all 6 equations are taken into account. The least square method gives 4 equations which take the form:

$$\frac{\partial \gamma}{\partial K_i} = 0 \qquad i = 1 \ldots 4 \tag{11}$$

$$\text{where: } \gamma = \sum_{|i-j|=1} \left(\frac{K_j}{1+K_i+K_j} + \frac{m_{ij}}{\omega_j^2} \right)^2$$

We can re-write them in the matrix form:

$$\begin{vmatrix} r_1^2+(1-r_2)^2 & r_1(r_1-1)+r_2(r_2-1) & 0 & 0 \\ r_1(r_1-1)+r_2(r_2-1) & (1-r_1)^2+r_2^2+r_3^2+(1-r_4)^2 & r_3(r_3-1)+r_4(r_4-1) & 0 \\ 0 & r_3(r_3-1)+r_4(r_4-1) & (1-r_3)^2+r_4^2+r_5^2+(1-r_6)^2 & r_6(r_6-1)+r_5(r_5-1) \\ 0 & 0 & r_6(r_6-1)+r_5(r_5-1) & (1-r_5)^2+r_6^2 \end{vmatrix} \times \tag{12}$$

$$\times \begin{vmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{vmatrix} = \begin{vmatrix} r_2(1-r_2)-r_1^2 \\ r_1(1-r_1)+r_4(1-r_4)-r_2^2-r_3^2 \\ r_3(1-r_3)+r_6(1-r_6)-r_4^2-r_5^2 \\ r_5^2(1-r_5)-r_6^2 \end{vmatrix}$$

where: $r_1 = -\frac{m_{12}}{\omega_1^2}$; $r_2 = -\frac{m_{21}}{\omega_2^2}$; $r_3 = -\frac{m_{23}}{\omega_2^2}$; $r_4 = -\frac{m_{32}}{\omega_3^2}$; $r_5 = -\frac{m_{34}}{\omega_3^2}$; $r_6 = -\frac{m_{43}}{\omega_4^2}$

The formulae (9) and solution of (12) give all model parameters.
The applied lumped element circuit describes the cavity passband by tridiagonal matrix. Usually, "reconstructed" matrix M is not the tridiagonal one, because of a finite accuracy of the model and the measurement errors. We can compute back eigenvector and eigenfrequencies of the resonant modes, taking into account only three diagonals of matrix M.. Differences ΔF_S between the measured values and the computed values will be used as corrections in further computations.

ADJUSTMENT OF THE CAVITY

The aim of the adjustment is to get for the highest resonant mode a distribution of the electric accelerating field on axes equal $A_0=(-1,1,-1,1)$ at the required Ω_0. Equations (3) take the following form:

$$\begin{aligned} m'_{11}-m'_{12}-2\Omega_0^2&=0\\ -m'_{21}-m'_{23}+m'_{22}-2\Omega_0^2&=0\\ -m'_{32}-m'_{34}+m'_{33}-2\Omega_0^2&=0\\ -m'_{43}+m'_{44}-2\Omega_0^2&=0 \end{aligned} \qquad (13)$$

As it was mentioned above the cell frequency adjustment is effected through a change of the cell shape (cell length). The change of length mainly influences electric field and then cause a variation of capacitors $c_1 \dots c_4$. Changing capacitor c_i we get change of ω_i and K'_i. The following formulae show the relation between these quantities:

$$\omega_i^2=\frac{1}{L_i c_i}=\frac{c_c}{L_i c_i c_c}=K_i\omega_{ci}^2 \qquad (14)$$

where: $\omega_{ci}^2=\dfrac{1}{L_i c_c}$, $i=1\dots 4$

are assumed to be the model parameters unchanged by the tuning. Combining (14) and (13) one gets 4 nonlinear equations with 4 unknowns $K'_1 \dots K'_4$:

$$\begin{aligned} &K'_1\left[2-\frac{K'_1}{1+K'_1+K'_2}\right]+\frac{K'_2K'_1}{1+K'_1+K'_2}=\frac{2\Omega_0^2}{\omega_{c1}^2}\\ &K'_2\left[2-\frac{K'_2}{1+K'_2+K'_3}-\frac{K'_2}{1+K'_1+K'_2}\right]+\frac{K'_1K'_2}{1+K'_1+K'_2}+\frac{K'_2K'_3}{1+K'_2+K'_3}=\frac{2\Omega_0^2}{\omega_{c2}^2}\\ &K'_3\left[2-\frac{K'_3}{1+K'_2+K'_3}-\frac{K'_3}{1+K'_3+K'_4}\right]+\frac{K'_2K'_3}{1+K'_2+K'_3}+\frac{K'_3K'_4}{1+K'_3+K'_4}=\frac{2\Omega_0}{\omega_{c3}^2}\\ &K'_4\left[2-\frac{K'_4}{1+K'_3+K'_4}\right]+\frac{K'_3K'_4}{1+K'_3+K'_4}=\frac{2\Omega_0^2}{\omega_{c4}^2} \end{aligned} \qquad (15)$$

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The solution of (15) gives the new values of $K'_1 \ldots K'_4$ and then from (14) required values of $\omega'_1 \ldots \omega'_4$.
We have now two sets of model parameters. The first one $K_1 \ldots K_4$ $\omega_1 \ldots \omega_4$ for the initial status of the cavity and the second one $K'_1 \ldots K'_4$ $\omega'_1 \ldots \omega'_4$ for the tuned cavity.
During adjustment of each cell we can observe a change of all 4 resonant frequencies. Replacing stepwise in (6) pairs (K_i, ω_i) by (K'_i, ω'_i) we get 4 matrixes the spectra of which show resonant frequencies after each cell has been adjusted. The computed frequencies values should be corrected with ΔF_s for $s = 1 \ldots 4$ We then can control easily adjustment procedure of each cell by simple frequency measurements. This procedure was tested on the copper model of the 500 MHz cavity. Usually one or two steps were necessary to reach flat field profile and required frequency of the accelerating mode.
An example of the whole tuning procedure of the copper model is given below.

STATUS OF THE CAVITY BEFORE TUNING

Figure 2 shows field profiles of all 4 modes and Table 2 contains their resonant frequencies before the tuning.

TABLE 2

Mode	Pi/4	Pi/2	3/4*Pi	Pi
F [MHz]	490.803	493.804	497.015	499.022

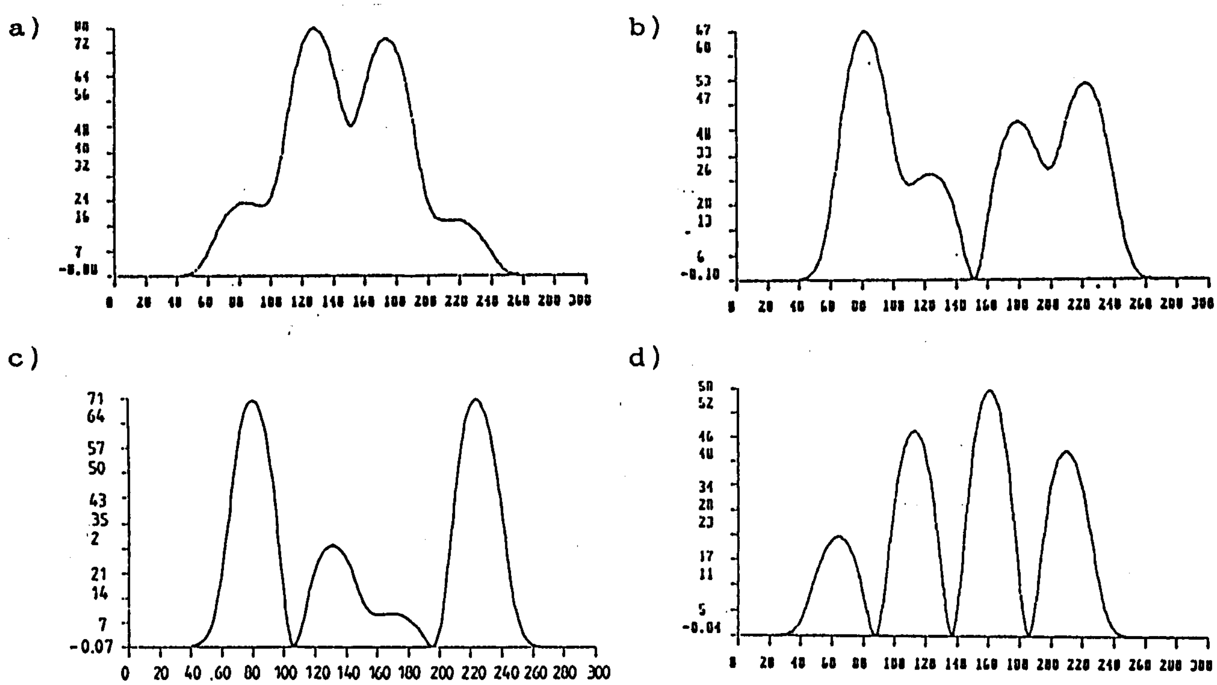

Fig.2 $(E_{acc})^2$ of 4 modes : Pi/4 (a),Pi/2 (b),3/4Pi (c),Pi (d).

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THE TUNING ADVISE (COMPUTED)

As the required Pi mode frequency we assumed F_o=499.584 MHz.

Resonant mode when the cell 1.is corrected

Resonans No = 1 is at the F = 490.929 (MHz)
Resonans No = 2 is at the F = 494.287 (MHz)
Resonans No = 3 is at the F = 497.665 (MHz)
Resonans No = 4 is at the F = 499.408 (MHz)
The highest mode norm. E_{acc} = (-1.0000, +0.9333, -0.7960, +0.5981)

Resonant mode when the cell 1 and 2 are corrected

Resonans No = 1 is at the F = 490.922 (MHz)
Resonans No = 2 is at the F = 494.282 (MHz)
Resonans No = 3 is at the F = 497.664 (MHz)
Resonans No = 4 is at the F = 499.402 (MHz)
The highest mode norm. E_{acc} = (-1.0000, +0.9312, -0.7965, +0.5994)

Resonant mode when the cell 1 , 2 and 3 are corrected

Resonans No = 1 is at the F = 490.919 (MHz)
Resonans No = 2 is at the F = 494.281 (MHz)
Resonans No = 3 is at the F = 497.663 (MHz)
Resonans No = 4 is at the F = 499.401 (MHz)
The highest mode norm. E_{acc} = (-1.0000, +0.9306, -0.7947, +0.5984)

Resonant mode when all cells are corrected

Resonans No = 1 is at the F = 491.007 (MHz)
Resonans No = 2 is at the F = 494.573 (MHz)
Resonans No = 3 is at the F = 498.110 (MHz)
Resonans No = 4 is at the F = 499.584 (MHz)
The highest mode norm. E_{acc} = (-0.9983, +0.9988, -1.0000, +0.9994)

STATUS OF THE CAVITY AFTER TUNING

Figure 3 shows the field profile of the Pi mode after tuning.

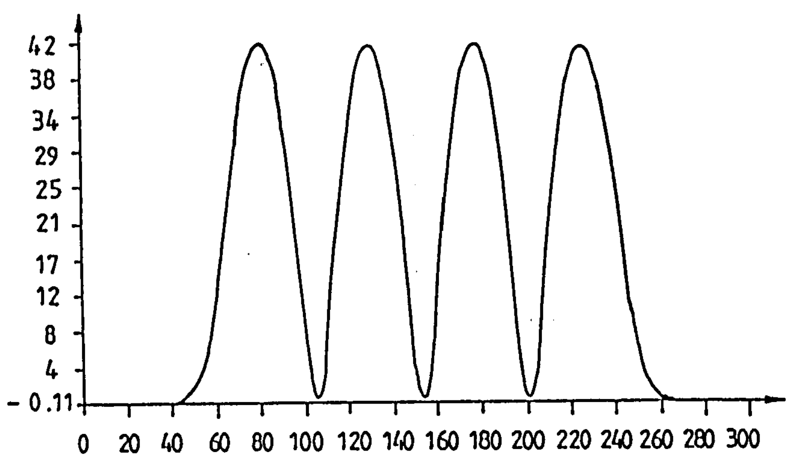

Fig.3 $(E_{acc})^2$ distribution of the Pi mode after the tuning.

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Table 3 gives measured frequencies of all modes and differences $\delta F = F_{measured} - F_{computed}$.

TABLE 3 : Frequencies of all modes (as measured) after tuning.

Mode	Pi/4	Pi/2	3/4*Pi	Pi
F [MHz]	491.027	494.539	498.105	499.583
δF[kHz]	20	-34	-5	-1

ACKNOWLEDGEMENT

We want to thank D. Proch and S. Kulinski for helpful discussion and remarks.