THEORY OF Q-DEGRADATION AND NONLINEAR EFFECTS IN Nb-COATED SUPERCONDUCTING CAVITIES

I.O.Kulik [a)], and V.Palmieri [b)]

[a] Department of Physics, Bilkent University, Ankara, Turkey

[b] Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Legnaro
Padova, Italy

Abstract- Amplitude-dependent absorption of RF power in superconducting cavity with a wall of normal metal (Cu) covered by a film of superconductor (Nb) is determined by three factors: (1)depairing effect of high-current density on superconducting energy gap; (2)increase of electromagnetic power penetrating to normal metal and Drude-absorbed in it, at higher RF amplitude; (3)heating of both superconducting coating and normal substrate resulting in increase of quasiparticle excitation density at higher RF power (P). Cavity Q-factor is calculated as a function of RF amplitude $A=\sqrt{P}$ and is shown to follow a relationship $\ln(Q/Q_0)=-const\cdot P^{\alpha}$ with α=1 at low temperature $T<T^*$ and α=1/2 at $T>T^*$ where T^* is characteristic temperature $T^*\approx T_C\delta/d$, δ is the London penetration depth and d the thickness of superconducting coating.

Superconducting cavities for particle accelerators allow to achieve values of the quality factors $Q\approx 10^{10}$ at RF electric field amplitudes $V_{ac}\approx 10MV/m$ [1,2]. Similar parameters are realized in cavities made by sputtering of Nb over the surface of Cu [3]. Nb-coated cavities can have even higher values of Q at small electric field but show more fast Q degradation with the increasing amplitude. In present paper, we try to understand factors which limit the ultimate performance of coated cavities. We show that depression of Q results from the depairing effect of the RF current, from the residual amplitude-dependent dissipation of a.c. electromagnetic power at the interface between N and S layers, and from the inhomogeneity and defects in a superconducting coating.

By using the local electrodynamics of superconductivity, an expression for the quality factor is received

$$Q=\frac{c^3}{8\delta_L^3\omega^2\sigma_2} \qquad (1)$$

showing the $1/\omega^2$ dependence on frequency ω [4]. δ_L is the London penetration depth of superconductor and σ_2 the imaginary part of complex conductivity $\sigma(\omega)=\sigma_1(\omega)+i\sigma_2(\omega)$. In a bilayer system, Q can be expressed through the reflection coefficient R of the RF power from a cavity wall

$$Q = \frac{\pi}{1-R}. \qquad (2)$$

Dependence of δ_L , σ_2 and R on the amplitude of RF field determine the amplitude dependence of the quality factor. For determination of R, we solve Maxwell equation for the electromagnetic field inside the cavity

$$-\frac{\partial^2 A}{\partial z^2} = \frac{4\pi}{c} j, \quad H = \frac{\partial A}{\partial z}, \quad E = -\frac{1}{c}\frac{\partial A}{\partial t} \qquad (3)$$

with the current density in a wall

$$j = -(n_s e^2 / mc)A + \sigma_2 E \quad \text{at} \quad z < d \qquad (4)$$

$$j = \sigma_n E \quad \text{at} \quad z > d$$

where d is the thickness of superconducting coating. Electromagnetic field inside the cavity at distance z from its surface equals $A = e^{ikz} - re^{-ikz}$ where r is the reflection amplitude, and $R = |r|^2$. We receive

$$r = \frac{(k_1 - k_2)e^{-kd_1}(1 - ik/k_1) - (k_1 + k_2)e^{k_1 d}(1 + ik/k_1)}{(k_1 - k_2)e^{-k_1 d}(1 + ik/k_1) - (k_1 + k_2)e^{kd_1}(1 - ik/k_1)} \qquad (5)$$

with

$$k_1 = \sqrt{\delta_L^{-2} - 2i\delta_{sk}^{-2}}, \quad k_2 = (1-i)\delta_{sn} \qquad (6)$$

where δ_{sk} is the skin penetration depth appropriate to normal conductivity of superconductor σ_2 , and δ_{sk} is skin depth of a normal metal

$$\delta_{sk} = (c^2 / 2\pi\sigma_2\omega)^{1/2}, \quad \delta_{sn} = (c^2 / 2\pi\sigma_n\omega)^{1/2}. \qquad (7)$$

Penetration of an a.c. field into superconductor is described with the introduction of a complex penetration depth δ according to

$$\frac{1}{\delta^2} = \frac{1}{\delta_L^2} - \frac{2i}{\delta_{sk}^2} \qquad (8)$$

Two factors determine the amplitude dependence of δ: (1)The depairing effect of finite current; (2)Increase of σ_2 due to the decreasing energy gap Δ.

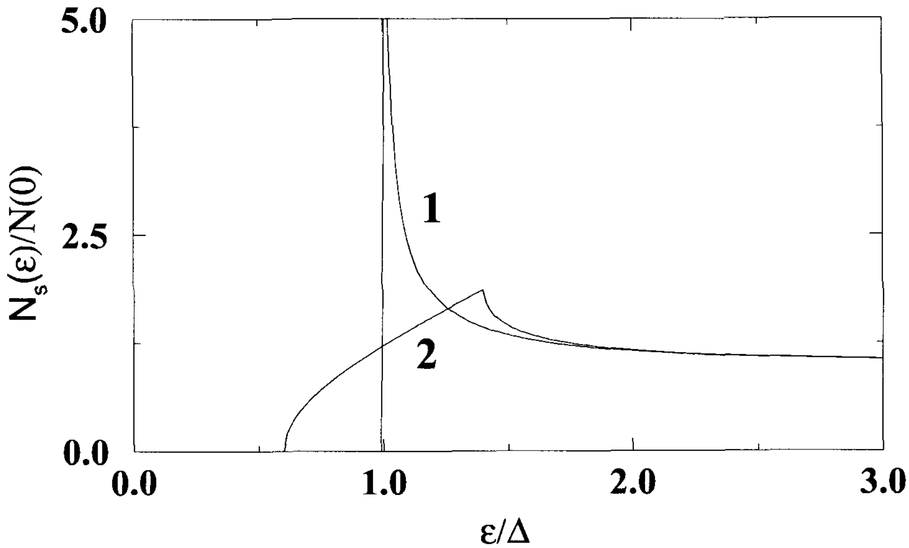

Fig.1. Density of states in the BCS superconductor at $j = 0$ (curve 1), and in a current-carrying state corresponding to $p_F v_s / \Delta = 0.4$ (curve 2).

In the current-carrying state, BCS density of states $N(\varepsilon)$ changes from the an expression

$$N(\varepsilon) = |\varepsilon| / \sqrt{\varepsilon^2 - \Delta^2}\, \vartheta(|\varepsilon| - \Delta)$$

(Fig.1,curve 1) to the dependence

$$\frac{N_s(\varepsilon)}{N(0)} = \begin{cases} [\sqrt{(\varepsilon + p_F v_s)^2 - \Delta^2} - \sqrt{(\varepsilon - p_F v_s)^2 - \Delta^2}\,]/2p_F v_s, & \varepsilon > \Delta + p_F v_s \\ \sqrt{(\varepsilon + p_F v_s)^2 - \Delta^2}\, / 2p_F v_s, & \Delta - p_F v_s < \varepsilon < \Delta + p_F v_s \\ 0, & \varepsilon < \Delta - p_F v_s \end{cases}$$

shown in curve 2. At the same time, the energy gap in the current-carrying state decreases according to the relation

$$\Delta = \Delta_0 - p_F v_s \qquad (9)$$

where the supervelocity v_s is related to the magnetic field at the surface of superconductor $v_s = eH\delta_L / mc$.

Since the concentration of quasiparticles exponentially decreases with the temperature

$$n_{qp} = \frac{2\Delta}{T} n e^{-\Delta/T} \ln\frac{T}{\omega}, \qquad (10)$$

this results in a decrease of the quality factor

$$Q_1 = Q_1^0 \exp(-e v_F H \delta_L / cT). \qquad (11)$$

Another source of RF power dissipation in the cavity arises from the a.c. electric field penetration to the normal substrate. Nevertheless of the fact that the electric field in the substrate comes to be
very small compared to the a.c. field at surface, it accelerates much greater amount of quasiparticle excitations thus making a comparable contribution to a.c. losses. We a quality factor Q_2 related to this effect

$$Q_2 = Q_2^0 \exp(-2d / \delta_L). \qquad (12)$$

The amplitude dependence of δ_L is determined through the relations

$$\delta_L = (\frac{mc^2}{4\pi n_s e^2})^{1/2}, \qquad n_s = n_s^0(1 - v_s^2 / v_c^2). \qquad (13)$$

In the above equations, Q_1^0, Q_2^0 are the factors

$$Q_1^0 \cong \frac{\lambda \delta_{an}^2}{8\delta_L^3} e^{\Delta/T}, \qquad Q_2^0 \cong \frac{\lambda \delta_{sn}}{32\delta_L^2} e^{2d/\delta_L}. \qquad (14)$$

Since losses related to both mechanisms add, we receive for the overall quality factor

$$Q^{-1} = Q_1^{-1} + Q_2^{-1} \qquad (15)$$

which results in an expression

$$Q = \left[\frac{1}{Q_1^0} \exp\frac{H_{ac}}{H_1} + \frac{1}{Q_2^0} \exp(\frac{H_{ac}}{H_2})^2\right]^{-1} \qquad (16)$$

in which H_1 and H_2 are characteristic amplitudes

$$H_1 = \frac{cT}{e v_F \delta_L}, \qquad H_2 = 1.76 \frac{cT_C}{e v_F \sqrt{d\delta_L}}. \qquad (17)$$

Second term in Eq.(16) is specific to the Nb-coated cavity and does not appear in a cavity of pure Nb. At low temperature, Q_1^0 is much larger than Q_2^0, then $\ln Q$ will linearly decrease with the increasing RF power whereas at low temperature it is expected to scale with the amplitude of RF field. The crossover temperature between these asymptotic regimes, T^*, is determined from the condition $H_1 = H_2$ thus giving

$$T^* = \Delta \frac{\delta_L}{2d} = 0.88 T_C \frac{\delta_L}{d}. \qquad (18)$$

The dependence of Q on RF power is presented in Fig.2. [It can be trusted only qualitatively at large P since vortex nucleation generally starts at currents much smaller than the depairing currents.]

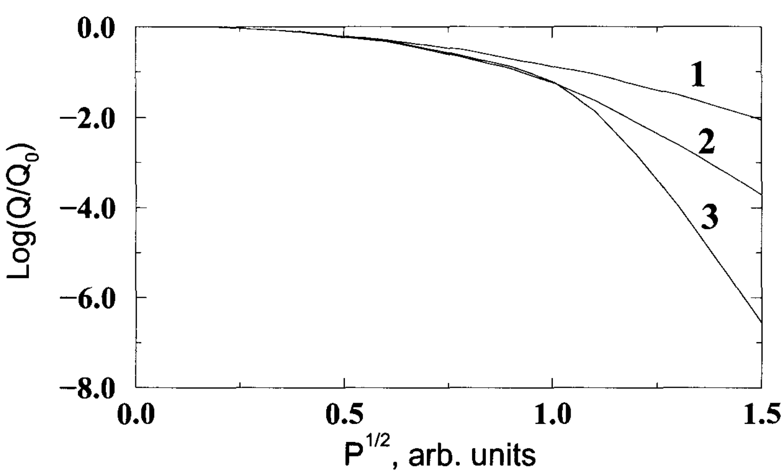

Fig.2. Cavity quality factor Q vs RF power P at $H_1 = 1.33 H_2$, at various temperatures. 1 - $T/T^* = 0.05$, 2 - $T/T^* = 0.1$, 3 - $T/T^* = 1.0$.

At large RF amplitude, the current becomes amplitude dependent owing to the depairing effect of v_s on n_s, Eq.(13). As a result, an a.c. current acquires odd harmonics of RF power 3ω, 5ω, etc. Since nonlinearity is small, only third harmonic may effectively contribute. Since this harmonic is not in resonance, the reflected power at 3ω will be excluded from further amplification and therefore will count to losses in Eq.(15). To estimate this effect, an equation for the vector potential inside a cavity

$$\frac{\partial^2 A}{\partial z^2} - \frac{mc^2}{4\pi n_s^0 e^2}(1 - A^2 / A_c^2) - \frac{4\pi\sigma_2}{c^2}\frac{\partial A}{\partial t} = 0 \qquad (19)$$

is first solved neglecting dissipation. This gives for the spatial dependence of the vector potential

$$A(z) = \frac{A(0)}{\cosh\frac{z}{\delta_L} + \sqrt{1 - A^2(0)/2A_c^2}\,\sinh\frac{z}{\delta_L}}. \qquad (20)$$

By differentiating $A(z)$ with respect to z and putting $z = 0$ we receive the boundary condition for A at the surface

$$(dA/dz)_{z=0} + \frac{A(0)}{\delta_L}\sqrt{1 - A^2(0)/2A_c^2} = 0. \qquad (21)$$

Including the dissipative term and solving perturbatively at $A << A_c$, we receive a contribution to $Q-$factor in Eq.(15)

$$Q_3^{-1} \approx 2(\frac{A(0)}{A_c})^2(\frac{c\delta_L}{\omega})^3. \qquad (22)$$

The last factor is of the order of the ratio of penetration depth to the wavelength of electromagnetic field in vacuum, a very small quantity. We conclude that, practically, there is no effect of harmonic contribution to losses, except possibly at very high frequencies.

The above mechanisms of Q degradation are applicable below the critical magnetic field which is of the order of the threshold field for vortex nucleation.

In a nonuniformly coated, mixed superconductor - normal metal cavities, another mechanism of Q degradation sets in. Assume a model in which superconducting film has small opening (Fig.3,a). Near the opening, supercurrent at small RF amplitude will bypass the normal region and therefore

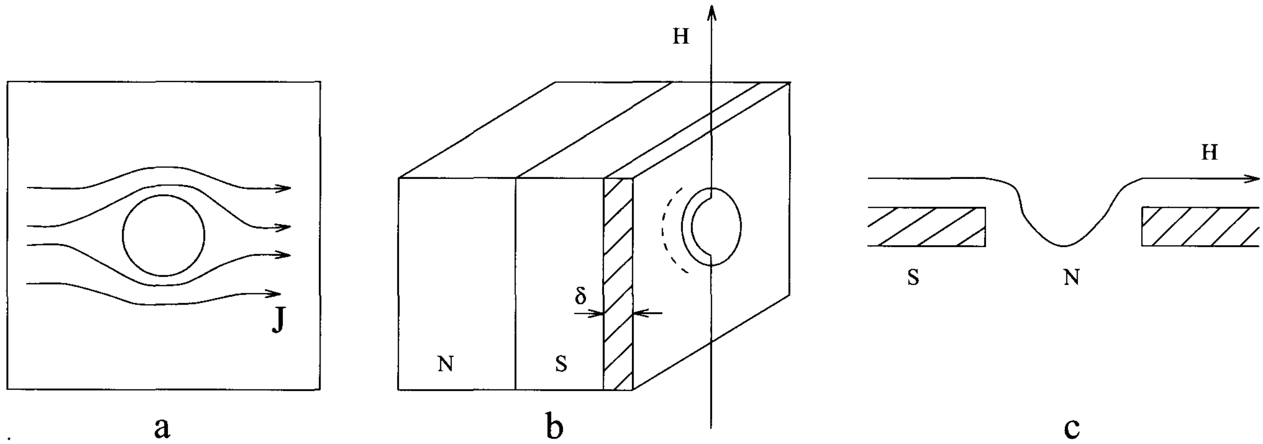

Fig. 3. Magnetic fields and currents near the opening in superconducting coating.
(a) Streamlines of supercurrent, (b,c) lines of force of magnetic field near orifice.

not have significant effect on cavity losses. At higher amplitude, this no longer will be true. The geometry of magnetic lines of force changes from parallel to inclined with respect to surface (Fig.3,c). Therefore the critical magnetic field near the opening will be lower than at the rest of surface. At increasing RF amplitude, this "weak spot" will allow passage of vortex lines resulting in destruction of superconductivity near the spot. The a.c. currents will start flowing through the spot rather than bypassing it resulting in an increase of Joule heating and in subsequent expansion of normal region to the adjacent parts of opening. Since the characteristic Q value of normal-metal cavity $Q \propto 10^5$ is much smaller than superconducting cavity value $Q \approx 10^{10}$, very small fraction of the "spotted" surface $\Delta S / S \approx 10^{-5}$ may significantly reduce the overall quality factor at the increased RF amplitude thus creating a main source of cavity Q degradation.

References

1. G.Fortuna, R.Pengo, A.Lombardi et al., Proc. European Particle Accelerator Conference, vol. **1**, p.43. Eds. P.Martin and P.Mandrillon, 1990.
2. I.Ben-Zvi, E.Chiavery, B.V.Elkonen et al., ibid, p.1103.
3. V.Palmieri, V.L.Ruzinov, S.Yu.Stark et al., Nuclear Instruments and Methods in Physics Research **A328**, 280 (1993).
4. M.Tinkham. Introduction to Superconductivity, 2nd edition, p.40. McGrow-Hill Inc., 1996.