ACCELERATION MECHANISM TO FORM A HOT-ELECTRON SHELL AROUND THE ECR-SURFACE IN AN ECRIS INJECTOR

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Abstract

Hot electron rings have been observed close to the ECR surface by x-ray and other techniques in both axi-symmetric (max-B) and non-axisymmetric (min-B) ECR ion sources (ECRIS). For optimization of ECRIS operation it is imperative to search an adequate theory that can predict what stochastic theory failed. Present paper discusses the ring's formation process, achievable energy, and thickness under an electrostatic (ES) wave theory using a cylindrical geometry and some numerical examples. We demonstrate the process to raise amplitude of the electron-trapping ES wave due to gradual increase of temperature of the background electrons which are accelerated and detrapped by the wave. This process was found to give a radial distribution of hot-electron temperature (T_e) over the interaction region. Our numerical example showed that the higher T_e results if the strength of mirror magnetic field (B_z) is larger in the bulk of ECR-zone, provided that the wave acceleration length (Δr) is same: e.g., T_e =344 keV if B_z =3.50 kG at Δr=0.4 cm. This trend agrees with experiments. Our analysis showed the origin why the shell thickness is in the order of Larmor radius. It is straightforward for present theory to explain those subjects in which stochastic theory has deficiencies: direction of particle rotation in ring, appearance of multiple rings, and a well-developed x-ray spectrum observable immediately after the application of rf-power.

1 INTRODUCTION

We have recently identified a hot-electron ring (or shell) in an ECRIS from a fine-structure radial distribution of the ion-confining negative potential-well which was derived using a set of experimental data of ion endloss current.[1] The shell's mean thickness was found to be 1.7 cm. A scenario given by Golovanivsky[3] was found good to explain existence of such shell. Present paper wishes to develop related theories for further amelioration of ECRIS.

2 STRENGTH OF ES-WAVE ELECTRIC FIELD

If the incident rf wave is an X-wave ($\mathbf{k}\perp\mathbf{B}_o$, $\mathbf{E}_1\perp\mathbf{B}_o$), an ES wave can be excited in the bulk of ECR-zone: EM-to-ES mode conversion. Here we estimate the strength or amplitude ($\hat{E}_1$) of an ES electron-wave of the form $E_1 = \hat{E}_1 e^{i(kr-\omega t)}$. In the limit of $\hat{E}_1 >> v_1 x B_z$ the electron equation of motion, equation of continuity, and Maxwell ($\nabla \bullet D = \rho$) equation may be linearlized, respectively, as

$$-im\omega v_1 = -eE_1 \qquad (1)$$

$$-i\omega n_{e1} = -ik \bullet n_{eo} v_1 \qquad (2)$$

$$ik \bullet \varepsilon_o E_1 = -en_{e1} \qquad (3)$$

Here, $n_e=n_{eo}+n_{e1}$, $v_e=v_o+v_1$, and $E=E_o+E_1$; but $\nabla n_{eo}=v_o=E_o=0$, and the perturbed electron density is $n_{e1} = \hat{n}_{e1} e^{i(kr-\omega t)}$. We can find that the frequency (ω) to satisfy the above set of equations ought to be the electron plasma frequency defined by n_{eo}: $\omega_p \equiv \sqrt{e^2 n_{eo}/\varepsilon_o m}$. Therefore, Eq. (1) gives

$$\hat{E}_1 = \frac{mv_1\omega_p}{e} \qquad (4)$$

This expression can be derived from Eq. (3) as well, if one uses $k=(\omega_{po}/v_1)(n_1/n_o)$ obtainable from Eq. (2). In order to estimate the magnitudes of $\hat{E}_1$ in ordinary condition we assign the average thermal velocity for v_1: i.e., $v_1= (2\kappa T_e/m)^{1/2}$. Then, Eq. (4) has the form:

$$\hat{E}_1 = \frac{\sqrt{2}\kappa T_e/e}{\sqrt{\varepsilon_o \kappa T_e/n_{eo} e^2}} \equiv \frac{\sqrt{2}\kappa T_e/e}{\lambda_D} \qquad (5)$$

Since the Debye wavelength is $\lambda_D(cm)\equiv 7.43x10^2 [T_e(eV)/ n_{eo}(cm^{-3})]^{1/2}$, Eq. (5) has only two unknowns n_{eo} and T_e:

$$\hat{E}_1(V/cm)) = 1.90x10^{-3}\sqrt{n_{eo}(cm^{-3})T_e(eV)} \qquad (6)$$

This tells that $\hat{E}_1$ is quite large even at T_e=100eV (initial T_e) when $n_{eo}\approx 10^{12}cm^{-3}$: $\hat{E}_1 \approx 19\,(kV/cm) = 1.9\,(MeV/m)$. Frequencies of the ES electron Bernstein (Bn) wave are $n\omega_c \le \omega_{Bn} \le (n+1)\omega_c$ if $\omega_c/\omega_p << 1$ for n=1, 2,... while $\omega_c \le \omega_{Bn} \le \omega_h \equiv (\omega_c^2+\omega_p^2)^{1/2}$ if $\omega_c/\omega_p \varepsilon 1$ for n=1. Thus, $\omega_{Bn}\rightarrow\omega_p$ in both extremities: $\omega_c/\omega_p<<1$ and $\omega_c/\omega_p >>1$.

3 ACCELERATION MECHANISM BY ES-WAVE

In order to fit the ECRIS geometry, we have extended the related formulae[3] into cylindrical expressions from the conventional Cartesian treatment. This enables us to consider a cylindrical wave of charged particles. The radial and azimuthal equations of motion of trapped electrons are:

$$\frac{dp_r}{dt} = F_r + \left[p_\theta \bullet \frac{d\theta}{dt}\right] = q[\mathbf{E}+\mathbf{v}x\mathbf{B}]_r + \gamma m_o v_\theta \bullet \frac{v_\theta}{r} \qquad (7)$$

$$\frac{dp_\theta}{dt} = F_\theta - \left[p_r \bullet \frac{d\theta}{dt}\right] = q[\mathbf{E}+\mathbf{v}x\mathbf{B}]_\theta - \gamma m_o v_r \bullet \frac{v_\theta}{r} \qquad (8)$$

Quantities within the large square-brackets in Eqs. (7) and (8) are due to the conversion into cylindrical coordinates. They are the centrifugal and Coriolis forces, respectively; which are small at a large radius. We will neglect both of them here as virtual forces for simplicity. This is justifiable for the case of min-B type ECRIS where interesting wave-particle interaction takes place only at $r\varepsilon r_{UHR}$.

If $\mathbf{E} \langle\langle \mathbf{v} \times \mathbf{B}$, the set of Eqs. (7) and (8) gives a trivial solution of gyration motion of the guiding center around $\mathbf{B}$ with Larmor radius ρ_L: $(r-r_o)^2+(\theta-\theta_o)^2=\rho_L^2$. Thus, we discuss here only the case $\mathbf{E} \rangle\rangle \mathbf{v} \times \mathbf{B}$, consistent with the previous assumption in Section 2, except for the interior to shell where thermalized particles gyrate becasue $\hat{E}_1=0$.

Consider an ES wave propagating in the positive r-direction perpendicular to $\mathbf{B}_z$. Stationary electrons within the phase of negative E_r can be trapped by the wave potential-well, $\phi= -\partial E_r/\partial r= iE_r/k$, and accelerated radially by the $-e\hat{E}_r$ force. Then, Eqs. (7) and (8) can be written as

$$\frac{dv_r}{dt} = -\frac{e}{\gamma m_o}[E_r + v_\theta \times B_z] \qquad (9)$$

$$\frac{dv_\theta}{dt} = -\frac{e}{\gamma m_o}[E_\theta - v_r \times B_z] \qquad (E_\theta = 0) \qquad (10)$$

Although the externally applied E_θ is zero, interaction of v_r with B_z induces a secondary E_θ. We assume a constant V_{ph} for v_r in Eq. (10): $v_r=V_{ph}$. Then, $dv_r/dt=0$ in Eq. (9), which gives $\Delta r(t) \equiv r- r_{UHR} = V_{ph}t$. And from Eq. (10),

$$v_\theta = \frac{e}{\gamma m_o} V_{ph} B_z t \equiv V_{ph}\omega_c^* t, \qquad (\omega_c^* \equiv \frac{\omega_c}{\gamma_\perp}) \qquad (11)$$

which gives

$$\theta = \frac{1}{r}\int v_\theta dt = \frac{V_{ph}\omega_c^* t^2}{2r} \qquad (\gamma_\perp = \frac{V_{ph}}{c}) \qquad (12)$$

Equation (11) indicates that v_θ increases with time and surpasses V_{ph} after one-cyclotron period (T_c), because

$$\frac{v_\theta}{V_{ph}} = \omega_c^* t \ge 1, \qquad \text{if} \qquad t \ge \frac{1}{\omega_c^*} \equiv \frac{T_c}{2\pi}. \qquad (13)$$

Substitution of Eq. (11) into Eq. (9) yields the equation of motion in the frame moving with the wave at V_{ph}:

$$\frac{dv_r}{dt} = -\frac{e}{\gamma m_o}[E_r + V_{ph}\omega_c^* B_z t] \qquad (14)$$

Note that the E_r which oscillates as $\hat{E}_r \cos(kr-\omega t)$ is pointing the negative r-direction during the outward electron acceleration. However, a dc electric field, $V_{ph}\omega_c B_z t$, induced by $v_\theta \times B_z$ interaction is in the positive r-direction. As a result, the radial size of negative-E_r domain shrinks with time, and the potential-well may tilt; thereby detrapping some of electrons from the beginning. The last electron shall be detrapped at the time

$$t_o = \frac{\hat{E}_r}{V_{ph}\omega_c^* B_z} \qquad \text{(Natural Detrapping Time)} \qquad (15)$$

Note that this t_o is independent of T_e since $\hat{E}_r \sim T_e^{1/2}$ and $V_{ph} \sim T_e^{1/2}$. The ES wave propagates farther ($V_{ph} \neq 0$) leaving the detrapped particles behind. Denoting the quantities in the moving frame by ('), the particle velocity after detrapping is $(v_r)' = -V_{ph}$. Therefore, $v_r = (v_r)' + V_{ph} = 0$ in the rest frame, according to the Galilee transform.

Beyond t_o, the last v_θ of detrapped particles must conserve until a collision, although $dv_\theta/dt = 0$ in Eq. (10) since $v_r=0$ after $t \ge t_o$. This azimuthally coasting velocity (v_θ^o) is the maximum one among the particles detrapped during their trip. Using the t_o into Eq. (11) we obtain

$$v_\theta^o = V_{ph}\omega_c^* \bullet \frac{\hat{E}_r}{V_{ph}\omega_c^* B_z} = \frac{\hat{E}_r}{B_z} \equiv v_{ExB} \qquad (16)$$

The v_θ^o after the first trip is known by $\hat{E}_1$ of Eq. (6) with initial T_e (=100 eV, say) since n_{eo} can be assumed quasi-constant. Equation (16) indicates that all particles are de-trapped at the moment when they have just acquired the ExB drift velocity $v_{ExB}(cm/s) = 10^8 \hat{E}_r(V/cm)/B_z(Gauss)$, which is independent of V_{ph}.

The velocity (v_θ) gained by the background particles parallel to the wavefront is irreversible to the wave motion of radial direction, thus heating the background electrons. If $V_{ph} \ge v_\theta^o$, the electron energy corresponding to v_θ^o is:

$$W_{eV}(MeV) = 0.511\left(\frac{1}{\sqrt{1-\beta_{//}^2}} - 1\right), \quad \beta_{//} = \frac{v_{ExB}}{c}. \qquad (17)$$

However, if $V_{ph} \le v_\theta^o$ we should use $W_{eV} = \kappa T_e = m(v_\theta^o)^2$.

Figure 1 shows a spatial evolution of T_e of detrapped

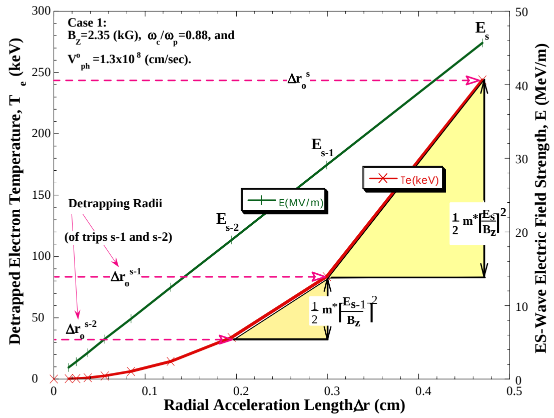

Figure1: Calculation of the detrapped-electron temperature as a function of radial distance.

particles, depicted by total 9-trips. The energy of Fig. 1 was calculated by Eq. (17) with $v_\theta^o = v_{ExB} = \hat{E}_r/B_z$ using the $\hat{E}_r$ upgraded trip by trip. The T_e raised by a trip enables to generate a new wave with a larger $\hat{E}_r$ from the upper hybrid resonance (UHR) surface located at Δr=0. The next trip will achieve a longer acceleration length (Δr). The Δr by the time t_o is, since $t_o=\Delta r_o/V_{ph}$, proportional to $\hat{E}_r$:

$$\Delta r_o \equiv r_o - r_{UHR} = V_{ph} t_o = \frac{\hat{E}_r}{\omega_c B_z} \qquad (v_\theta^o = \omega_c^* \Delta r_o) \qquad (18)$$

On the other hand, time evolution of the T_e of detrapped particles is obtainable by plotting their energy as a function of t_o (=0.123 ns for Case 1) as shown in Fig. 2 of next page.

Let us now consider the case when $\hat{E}_r$ has well grown, so that the natural detrapping radius (r_o) could exceed the ECR radius: $r_o \equiv \Delta r_o + r_{UHR} \geq r_{ECR}$. In such a case, however,

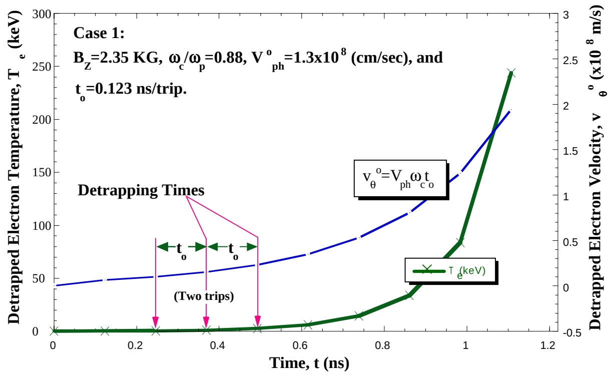

Figure 2: Time development of hot-electron temperature.

the $\hat{E}_r$ is forced to be damped at $r=r_{ECR}$ because Bn-waves are resonant there satisfying: $\omega_{Bn}=\omega_c^{ECR}/q$, where q=1,2, ... Since $\hat{E}_r \to 0$ in Eq. (14), all the particles must be detrapped at once at r_{ECR}. The particle velocity is then $(v_r)'=0$ in the moving frame "moving" at $V_{ph}=0$. Therefore, $v_r=(v_r)'+V_{ph}=0$ in the rest frame. Then, the maximum acceleration time available for charged particles is given by

$$t_{max} = \frac{r_{ECR} - r_{UHR}}{V_{ph}} \quad \text{(Forced Detrapping Time)} \quad (19)$$

This and Eq. (11) give $v_\theta^{max} = V_{ph}\omega_c^* t_{max}$ at ECR surface:

$$v_\theta^{max} = \omega_c^*(r_{ECR} - r_{UHR}) \equiv \omega_c^* \Delta r_o^{max} \qquad (20)$$

Note that the velocity v_θ^{max} is not of the last single particle, but of the all particles started from the UHR-surface satisfying the condition: $\omega_{rf}^2 = \omega_h^2 \equiv \omega_c^2+\omega_p^2$, where ω_c and ω_p take their local values. The energy of $<v_\theta^{max}>^2$ will be deposited around the radius r_{ECR} as $<v_{th}>^2$. Thermalized particles must depict a gyration motion with $\rho_L= v_{rms}/\omega_c \approx <r_{ECR} - r_{UHR}>/\gamma_\perp$, where $v_{rms}\equiv(3\kappa T_e/m)^{1/2}$. This explains theoretical aspect of the shell thickness experimentally observed,[4] and T_e of hot electrons can be estimated from the formula:

$$T_e(\text{eV}) = 0.058\, B_z^2(\text{Gauss})\, \rho_L^2(\text{cm}). \qquad (21)$$

This tells that T_e=530keV for an ECRIS (Constance-B) whose ρ_L=1.7/2 = 0.85 cm as we have derived.[1]

For the evaluation of t_o and t_{max} the information of V_{ph} is essential. It can be calculated from the dispersion relation of Bn-waves[5] in the limit of $k\rho_L$<<1:

$$\omega^2 \approx \omega_h^2 - (k\rho_L)^2\omega_p^2 \equiv \omega_c^2 + \omega_p^2\left\{1-(k\rho_L)^2\right\} \quad (\omega_c/\omega_p>>1) \quad (22)$$

$$\approx 4\omega_c^2\left\{1-\frac{3}{4}(k\rho_L)^2\right\}, \quad \rho_L \equiv \frac{v_{th}}{\omega_c} \quad (\omega_c/\omega_p<<1) \quad (23)$$

Respective phase velocities (ω/k) are given by

$$V_{ph} \approx v_{th}\frac{\omega_c}{\omega_p}\sqrt{1-\frac{\omega_p^4}{\omega_c^4}} \approx v_{th}\left(\frac{\omega_c}{\omega_p}\right) > v_{th} \quad (\omega_c/\omega_p>>1) \quad (24)$$

$$V_{ph} \approx v_{th}\frac{2\omega_c}{\omega_p}\sqrt{1-\frac{3}{4}\left(\frac{\omega_p}{\omega_c}\right)^2} \leq v_{th} \quad (0.86\leq\omega_c/\omega_p<<1) \quad (25)$$

Here, $v_{th}\equiv\sqrt{\kappa T_e/m} = 4.19\text{x}10^7\sqrt{T_e(\text{eV})}$ (cm/s) and we have assumed $k\approx k_D\equiv 1/\lambda_D \equiv \omega_p/v_{th}$. Note that in an under-dense plasma ($\omega_p \leq \omega_{rf} = \omega_c^{ECR}$) either V_{ph} of Eqs. (24) or (25) is likely because the ω_c is that at UHR-surface.

4 NUMERICAL EXAMPLES

Numerical examples were performed for the three cases shown in Table 1. It was found that at least 0.2 nsec is needed before forming a hot-electron shell. Figure 3 shows

Table 1: 3-cases considered and result of hot electron energy.

	Case 1	Case 2	Case 3
B_z(kG) in the core:	2.35	3.41	3.50
ω_c(rad/s) in the core:	4.14x10^{10}	6.0x10^{10}	6.16x10^{10}
n_e(cm^{-3}) in the core:	6.9x10^{11}	1.1x10^{11}	4.8x10^{10}
ω_c/ω_p in the core:	0.88	3.21	4.96
V_{ph}^o(cm/sec) in initial:	1.31x10^8	1.35x10^9	2.10x10^9
W_{eV}(keV) at Δr=0.4 cm:	156(178)	326	344

Here used: v_{th}^o=4.2X10^8 (cm/s) and $\omega_{rf}^2 = \omega_h^2 \equiv \omega_c^2+\omega_p^2 = 2\pi\text{x}10^{10}$.

the orbit of trapped-electrons plotted by T_c/10 time-step: $t^*\equiv t/T_c$ =0.1, 0.2, ... until 0.8 when the detrapping takes place for every trip in this Case 1. The radial and angular positions, $r(t^*)=T_cV_{ph}t^*+r_{UHR}$ and $\theta_s(t^*)= \pi T_c V_{ph}(t^*)^2/r + \theta_{s-1}$, advance gradually as the V_{ph} increases trip by trip.

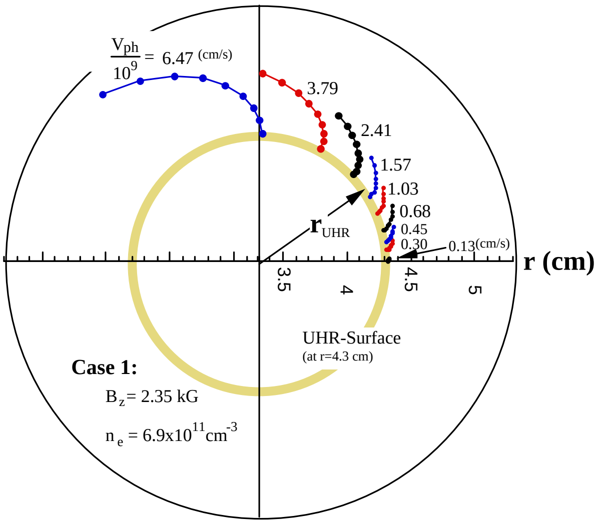

Figure 3: Trapped electron trajectories for increasing V_{ph}.

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