BEAM-ION INSTABILITY IN A MAGNETIC FIELD

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Abstract

A two-beam instability due to ion-beam interactions is considered to be reduced by placing a clearing gap in the bunch train. Since the clearing gap is finite, resdidual ions may have some effects on the instability. Especially, in a magnetic field, ions may not be cleared as expected.

We discuss the motion of ions in a magnetic field and the effect on the beam.

1 INTRODUCTION

A two-beam instability due to ions has been studied numerically during the last three years[1]. We investigated the two-beam instability in a magnetic field by using a rigid Gaussian bunch model[2]. We discuss the instability for KEKB-HER. The parameters of the ring are given in Table 1. Carbon mono-oxide (CO) for ion species have been conseidered.

Table 1: *Parameters of KEKB-HER.*

	KEKB-HER
Energy (GeV)	8.0
Current (A)	1.1
Number of e^- in a bunch	1.4×10^{10}
Emittances (x/y)	$1.8 \times 10^{-8}/3.6 \times 10^{-10}$
Bunch spacing (ns)	2

In the rigid Gaussian bunch model, the beam was described by a series of bunches, and ions were represented by macro-particles. The equations of motion for bunches ($\bar{\boldsymbol{x}}_{-,a}$) and ions ($\bar{\boldsymbol{x}}_{i,j}$) are expressed by

$$\frac{d^2\bar{\boldsymbol{x}}_{-,a}}{ds^2} + K(s)\bar{\boldsymbol{x}}_{-,a} = \frac{2r_e}{\gamma}\sum_{j=1}^{N_i}\boldsymbol{F}_G(\bar{\boldsymbol{x}}_{-,a} - \boldsymbol{x}_{i,j}), \quad (1)$$

$$\begin{aligned}\frac{d^2\boldsymbol{x}_{i,j}}{dt^2} &= \frac{2N_e r_e c^2}{M_i/m_e}\boldsymbol{F}_G(\boldsymbol{x}_{i,j} - \bar{\boldsymbol{x}}_{-,a}) \\ &+ \frac{e}{M_i}\left(\frac{d\boldsymbol{x}_{i,j}}{dt}\times\boldsymbol{B} - \frac{\partial\phi}{\partial\boldsymbol{x}_{i,j}}\right). \end{aligned} \quad (2)$$

where F_G is written using the Bassetti-Erskine formula. Bunches and ions interact with each other when $t_-(s) = t_i(s)$ or $s_i(t) = s_-(t)$. To simplify the model, all of the ions were located at the same longitudinal position ($s_i(t) = const$). We neglected the electric force between ions, because the density of ions is much smaller than that of the beam; the neutralization factor is less than 1.

2 MOTION OF ION

Ions move along an elliptical trajectory in their phase spaces, (x, v_x) and (y, v_y), during the passage of a bunch train. In a linear approximation, the motion of ions is described by a matrix transformation,

$$\begin{pmatrix} 1 - T_b K_{x(y)} & T_b \\ -K_{x(y)} & 1 \end{pmatrix}, \quad (3)$$

where T_b is the bunch spacing represented by time and $K_{x(y)}$, which is the kick by a bunch, is expressed by

$$K_{x,(y)} = \frac{2N_e r_e c}{M_i/m_e}\frac{1}{\sigma_{x(y)}(\sigma_x + \sigma_y)}. \quad (4)$$

Here N_e, σ's , M_i and m_e are the number of electrons in a bunch, the beam sizes, the mass of an ion and the mass of an electron, respectively. Ions with an initial amplitude of σ_x have a maximum velocity of $\sim 10^4 m/s$. If the trace of the matrix in Eq.(3) is less than 2, ions oscillate in the electric force due to a bunch train with a frequency of

$$\begin{aligned}\omega_i &= \cos^{-1}(1 - T_b K_{x(y)}/2)/T_b \\ &= \sqrt{\frac{2N_e r_e c}{T_b M_i/m_e}\frac{1}{\sigma_{x(y)}(\sigma_x + \sigma_y)}}. \end{aligned} \quad (5)$$

After the passage of a bunch train, ions are driven by only the Lorentz forces, which is the 2nd term on the RHS of Eq.2. An ion in a magnetic field moves along a circular orbit with a Larmor radius (ρ_c) of

$$\rho_c = \frac{M_i v}{eB}. \quad (6)$$

Typically, a CO^+ ion with $v = 10^4 m/s (E = 15eV)$ moves with a Larmor radius of $\sim 3mm$ in a magnetic field (B) of $1T$. Eq.2 without a beam force can be solved exactly in a drift space or a bending magnet, and presents a straight or circular trajectory in real coordinate space. In a quadrupole magnet, though the equation can not be solved exactly, we obtained an appriximate solution by a 4-th order Runge-Kutta method. The conservation of the constants of motion (energy) is broken, since the method does not satisfy the exact symplectic condition. The integration step of Runge-Kutta was determined as satisfying the conservation numerically in our model.

We first consider only the motion of ions (Eq.(2)). A beam, which is constructed from a train of 500 bunches in $2nsec$ spacing and 100 empty buckets ($200nsec$), passes through the center of the chamber ($\bar{\boldsymbol{x}}_{-,a} = 0$). The transverse beam size is $(\sigma_x, \sigma_y) = (0.42mm, 0.06mm)$. Fig.1

shows the trajectories in drift space, a bending (B=1T) and a quadrupole magnetic field (B'=10T/m). Ions in a bending field move along the cyclotron orbit with a radius of $\sim 3mm$, as predicted. There was no remarkable difference between the motions in the drift space and the quadrupole field.

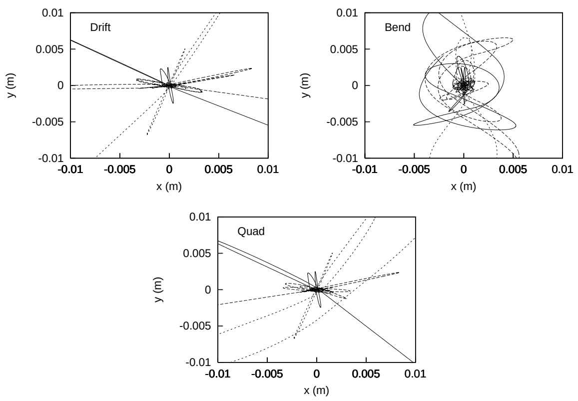

Figure 1: Trajectory of ions in drift space, bending and quadrupole magnetic field. Three ions with different inital randum seeds are tracked.

We now investigate the ion motion in a more realistic model. KEKB-HER is considered to be operated with $8 \sim 10$ bunch trains having a length of ~ 500 bunches. The trains are separated from each other by about ~ 100 empty buckets ($\sim 200ns$) to clear ions trapped in the train. Ions are produced at the beam position by the Gaussian distribution with the same deviation as that of the beam. By repeating the passage of an bunch train and clearing gap, an equilibrium distribution of ions is formed in the beam chamber. Fig.2 shows the equilibrium ion distributions after 1000 repetations. We find the characteristic distribution in each case; a narrow horizontal distribution in a bend and a four-arm tail in quadrupole. The neutralization factors for bending and quadrupole field were estimated to be $1.7\times$ and $2.1\times$ of that for the drift space, respectively. A magnetic field prevented the clearing ions.

3 ION-BEAM INSTABILITY

The two-beam instability is caused by coupling between the corrective motions of the ion cloud and bunch series. The frequency of the ion barycenter and bunch series are equal to each other, with the result that both the amplitude of the ion barycenter and bunches grow. The two-beam instability was investigated by solving both Eqs.(1) and (2). Actually it is equivalent that the rigid bunches are tracked in the ion cloud. The tracking was performed according to the following steps:

1. Initialize all bunches ($\boldsymbol{x}_{-,a} = 0, a = 1, h$)
2. Create macro-ions ($\boldsymbol{x}_{i,j}, j = 1, N_i$)
3. Calculate F_G and kick over the macro-ions

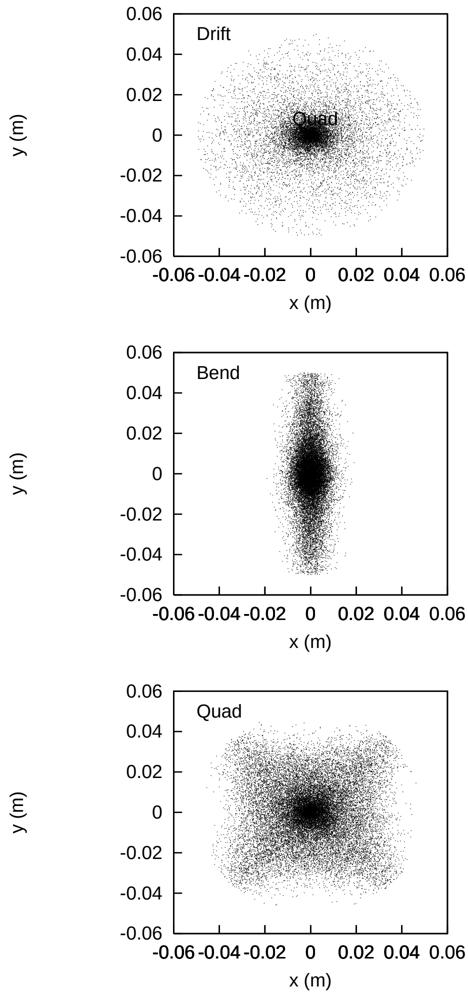

Figure 2: Equilibrium distributions of ions in drift space, bending and quadrupole magnetic field.

4. Kick over the first bunch ($\boldsymbol{x}_{-,1}$) by $\sum F_G$
5. Repeat N_{bunch} times for the second, third... bunches process 2 to 4
6. Revolution of bunches; multiply the revolution matrix by $\boldsymbol{x}_{-,a}$
7. Repeat the process 2 to 6

We consider a 1/8 model of KEKB-HER; $C = 370m$, $h = 640$. It is assumed that 540 buckets are filled and 100 buckets are empty. The vacuum pressure is $10^{-9}Torr$, with the result that ions are created $100m^{-1}$ during every bunch passage.

Fig.3 shows the bunch correlation pattern after 1000 revolutions in each case. The vertical bunch amplitudes observed at a position of the ring are plotted in the Figure. The vertical motion enhances for the horizontal one in ion the instability, because of a smaller vertical beam size than the horizontal one. Taking into acount $2ns$ per bunch, the frequency of the ion barycenter and bunches was about

$14MHz$, which was equal to that of Eq.(5). In the figure, the bunch pattern for the pure fast ion case, in which ions are assumed to be completely cleared by the empty bucket, is also plotted. There is no remarkable difference between them.

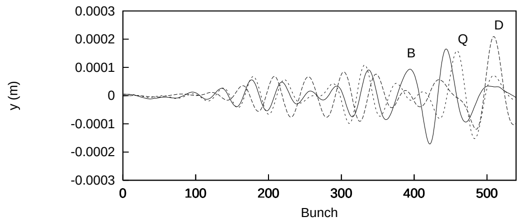

Figure 3: Bunch correlation pattern after 1000 revolutions. Vertical bunch amplitudes observed at a position in the ring are plotted.

Fig.4 shows the growth of the vertical amplitudes of bunches. The growth rate of the fastest bunch was estimated to be about 100 turns in this case.

4 SUMMARY

We have studied the effects of the clearing gap and magnetic field for the two-beam instability due to ions. We consider the $1/8$ model of KEKB-HER. It is assumed that 540 buckets are filled and 100 buckets are empty. The vacuum pressure is $10^{-9} Torr$ and ions are created $100 m^{-1}$ in every bunch passage. In this model, the equilibrium distribution of the ion cloud reflects the characteristics of the magnetic field of the magnet (bend, quad). However the beam amplitude growth due to the ion instability changed little due to the magnetic field. The growth time was about 100 turns, close to the growth time in the case that ions completely cleared (pure fast ion instability).

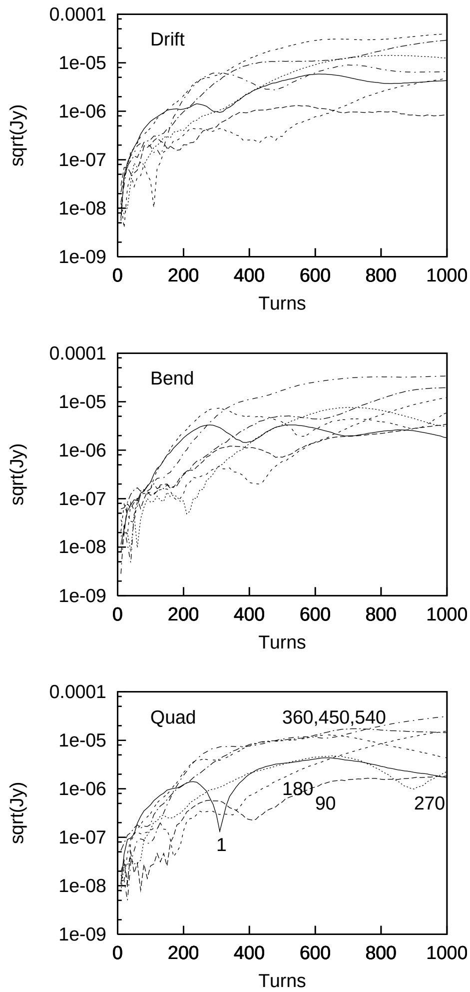

Figure 4: Growth of vertical bunch amplitudes in magnetic field.

5 REFERENCES

[1] T. Raubenheimer and F. Zimmermann, Phys. Rev. **E52**, 5487 (1995).

[2] K. Ohmi, Phys. Rev. **E55**, 7550 (1997).