HALO-FREE BEAM TRANSPORT IN NONLINEAR FOCUSING CHANNEL

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Abstract

High brightness beam transport in a non-uniform focusing channel, while avoiding beam emittance growth and halo formation, is considered. Two approaches to self-consistent problem for matched beam distribution are treated: (i) focusing field is determined by given beam distribution, and (ii) matched beam distribution is defined by focusing potential of the structure. Attained solutions provide theoretical basis for choosing parameters of the space charge dominated beam transport with suppressed halo.

1 INTRODUCTION

Prevention of emittance growth and halo formation in high-intensity beams are a key problem for proposed particle accelerators for heavy ion fusion, spallation neutron sources, and radioactive waste transmutation. Beam with a nonuniform profile is mismatched within a linear focusing channel. It results in a beam emittance growth and halo formation (see Fig. 1).

To prevent emittance growth and halo formation, beam has to be matched with the channel. Beam distribution function, $f(x, p_x, y, p_y)$, expressed as a function of Hamiltonian, H, is a constant of motion in a uniform, time-independent focusing channel

$$f = f(H), \quad H = \frac{p_x^2 + p_y^2}{2 m \gamma} + q U_{ext} + q \frac{U_b}{\gamma^2}, \quad (1)$$

where x and y are particle positions, p_x and p_y are transverse particle momentum, q and m are charge and mass of the particles, respectively, γ is a particle energy, U_{ext} is a potential of focusing field, and U_b is a space charge potential of the beam. Matched beam distribution function (1) obeys self-consistent Vlasov-Poisson equations:

$$\frac{df}{dt} = \frac{1}{m\gamma}(\frac{\partial f}{\partial x} p_x + \frac{\partial f}{\partial y}p_y) - q(\frac{\partial f}{\partial p_x} \frac{\partial U}{\partial x} + \frac{\partial f}{\partial p_y} \frac{\partial U}{\partial y}) = 0, \quad (2)$$
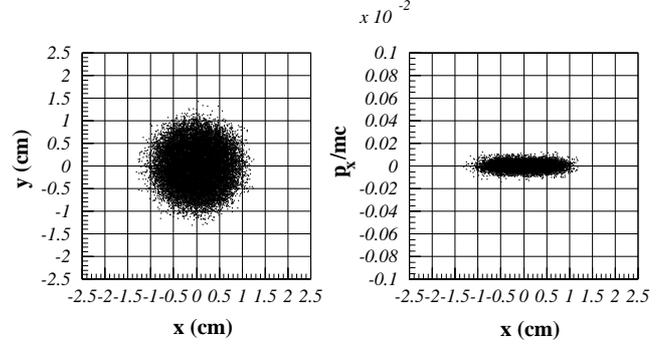
$$\frac{1}{r}\frac{\partial}{\partial r}(r \frac{\partial U_b}{\partial r}) = -\frac{q}{\varepsilon_o} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, p_x, y, p_y) \, dp_x dp_y, \quad (3)$$

where $U = U_{ext} + U_b \gamma^{-2}$ is a total potential of the structure. There are two formulations of the self-consistent problem:

1. Starting with given beam distribution function to find the required focusing potential, which maintains this distribution in the channel.

2. Starting with given potential of the focusing structure to find the matched beam distribution function.
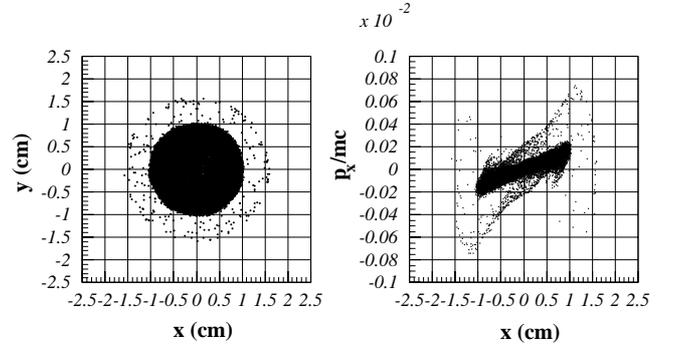
z = 0

z = 80 cm

Fig. 1. Emittance growth and halo formation of 150 keV, 0.5A, 0.07 π cm mrad proton beam with distribution function, Eq. (4), in linear focusing channel.

2 FOCUSING FIELD FOR BEAM EQUILIBRIUM

General method to solve the first problem is to substitute beam distribution function into Vlasov's equation and find total potential of the structure U [1]. Required focusing field is then found as a difference between the total potential and known space charge potential of the beam $U_{ext} = U - U_b\gamma^{-2}$. The same relationship is valid for electrical field $E_{ext}(r) = E(r) - E_b(r) \gamma^{-2}$. If distribution function provides elliptical phase space projections, total field of the structure has to be linear function of radius [1].

Consider beam with distribution function

$$f = f_o \exp(-2 \frac{p_x^2 + p_y^2}{p_o^2} - \frac{(x^2 + y^2)^2}{R_o^4}), \quad (4)$$

which does not possess elliptical symmetry. Substitution of distribution function (4) into Vlasov's equation (2) provides expression for total field of the structure

$$E(r) = -\frac{1}{\gamma} \frac{mc^2}{q R_o}(\frac{\varepsilon^*}{R_o})^2 (\frac{r^3}{R_o^3}), \quad (5)$$
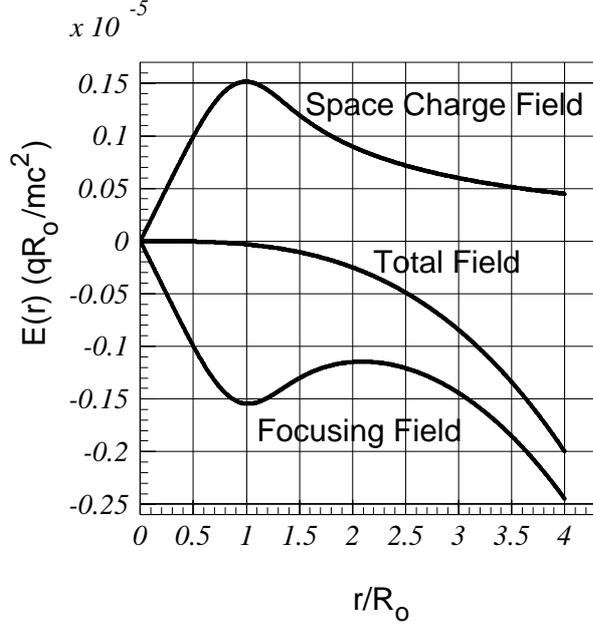
Fig. 2. Total field of the structure, required focusing field and space-charge field of the beam with distribution function, Eq. (4).

where $\varepsilon^* = R_o p_o/(mc)$ is an effective normalized beam emittance. Space charge field of the beam is obtained from the Poisson's equation (3):

$$E_b(r) = \frac{I}{2\pi\varepsilon_o \beta c\, r}\, erf(\frac{r^2}{R_o^2})\,, \qquad (6)$$

where I is a beam current and β is a particle velocity. Combination of total field, Eq. (5), and space charge field, Eq. (6), gives the expression for the required focusing field of the structure to maintain beam distribution:

$$E_{ext}(r) = -\frac{1}{\gamma}\frac{mc^2}{q\,R_o}[(\frac{\varepsilon^*}{R_o})^2(\frac{r^3}{R_o^3}) + \frac{2I}{I_c\,\beta\gamma}(\frac{R_o}{r})\, erf(\frac{r^2}{R_o^2})]\,, \quad (7)$$

where $I_c \equiv 4\pi\varepsilon_o mc^3/q$ is a characteristic value of beam current. Fig. 2 illustrates relationship between space-charge field, total field and focusing field of the structure. In contrast with distribution functions with elliptical symmetry (for example, Gaussian distribution), in the considered case total field is not a linear function of radius, but is essentially nonlinear function $\sim r^3$. In Fig. 3 results of beam dynamics study with conserved beam distribution function in the channel with focusing field, Eq. (7), are presented. Required focusing field can be created in plasma lens.

Important point is stability of beam equilibrium in nonlinear focusing field. Sufficient condition for stability is given by Newcomb-Gardner theorem [2], which states, that monotonically decreasing equilibrium distribution function of Hamiltonian $\partial f/\partial H < 0$ is stable with respect to perturbations. Distribution (4) as well as most of realistic beam distributions satisfies stability condition.
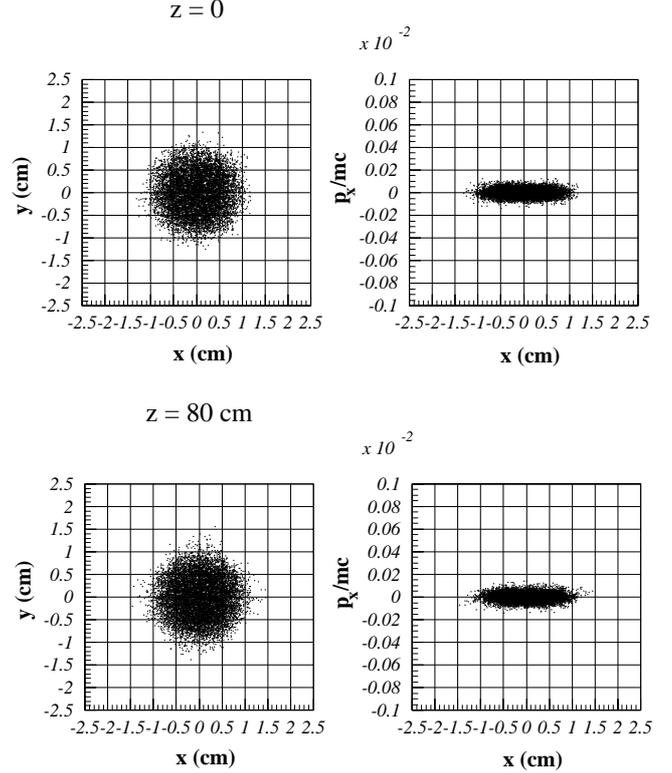
z = 0

z = 80 cm

Fig. 3. Conservation of 150 keV, 0.5A, 0.07 π cm mrad proton beam with distribution, Eq. (4), in nonlinear focusing field, Eq. (7).

3 SELF-CONSISTENT BEAM DISTRIBUTION IN CONTINUOUS FOCUSING CHANNEL

Inverse self-consistent problem is to find unknown distribution function via given focusing potential of the structure. In continuous channel with linear focusing field all self-consistent solutions tend to uniform beam in the limit of high brightness beam [3]. In Ref. [4] this result was generalized for the case of an arbitrary applied focusing field. It was found that the self-consistent stationary particle distribution has such a shape that the space charge beam potential U_b is opposite to external focusing potential U_{ext}:

$$U_b = -\gamma^2\frac{kb}{1 + kb}\, U_{ext}\,, \qquad (8)$$

where $b \equiv 2IR^2/(\beta\gamma I_c\varepsilon^2)$ is a dimensionless value of beam brightness and k is a beam profile parameter (k = 1 for uniform beam and k = 2 for Gaussian beam).

Space charge density of the matched high-brightness beam is determined by Poisson's equation $\rho_b = -\varepsilon_o\Delta U_b$. Matched beam profile is defined mostly by focusing potential function and is a weak function of particle distribution in phase space. Time-dependent focusing potential $U(r,\varphi,t) = U_o(r,\varphi)\cdot\cos\omega_o t$ can be substituted by an effective potential

$$U_{ext}(r,\varphi) = \frac{qE^2(r,\varphi)}{4m\gamma\omega_o^2}\,, \qquad (9)$$

if phase advance of particle oscillations per period is small enough (smooth approximation).

4 BEAM TRANSPORT IN QUADRUPOLE CHANNEL WITH OCTUPOLE COMPONENT

Matched conditions for nonuniform beam requires the focusing field to be highly nonlinear function of radius. Consider an uniform four-vanes quadrupole focusing structure with octupole field component. Electrostatic potential of such structure is given by

$$U(r,\varphi,t) = (\frac{G_2}{2} r^2 \cos2\varphi + \frac{G_4}{4} r^4 \cos4\varphi) \sin \omega_o t , \quad (10)$$

where G_2 is a quadrupole field gradient, G_4 is an octupole field component. Oscillating field (10) creates the effective scalar potential

$$U_{ext}(r,\varphi) = \frac{mc^2}{q} \frac{\mu_o^2}{\lambda^2} [\frac{1}{2} r^2 + a_4 r^4 \cos2\varphi + \frac{a_4^2}{2} r^6], \quad (11)$$

where $\mu_o \equiv q\, G_2 \lambda^2/(\sqrt{8}\, \pi\, mc^2)$ is a smoothed transverse oscillation frequency, $\lambda = 2\pi c/\omega_o$ is a wavelength, and $a_4 = G_4/G_2$ is a ratio of the two field components. Equipotential lines of the effective potential (11) are transformed from circles at small values of radius (r), to distorted ellipse at larger r.

Space charge density of the matched high-brightness beam in the considered channel is given by:

$$\rho_b(r,\varphi) = \rho_o[1 + 6\, a_4\, r^2 \cos2\varphi + 9a_4^2 r^4] . \quad (12)$$

Beam with distribution of (12) is maintained in the channel with effective potential (11). Due to the term $\cos2\varphi$, space charge density of (12) is a decreasing function with x-coordinate, but increasing function with y-coordinate, being $r^2 = x^2 + y^2$.

Realistic beam distribution has monotonically decreasing density function with radius. Good approximation to realistic beams is a distribution of the form $\rho_b = \rho_o [1 - (r/R)^2]^2$, which is close to the truncated Gaussian distribution. Realistic beam is expected to be matched in x-coordinate, but mismatched in y-coordinate. After sufficiently long transport, beam will be mismatched with such channel due to coupling between x and y coordinates. Therefore, utilizing octupole component in a quadrupole channel is not enough to provide beam matching.

5 BEAM TRANSPORT IN QUADRUPOLE CHANNEL WITH DUODECAPOLE COMPONENT

Better results were observed in computer simulation of the beam in a quadrupole structure with duodecapole field component G_6 (see Refs. [1], [4]):

$$U(r,\varphi,t) = (\frac{G_2}{2} r^2 \cos2\varphi + \frac{G_6}{6} r^6 \cos6\varphi) \sin \omega_o t . \quad (13)$$

In that case the effective potential and matched beam profile are symmetric functions with y as well as x coordinates:

$$U_{ext}(r,\varphi) = \frac{mc^2}{q} \frac{\mu_o^2}{\lambda^2} [\frac{1}{2} r^2 + a_6 r^6 \cos4\varphi + \frac{a_6^2}{2} r^{10}] , \quad (14)$$

$$\rho_b = \rho_o (1 + 10\, a_6\, r^4 \cos 4\varphi + 25\, a_6^2\, r^8) , \quad (15)$$

where $a_6 = G_6/G_2$ is a ratio of field components. Equipotential lines $U_{ext}(r,\varphi) = C$ are close to square. Matched beam density, Eq. (15), is a decreasing function of x and y coordinates, where $\varphi = 0, 90^o$, but increasing function of intermediate x-y direction, where $\varphi = 45^o$. Equipotential line truncates beam in such way, that realistic beam with monotonous decreasing density function of radius remains approximately matched with quadrupole-duodecapole focusing structure (see Refs. [1] and [4] for more details).

6 MATCHED BEAM IN QUADRUPOLE CHANNEL WITH HIGHER ORDER COMPONENTS

Including of higher order terms in quadrupole channel makes an analysis complicated. Consider structure with potential

$$U(r,\varphi,t) = (\frac{G_2}{2} r^2 \cos2\varphi + \frac{G_6}{6} r^6 \cos6\varphi + \frac{G_{10}}{10} r^{10} \cos10\varphi$$
$$+ \frac{G_{14}}{14} r^{14} \cos14\varphi) \sin \omega_o t . \quad (16)$$

Matched beam distribution is given by

$$\rho(r,\varphi) = \rho_o[1 + 10\, r^4 a_6 \cos4\varphi + r^8(18a_{10}\cos8\varphi + 25a_6^2)$$
$$+ 169r^{24}a_{14}^2 + r^{12}(26a_{14}\cos12\varphi + 90\, a_6\, a_{10} \cos4\varphi)$$
$$+r^{16}(81\, a_{10}^2 + 130\, a_6\, a_{14}\cos8\varphi)+234r^{20}a_{10}\, a_{14}\cos4\varphi], (17)$$

where $a_{10} = G_{10}/G_2$ and $a_{14} = G_{14}/G_2$. Higher order field components result in more round matched beam boundaries, but in the same time in more non-uniform matched beam profile with increasing - decreasing functions of azimuth angle. Realistic beam profile becomes essentially mismatched with the function, Eq. (17), at the beam boundary, where effect of high order field components makes beam matching difficult. Including of higher order, than duodecapole, field component does not improve matching of realistic beam with the structure.

7 REFERENCES

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