TRANSVERSE-PROFILE EQUILIBRIUM IN A SPACE-CHARGE-DOMINATED BEAM

N. Pichoff[†], G. Haouat[‡], P.Y. Beauvais[†], J.M. Lagniel[†], R. Ferdinand[†], S. Nath[*], S. Joly[‡]
[‡] CEA/DAM/DPTA/SP2A, Bruyères-le-Châtel, BP 12, 91680, Bruyères-le-Châtel, France.
[†] CEA/DSM/GECA, Saclay, 91191, Gif-sur-Yvette Cedex, France.
[*] LANL, MS-H817, Los Alamos, NM - 87545, USA.

Abstract

Evolution of the transverse distribution of an intense charged-particle beam crossing a periodic focusing channel is analysed. Theoretical considerations are given showing that, under combined effects of the confinement and space-charge forces, the profile of a space-charge-dominated beam evolves toward a stationary equilibrium, solution of the Vlasov equation. The equilibrium shape is shown to depend on both channel characteristics and beam parameters. To check these predictions an experiment on a 0.5-MeV proton beam transported through a 29-cell quadrupole FODO channel is presented. Beam transverse distribution is measured at the entrance and exit of the channel for currents from 2 to 37 mA. The measurements are compared with the results of a dedicated 2D simulation code.

1 INTRODUCTION

Intense, high-brightness beams envisioned for many advanced accelerator applications require minimal emittance and lowest particle loss throughout their acceleration and transport, although the dynamics of such beams is significantly affected by strong self forces due to space charge.

Non-uniform charge distribution, mismatch and misalignment of space-charge-dominated beams are known to undergo emittance growth and halo formation; these effects are attributed to the conversion of excess free energy for non-stationary initial distributions into random kinetic energy [1]. Proper matching of the beams at every transition of accelerators or transport systems is thus necessary to preserve emittance and to avoid beam loss. A question remains however: how evolves the dynamics of intense beams r.m.s. matched with their transport channels? Theoretical and numerical work has been carried out on the problem of self-consistent particle phase space distributions for continuous and periodic channels [2]. But the studies only consider r.m.s. values of the beam parameters, and cannot yield any information on the low-density halo population.

The objective of the present work is the analysis of the transverse profile and phase-space distribution of a proton beam transported through the Saclay 29-cell FODO channel, in a range of beam currents extending from weak to strong space-charge effects [3]. It is intended to check theoretical predictions for beam r.m.s. parameters and to latter investigate halo formation. After a short recall of the background theory and a description of the simulation tools developed for this work, the experimental set-up is presented and measured data are reported and discussed.

2 THEORETICAL BACKGROUND

The problem involved is the transport of a charged-particle beam through a periodic channel of period length S. The evolution of the beam density distribution $f(\vec{r},\vec{p},t)$ is described by Vlasov's equation. Let us consider the equilibrium distribution $f_e(\vec{r},\vec{p},t)$ which has the same periodicity as the focusing channel:

$f_e(\vec{r},\vec{p},t+T) = f_e(\vec{r},\vec{p},t)$, ($T = S/v$, v: particle velocity).

The solution of Vlasov's equation for this distribution yields:

$$\left\langle \vec{p}\cdot\vec{\nabla}_{\vec{r}} f_e + \left(\vec{F}/m\right)\cdot\vec{\nabla}_{\vec{p}} f_e \right\rangle_T = 0 .$$

The distribution $f_e(\vec{r},\vec{p},t)$ can be defined as the sum of a stationary distribution and a periodic perturbation:

$f_e(\vec{r},\vec{p},t) = f_0(\vec{r},\vec{p}) + \delta f_P(\vec{r},\vec{p},t)$. We have then:

$$\vec{p}\cdot\vec{\nabla}_{\vec{r}} f_0 + \left(\vec{F}/m\right)\cdot\vec{\nabla}_{\vec{p}} f_0 = 0 \text{ and } \left\langle \delta f_P(\vec{r},\vec{p},t)\right\rangle_T = 0 .$$

The function f_0 represents the stationary state of the beam distribution in the continuous focusing channel equivalent to the periodic channel. Equivalence means that both channels have the same mean focusing force, *i. e.*, the particles have the same phase advance per meter. Evolution of this stationary distribution can be studied as a function of the beam parameters (intensity, initial distribution, r.m.s. emittance and r.m.s. size) and the focusing channel characteristics.

The stationary distribution f_0, solution of the steady-state Vlasov equation, depends explicitly on the hamiltonian H of the motion of the beam particles:

$f_0(\vec{r},\vec{p}) = f_0(H(\vec{r},\vec{p}))$, where:

$$H(\vec{r},\vec{p}) = E_k(\vec{p}) + W_c(\vec{r}) + W_{sc}(\vec{r}) = E_k(\vec{p}) + W(\vec{r}) ,$$

with $E_k(\vec{p})$ the kinetic energy, $W_c(\vec{r})$, $W_{sc}(\vec{r})$ and $W(\vec{r})$ the confinement, space-charge and total potential energies, respectively. Contour plot of the steady-state phase space distribution corresponds to the curves: $H(\vec{r},\vec{p}) = C^{te}$.

Evolution of the beam envelope R is described by the equation:

$$d^2R/dz^2 + F_c(R) - F_{sc}(\Re) - \varepsilon_R^2/R^3 = 0 ,$$

where R and ε_R stand for the beam-envelope size and the emittance, in the three spatial co-ordinates, X, Y, Z, and $\Re$ is a function of the transverse X, Y co-ordinates for a continuous beam, and of the three co-ordinates for a bunched beam. $F_c(R)$ denotes the confinement force, and $F_{sc}(\Re)$ the space-charge force.

In the case where the beam is matched ($d^2R/dz^2 = 0$), the particle distribution is stationary with $R = R_m$.

We consider the parameter $\zeta_R = F_{sc}(\Re_m)\cdot R_m^3/\varepsilon_R^2$ which is the ratio between the space-charge force and the emittance force acting on the beam. It is related to the tune depression factor η_R by: $\zeta_R = \eta_R^{-2} - 1$. If $\zeta_R >> 1$, the beam is space-charge dominated, if $\zeta_R << 1$, the beam is emittance dominated. Let us examine these two cases.

- **$\zeta_R << 1$**. The contribution of the space-charge potential energy to the hamiltonian can be neglected. We have $H(\vec{r},\vec{p}) \approx E_k(\vec{p}) + W_c(\vec{r})$. If external confinement is linear in all directions, the particle potential energy is proportional to r^2 and, as the particle kinetic energy is proportional to p^2, all beam particles are moving in a harmonic potential well and the iso-density curves are ellipses in the phase sub-spaces. The stationary beam profile depends mainly on the initial density distribution.

- **$\zeta_R >> 1$**. To maintain beam confinement, the focusing force must oppose the large repulsive space-charge force. Since $W_c(\vec{r})$, $W_c(\vec{r}) >> (H(\vec{r},\vec{p}) - E_k(\vec{p}))$, we have $W_c(\vec{r}) \approx -W_{sc}(\vec{r})$. If external confinement is linear in all directions, so is the space-charge force, and the stationary particle distribution inside the beam is homogeneous. For increasing intensities, beam particles are moving in a potential well which is no longer harmonic but becomes more and more flat in the central region of the beam and has a sharp increase at the beam boundary. This is a "reflective-wall potential" since the particles do not feel any force inside the beam, move freely with constant momentum and are reflected at the boundary. The phase-space distribution becomes rectangular [2].

3 COMPUTER SIMULATIONS

Beam dynamics of unbunched beams has been calculated in a periodic focusing quadrupole channel applying the 2D particle-in-cell simulation code MONET [3]. It has been developed to simulate beam transport on the Saclay FODO channel, to interpret experimental results and to validate theoretical predictions. The particle dynamics is self-consistently computed. To account for the space-charge force, the transverse charge distribution is divided into many homogeneous elliptical rings, from which the electric fields are analytically computed at the mesh points of a two-dimensional lattice in the transverse space. The transport channel includes quadrupoles, drift spaces and apertures. Several initial phase-space particle distributions (uniform, Gaussian, parabolic or measured data) can be considered.

Simulations with linear external confinement show that the final, steady-state core profile of a space-charged-dominated beam is homogeneous whatever the initial distribution is. However, a halo of particles surrounding the core is produced, the shape and amplitude of which strongly depend on the initial beam distribution and the excess Coulomb-field energy. Numerical results are compared below to experimental data.

4 EXPERIMENT

The experiment has been carried out in the FODO channel at Saclay. The experimental set-up, described in detail elsewhere [3], consists of a proton-beam injector, a 10.4-m long transport channel with periodic focusing, and an exit section. The injector comprises a duoplasmatron source housed in a high-voltage terminal and a matching section with a quadrupole-triplet close to the source exit and a quadrupole-quintuplet in front of the FODO channel. The transport channel is the focusing system of an old 20-MeV Alvarez linac [4]. This transport line includes 58 magnetic quadrupoles arranged in a 29-periods FODO channel; because of the structure design of the Alvarez linac, the period length is not constant but increases gradually from entrance to exit.

In the experiment, the source produces a pulsed proton beam of 500 keV energy, 300 µs bunch length, 1 Hz repetition rate and bunch current ranging from 2.2 to 37 mA. The beam from the source is focused by the matching-section triplet and quintuplet, and injected into the transport channel in such a way that mismatching is minimized, that is, its envelope is periodic to good approximation. It is also accurately centered on the FODO-channel axis to reduce misalignment effects.

Beam diagnostics are located in the matching and exit sections. Beam intensity and bunch length are monitored by current transformers connected to a digitizing signal analyzer. Beam position, profile and emittance are accurately measured with a pinhole/profile-harp system at the front end of the channel, and with a pepper-pot/phosphor-screen device with beam-image acquisition system at the back end [4].

5 RESULTS AND DISCUSSION

A series of measurements has been performed on proton beams for nine different intensities (given in Table 1), with the FODO channel tuned to a phase advance without space charge $\sigma_o = 60°$.

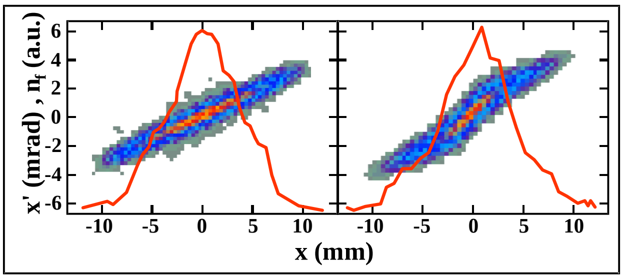

Figure 1: *Phase-space distribution and profile measured in front of the FODO channel at beam currents of 2.2 mA (left) and 37 mA (right).*

The experimental results reported here concern only beam-core observations; lower-density halo data will be presented later on. The adopted experimental procedure was as follows. For each proton-beam current, the phase-space distribution is first measured in the matching section, at the exit of the source. Typical (x',x) phase-space distributions are shown in Fig. 1 for the two extreme currents, 2.2 and 37 mA, corresponding to weak and marked space-charge effects, respectively. Although these distributions yield emittances of slightly different values (larger at higher current), they exhibit a similar pattern. This is more evident in the beam transverse profiles displayed on the same figure and obtained by projecting the (x',x) distributions onto the x axis. These profiles, normalized to the same maximum value, are alike with a peaked, rather Gaussian, shape.

Table 1: *Beam currents and transport parameters σ, η and ζ.*

I (mA)	2.2	3.8	7.0	10	14	21	26	33	37
σ (deg)	56	52	46	41	35	28	24	20	19
η	.93	.87	.76	.68	.59	.46	.40	.34	.31
ζ	.17	.32	.71	1.2	1.9	3.6	5.2	7.8	9.5

The beam is then matched to the channel by supplying the triplet and quintuplet quadrupoles with currents which are determined from the phase-space data and the channel parameters by using the transport code MONET; the beam parameters σ (phase advance with space charge), η and ζ are also determined (Table 1).

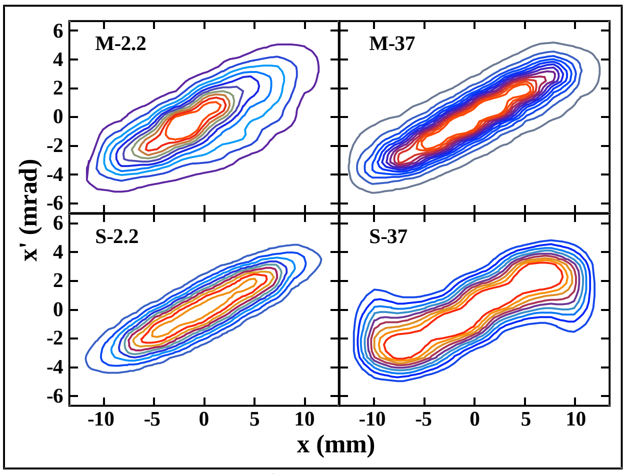

Figure 2: *Comparison of measured (above) and simulated (below) phase-space contour plots for beam currents of 2.2 mA (left) and 37 mA (right).*

Finally, the phase-space distribution is measured in the exit section on the matched beam after its transport through the FODO channel. In Fig. 2 are displayed contour plots of the (x',x) distributions for the same currents as in Fig. 1. Also shown in this figure are the corresponding simulation results obtained by using the phase-space distributions measured in the matching section (Fig. 1) as input parameters for the code MONET. It appears clearly that the measured equal-density contours are much closer to rectangular shapes for the 37-mA data than they are for the 2.2-mA data, a result which is consistent with the theory. The structural agreement between experiment and simulation is very good, indicating that the simulation accurately represents the actual experiment.

Figure 3 shows the transverse profiles at the end of the channel for the nine beam currents of the experiment. They were obtained by selecting in the image of the beamlets from the pepper-pot-mask, given by the phosphor screen, the closest horizontal beamlet raw to the beam center, and by processing the data using an interpolation technique.

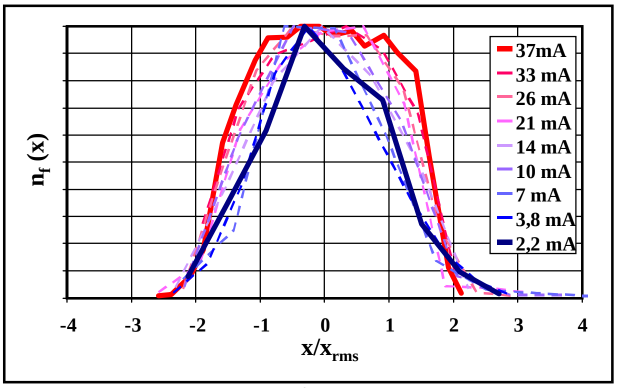

Figure 3: *Measured beam profiles at the FODO-channel exit.*

One observes that, at low beam current, the output profile is very similar to the initial one, and as beam intensity increases, the output profile becomes progressively closer to a square shape, implying that the proton density in the beam becomes almost uniform.

6 CONCLUSION

The dynamics of a proton beam transported through a quadrupole FODO channel has been analyzed as a function of the space-charge-induced self force. Transverse real- and phase-space distributions measured on the beam matched to the channel, tuned at $\sigma_o = 60°$, agree fairly well with numerical simulations and reveal that the beam reaches a stationary equilibrium with a profile remaining unchanged at low current, and evolving toward a more homogeneous distribution as the space-charge force increases. These results confirm theoretical predictions which attribute the charge-density homogenization to the transformation of field energy into kinetic and potential beam energy.

7 REFERENCES

[1] M. Reiser, J. Appl. Phys. **70**, 1919 (1991).

[2] J. Struckmeier, I. Hofmann, Part. Accel. **39**, 219 (1992).

[3] N. Pichoff, "Etude théorique et expérimentale du halo d'un faisceau intense de particules chargées dans un accélérateur", Ph. D. thesis, Université Paris XI-Orsay, France, 1997.

[4] G. Haouat, N. Pichoff, P.Y. Beauvais and R. Ferdinand, Proc. of the 5th Europ. Part. Acc. Conf., EPAC96 (1996) p. 1206.