

THE COMPUTATION OF THE BUNCHING SYSTEM OF INTENSE ION BEAM BY MOMENTS METHOD

N. Kazarinov[†], I. Kalagin

Joint Institute for Nuclear Research, 6, Joliot-Curie, 141980, Dubna, Russia

Abstract

The computation of the bunching system of intense ion beam based on the moments method is presented.

Longitudinal $\rho_{||}$ and transverse densities ρ_{\perp} are equal to:

$$\rho_{||}(z) = \frac{1}{\sqrt{2\pi}\sigma_z} \sum_{n=-\infty}^{\infty} \exp\left(-\frac{(z-n\lambda)^2}{2\sigma_z^2}\right) \quad (2.1)$$

$$\rho_{\perp}(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) \quad (2.2)$$

INTRODUCTION

Within the framework of the Multi-Component Ion Beam code (MCIB04) [1] the program for 3D simulation of the intense charge particle beam dynamics is created.

Fast analysis and study of the averaged beam characteristics, such as root-mean-square (RMS) dimensions, is performed by the moments method [2].

The main advantage of the moments method in comparison with macro particle one is fast calculation and therefore applicability for transport line optimization.

The model describing the charge density of the bunched beam is introduced. The external electromagnetic fields are assumed to be linear.

The fitting procedure based on minimization of a quadratic functional at any point of the beam line by using either gradient or simplex-method is available [3].

The simulation of the bunching system of DC-350 cyclotron axial injection beam-line [4] was fulfilled by using created 3D version of MCIB04 code.

According to formula (2.1) longitudinal density is periodical function $\rho_{||}(z) = \rho_{||}(z + \lambda)$ with a constant number of particles at period λ :

$$\int_{-\lambda/2}^{\lambda/2} \rho_{||}(z) dz = N \quad (3)$$

In the case $\sigma_z \gtrsim \lambda$ this model describes the beam with constant density and for $\sigma_z \ll \lambda$ gives Gaussian beam. The dependencies on z of the longitudinal beam density for various values of ratio λ/σ_z are shown in Fig.2.

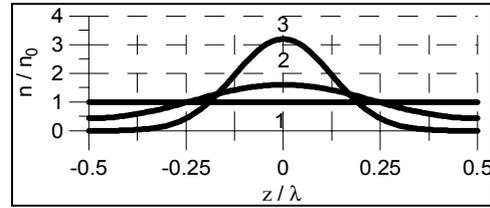


Figure 2: Longitudinal beam density
Curve 1 – $\lambda/\sigma_z = 1$, 2 – $\lambda/\sigma_z = 4$, 3 – $\lambda/\sigma_z = 8$

BEAM MODEL

Let consider the train of bunches (Fig.1), moving with average velocity $\beta_0 c$ with distance between its center-of-mass $\lambda = \beta_0 \lambda_0$. Here λ_0 is cyclotron RF field wave length.

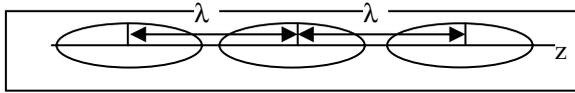


Figure 1.

The beam density may be defined as:

$$\rho(x, y, z - \beta_0 ct) = N \rho_{||}(z - \beta_0 ct) \rho_{\perp}(x, y) \quad (1)$$

where $N = \frac{I\lambda}{Ze\beta_0 c}$ – the number of particle at spatial

period λ , I – beam current, Ze – ion charge.

BEAM SELF FIELD

By using formulae (1, 2) the beam self field may be represent in the following form [5]:

$$E_x \cong 2\pi Ze N \rho_{||}(z - \beta_0 ct) \sigma_x \sigma_y \int_0^{\infty} \frac{x}{(\sigma_x^2 + s)R(s)} \rho_{\perp}(T) ds$$

$$E_y \cong 2\pi Ze N \rho_{||}(z - \beta_0 ct) \sigma_x \sigma_y \int_0^{\infty} \frac{y}{(\sigma_y^2 + s)R(s)} \rho_{\perp}(T) ds \quad (4)$$

$$T = \frac{x^2}{\sigma_x^2 + s} + \frac{y^2}{\sigma_y^2 + s} \quad ; \quad R(s) = \sqrt{(\sigma_x^2 + s)(\sigma_y^2 + s)}$$

$$E_z \cong 2Ze N \rho'_{||}(z - \beta_0 ct) \left(\ln \frac{b}{a} + \frac{1}{2} - \frac{x^2 + y^2}{2a^2} \right) \quad , \quad x^2 + y^2 \leq a^2$$

Here $a = \sqrt{2(\sigma_x^2 + \sigma_y^2)}$ – RMS radius of the beam, b – vacuum pipe radius and prime denotes derivative with respect to z .

[†]nyk@lnr.jinr.ru

MOMENTS EQUATIONS

Let us define the second order moments M of the beam distribution function f :

$$M = \overline{YY^T} = \frac{1}{N} \int YY^T f dy \quad (5)$$

where superscript T denotes transpose vector or matrix, $Y^T = (x, y, x', y', z - \beta_0 ct, \delta) = (X^T, V^T, Y_{||}^T) = (Y_{\perp}^T, Y_{||}^T)$ – vector of phase space coordinates of the particle, $\delta = (\beta - \beta_0) / \beta_0$ – relative momentum spread. Integration in (4) is fulfilled over all phase space occupied by bunch particles (at one spatial period), prime denotes derivative with respect to longitudinal coordinate of the bunch center-of-mass.

The equations for transverse second order moments $M_{\perp} = \overline{Y_{\perp} Y_{\perp}^T}$ do not differ significantly in comparison with the case of non-bunched beam [2]. This difference leads to replacement of the beam current I by its effective value $k_{\perp} I$, where the bunching factor k_{\perp} is connected with changing of the transverse beam self fields due to changing of the longitudinal density:

$$k_{\perp} = \lambda \int_{-\lambda/2}^{\lambda/2} \rho_{||}^2(z) dz = \sqrt{\frac{z_0^2}{z^2}} F_{\perp} \left(\frac{z^2}{z_0^2} \right) \quad (8)$$

Here $\sqrt{z^2}$ is longitudinal RMS dimension of the bunch:

$$z^2 = \int_{-\lambda/2}^{\lambda/2} z^2 \rho_{||}(z) dz \quad (9)$$

and $\sqrt{z_0^2} = \lambda / \sqrt{3}$ its value for non-bunched beam. The plot of function $F_{\perp}(x)$ is shown in Fig.3.

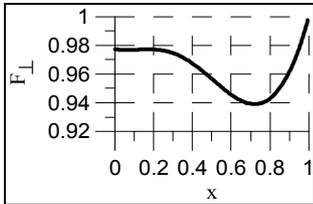


Figure 3.

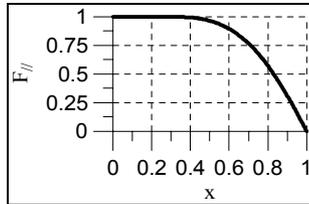


Figure 4.

As may be seen from Fig.3 function $F_{\perp}(x)$ is approximately equal to unity with difference does not greater than 6%. In the program this function is represented as the sixth order polynomial.

The equations for the longitudinal second order moments $M_{||}$ has the following form:

$$M_{||} = \overline{Y_{||} Y_{||}^T} = \left(\frac{z^2}{z \delta} \quad \frac{z \delta}{\delta^2} \right) \quad (10.1)$$

$$\left(\frac{z^2}{z \delta} \right)' = 2 \frac{z \delta}{\delta^2} \quad (10.2)$$

$$\left(\frac{z \delta}{\delta^2} \right)' = \delta^2 + \frac{Ze}{Am \beta_0^2 c^2} \frac{z E_z}{z \delta} \quad (10.3)$$

$$\left(\frac{z \delta}{\delta^2} \right)' = \frac{Ze}{Am \beta_0^2 c^2} \frac{z E_z}{z^2} \frac{z \delta}{z \delta} \quad (10.4)$$

Computation of average $\frac{z E_z}{z \delta}$ in accordance with formulae (4, 5) results in:

$$\frac{Ze}{Am \beta_0^2 c^2} \frac{z E_z}{z \delta} = k_{||} \frac{Z}{A} \frac{I}{I_A} \frac{1}{\beta_0^3} \left(\ln \frac{b}{\sqrt{2(\sigma_x^2 + \sigma_y^2)}} + \frac{1}{4} \right) \quad (11)$$

where A – ion mass, $I_A = mc^3/e$ – Alfvén's current, the bunching factor of the longitudinal motion $k_{||}$ is defined by formula:

$$k_{||} = \lambda \int_{-\lambda/2}^{\lambda/2} [\rho_{||}^2(z) - \rho_{||}^2(\lambda/2)] dz = k_{\perp} F_{||} \left(\frac{z^2}{z_0^2} \right) \quad (12)$$

The plot of function $F_{||}(x)$ is shown in Fig.4. In the case $x \sim 1$ function $F_{||}$ is close to zero because of the longitudinal electric field of non-bunched beam is equal to zero. For the well bunched beam ($x \ll 1$) due to small longitudinal density at point $z = \lambda/2$ formulae (11) and (12) become identical and function $F_{||}$ is close to unity. In the program function $F_{||}(x)$ is approximated by the fifth order polynomial for all values of x .

BUNCHING SYSTEM COMPUTATION

The simulation of the bunching system of the DC350 cyclotron axial injection beam-line [4] was fulfilled by using created 3D version of MCIB04 code [6].

The bunching system consists of linear and sinusoidal bunchers. The linear buncher is placed at 275 cm and sinusoidal – at 80 cm from median plane of the cyclotron. The parameters of the beam are contained in Table. 1.

Table 1: ^{48}Ca beam initial parameters

Injected beam	$^{48}\text{Ca}^{6+}$
Mass, A	48
Charge, Z	2÷8
Injected current, μA	0÷190
Ca beam current, μA	0÷700
He beam current, μA	200
$^{48}\text{Ca}^{6+}$ kinetic energy, keV/u	3.1375
Diameter, mm	8
Emittance, π mm×mrad	142

The initial conditions for the moments were defined at the entrance of the linear buncher and were found by macro-particle simulation. Charge state distribution for ion beam and its self fields were taken into account in this simulation.

The beam focusing is provided by two solenoids. The longitudinal magnetic field of the cyclotron is considered also.

In the simulation all bunchers were replaced by infinitesimal width gap with variable voltage. The dependencies on longitudinal coordinate z (or time) of voltages for two type bunchers have the following form:

$$U(z) = \begin{cases} 2U_0 \frac{z}{\lambda}, & |z| \leq \lambda/2 - \text{linear} \\ 2U_0 \sin(\frac{2\pi}{\lambda}z), & |z| \leq \lambda/2 - \text{sinusoidal} \end{cases} \quad (13)$$

The influence of the bunchers leads to the sudden change of the longitudinal moment $\overline{z\delta}$ and momentum spread at the place of bunchers location:

$$\overline{z\delta} = \overline{z\delta_0} + \frac{z\overline{U}}{2U_{ECR}}; \delta^2 = \delta_0^2 + \frac{z\delta_0}{z_0^2} \frac{z\overline{U}}{U_{ECR}} + \frac{\overline{U^2}}{4U_{ECR}^2} \quad (14)$$

$$\overline{zU} = \int_{-\lambda/2}^{\lambda/2} zU(z)\rho_{||}(z)dz; \overline{U^2} = \int_{-\lambda/2}^{\lambda/2} U^2(z)\rho_{||}(z)dz$$

Here subscript "0" denotes the values of the moments at the bunchers entrance. Besides the longitudinal emittance $\varepsilon_{||}$ increases at the buncher:

$$\varepsilon_{||}^2 = \overline{z^2\delta^2} - (\overline{z\delta})^2 = \varepsilon_{||0}^2 + \frac{z_0^2\overline{U^2} - (\overline{zU})^2}{4U_{ECR}^2} \quad (15)$$

In accordance with Cauchy-Schwarz inequality magnitude $\overline{z^2U^2} - (\overline{zU})^2$ is not negative and is equal to zero in the case of linear buncher. Thereby nonlinearity of the buncher voltage leads to the growth of the longitudinal emittance and minimal achievable beam RMS dimension.

The matching condition at the entrance of the spiral inflector corresponds to the steady state of the beam (without envelopes oscillation) in the uniform magnetic field with magnitude to be equal to the field in the cyclotron center. The amplitude of the voltage at linear buncher was found to provide the equality $k_{\perp} = 2$ at the entrance of sinusoidal buncher. The amplitude of voltage at sinusoidal buncher corresponds to minimum longitudinal beam RMS dimension.

The beam envelopes near spiral inflector of the cyclotron are shown in Fig.5. The voltages at the bunchers for various $^{48}\text{Ca}^{6+}$ beam current are shown in Fig.6.

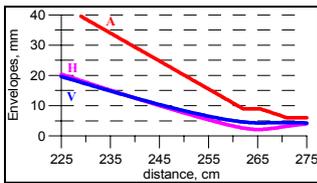


Figure 5: Aperture (A), horizontal (H) and vertical (V) $^{48}\text{Ca}^{6+}$ beam envelopes near inflector.

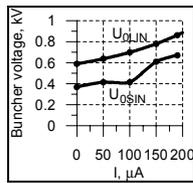


Figure 6: Buncher voltage for various $^{48}\text{Ca}^{6+}$ beam current

Let define the bunching efficiency as ratio of the number of particles within RF phase interval $|\Delta\phi| \leq 15^\circ$ to non-bunched beam one. This quantity shows the possible

increasing of the number of particle captured into acceleration in the cyclotron due to the bunching system.

The dependencies of the bunching efficiency on distance along the beam line for various $^{48}\text{Ca}^{6+}$ beam current are I shown in Fig.7.

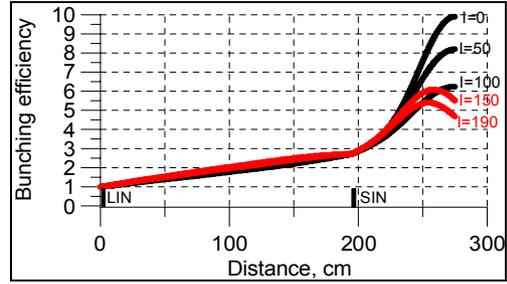


Figure 7: Bunching efficiency along distance. Beam current in μA .

The dependence of the bunching efficiency on the $^{48}\text{Ca}^{6+}$ beam current at the exit of the system is shown in Fig.8.

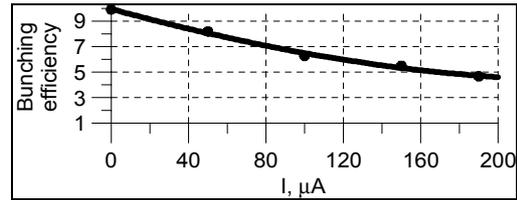


Figure 8: Bunching efficiency versus beam current

REFERENCES

- [1] V. Aleksandrov, N. Kazarinov, V. Shevtsov, Multi-Component Ion Beam code-MCIB04. Proceedings of XIX Russian Particle Accelerator Conference (RuPAC-2004), Dubna, Russia, 2004, p.201.
- [2] N.Yu. Kazarinov, E.A. Perelstein, V.F. Shevtsov. Moments method in charged particle beams dynamics. Particle Accelerators, v.10, 1980, p. 33-48.
- [3] N.Yu. Kazarinov, V.F. Shevtsov. Optimization of transportation channel parameters for beam with big spatial charge. JINR communication, P9-2002-148, Dubna, 2002.
- [4] G. Gulbekyan et al. Axial injection channel of the DC-350 cyclotron. Proceedings of 18-th International Conference on Cyclotron and their Application. 30 September – 5 October, Giardini Naxos, Italy, 2007, MOPPRB13.
- [5] A.W.Chao. Physics of Collective Beam Instabilities in High Energy Accelerators. Wiley Series in Beam Physics and Accelerator Technology, ISBN 0-471-55184-8, New-York, 1993.
- [6] N. Kazarinov, V. Aleksandrov, V. Shevtsov. 3D intense beam dynamics simulation by using moments method. In: Proceedings of VII International scientific workshop to the memory of V.P.Sarantsev., to be published.