STUDY ON SPACE CHARGE EFFECT IN THE SPIRAL INFLECTOR

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Abstract

This paper analyzes space charge effect of the beam in spiral inflector in order to increase the injection efficiency of the compact cyclotron. In the process, we used an ideal cylindrical beam which was approximately the same as the real beam for the calculation of the charged particle dynamics. The beam has azimuthal symmetry and infinite extent in the beam direction. According to the distribution difference of beam density, two modes were used to study the space charge effect, uniform distribution and gauss distribution. As a result, we developed a modified version of CASINO to calculate the space charge effect in the spiral inflector. For double check, a modification of TRACE3D to include the spiral inflector for the injection matching was implemented. The results comparison will be given in this paper too.

INTRODUCTION

In the field of high intensity accelerators the compact cyclotron has drawn an increasing attention due to its extensive and important application in recent years. Spiral inflector is a key part for this kind of machine, but the in-depth study on the space charge effect of the spiral inflector has rarely been carried out so far.

The matrix-based TRACE3D and TRANSOPTR can be easily applied to optics matching of the injection line, but compared to the software based on trajectory tracing, this kind of software is not sufficient to study in detail the beam optical quality of the spiral inflector that is based on numerical analysis including fringe field. However, the currently available software for spiral inflector design using trajectory tracing method could not meet the requirement to calculate the space charge force, including the software of CASINO of TRIUMF. As a result, the existing method is not enough to study the spiral quality in the condition of high beam injection.

Given this limit, we have tried some tentative work of improving on the CASINO to calculate the space charge effect, thus introducing software CASINO_SCE. In fact, Dragan had done some tentative work to solve the space charge effect problem of the spiral inflector in the RIKEN laboratory of Japanese. In his study, he put forward a simplified model of a “straight” cylindrical beam [1][2]. We developed a method to study the space charge effect with uniform distribution and gauss distribution respectively, and we noticed the difference of the result with Dragan’s.

To explain the difference of the two results and to verify our calculation as well, we apply the matrix of the spiral inflector to TRACE3D. In that case, TRACE3D can be used to match the injection line including spiral inflector, and this mended software was named TRACE3D_INF. Based on this work, we test the space charge effect of the CASINO_SCE excluding fringe field with software TRACE3D_INF and TRANSOPTR. By comparing the results, we can conclude that the result of the CASINO_INF is reasonable.

THEORETICAL ANALYSIS OF THE SPACE CHARGE FIELD

To describe particle motion in high current beams, we must have the electric and magnetic fields generated by the particles. So in the space charge effect calculation, we use an ideal cylindrical beam approximate to the real beam (Fig.1). The cylindrical beam can be considered asuniform or gauss distribution, at radius r, beam density ρ, beam intensity I_0, and the maximum beam radius is a. Ideally the cylindrical beam is axially linear, while strictly speaking, the beam should be spiral due to the function of the spiral inflector.

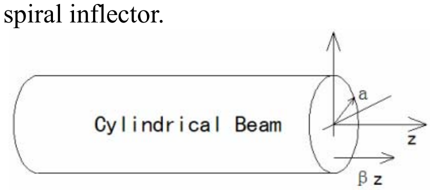

Figure 1: Cylindrical beam propagating in the z direction.

The axial beam that travels along the central ion trajectory through the spiral inflector can be expressed by the formulae[3]：

$$x(b) = \frac{A}{2}\left[\frac{2}{1-4K^2} + \frac{\cos[(2K-1)]b}{2K-1} - \frac{\cos[(2K+1)b}{2K+1}\right]$$

$$y(b) = \frac{A}{2}\left[\frac{\sin[(2K+1)b]}{2K+1} - \frac{\sin[(2K-1)b]}{2K-1}\right]$$

$$z(b) = A - A\sin(b)\ ,\ 0 \le b \le \pi/2$$

Uniform Distribution

The electric field generated by the space charge effect has only a radial component E^{sc} and is given by the formula:

$$E^{sc} = \frac{\rho_0 r}{2\varepsilon_0}\hat{e} = \frac{I_0 r}{2\pi\varepsilon_0 a^2 v_0}\hat{e}_r, r \le a$$

The self-magnetic field B^{sc} has only an azimuthal component and is given by the following formula:

$$B^{sc} = \mu_0 \frac{I_0 r}{2\pi a^2}\hat{e}_\varphi, r \le a$$

where I_0 is the maximum beam current, v_0 is the beam velocity, a is the beam radius, ε_0 is permittivity of vacuum, μ_0 is permeability of vacuum, $\hat{e}_r$ is the unit vector in radial direction and $\hat{e}_\varphi$ is unit vector in azimuthal direction.

Gauss Distribution

When the beam is in gauss distribution, according to gauss distribution formula, we can get:

$$J_0\int_0^a\left(\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{r^2}{2\sigma^2}}\right)2\pi r dr = I_0$$

From the equation above, we can get the parameter J_0:

$$J_0 = \frac{I_0}{\sqrt{2\pi}\sigma\left(1-e^{-\frac{a^2}{2\sigma^2}}\right)}$$

So we can get the beam current at radius r:

$$I = J_0\sqrt{2\pi}\sigma\left(1-e^{-\frac{r^2}{2\sigma^2}}\right)$$

Thus we can get the self-fields equations:

$$E^{sc} = \frac{I}{2\pi r\varepsilon_0 v_0}\hat{e}_r, r \le a$$

$$B^{sc} = \mu_0\frac{I}{2\pi r}\hat{e}_\varphi, r \le a$$

NUMERICAL SIMULATION

Based on the formulae above we modified the program CASINO [3] to calculate the space charge of the spiral inflector by tracing a single particle. In our calculation with CASINO_SCE, we have taken into consideration the spiral inflector which is likely to be used in the 100MeV high intensity cyclotron being constructed at CIAE (CYCIAE-100). For the parameters of the spiral inflector, A=4.0cm, Rm=3.9695cm, k'=-0.74. The electric field distribution in the spiral inflector is an ANALYTIC field, the magnetic field comes from CYCIAE-100, and the injection energy is 40keV. In order to compare the results between uniform and gauss distribution, we choose the full width at half-maximum (FWHM) of gauss distribution as the maximum width of the gauss distribution, the integral area of uniform distribution is equal to the integral area of gauss distribution. In this case, the beam radius of the uniform distribution is 2mm, while the beam radius of the gauss distribution is 3mm, and σ=4.5mm. From these initial conditions, we can get the space charge effect on the particle for the two distributions as is shown in Fig.2, in which the abscissa represents RF time, and the ordinate represents the displacement in the ur and hr directions. Three different values at different beam intensity through the spiral inflector have been chosen in the calculation: I=0mA, 5mA and 10mA.

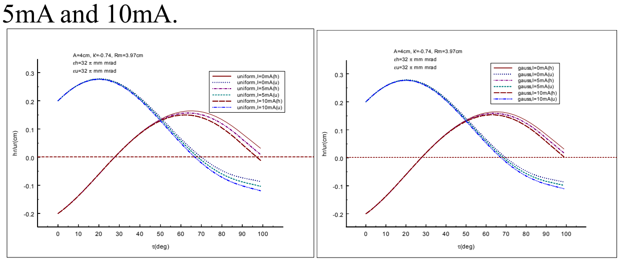

Figure 2: space charge effect of uniform distribution (left) and gauss distribution (right)

It can be seen from the figures that whether for gauss distribution or uniform distribution, the space charge effect is stronger in the hr direction than in the ur direction.

To make it clear, in order to get the difference of the space charge effect between gauss distribution and uniform distribution, we select the beam current I=0mA and 10mA to calculate the two case. Based on the results of Fig.2, we can get the comparison result shown in Fig.3, from which we can conclude that the space charge effect of uniform distribution is stronger than the that of the gauss distribution.

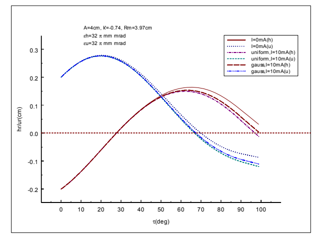

Figure 3: the comparison for space charge effect between gauss and uniform distribution

RESULT COMPARISON WITH DRAGAN'S

Dragan had done some tentative work to solve the space charge effect of the spiral inflector in the RIKEN laboratory of Japanese. In his study, the injection ion is N_{14}^{5+} of which the charge-to-mass ratio is 0.375, and the injection energy is 50keV. The parameters for the selected spiral inflector are: A=2.2cm , Rm=1.6cm , $k' = 0.3$. The electric filed in the spiral inflector is computed from an electric map produced by RELAX3D, and the magnetic field is uniform field with a magnitude of 1.5T. The inflector entrance emittances are $\varepsilon_u = 140\pi mm \cdot mrad$ and $\varepsilon_h = 130\pi mm \cdot mrad$ [2][4]. In his calculation, when the beam current varies from 10μA to 40μA, the maximum displacement in the ur is about Δu_r=0.3mm and the maximum displacement in the ur is about Δh_r=4.8mm.

According to Dragan's inputting parameters, we do the same calculation using our program, and we find there is a difference between the results, as is shown in Table 1.

Table 1: the result comparison of space charge effect

	Dragan	Our calculation
Δh_r (mm)	~0.48	0.00695
Δu_r (mm)	~0.3	0.00155

AMENDMENT OF TRACE3D & VERIFICATION OF SPACE CHARGE EFFECT RESULT

In order to verify the computation of the space charge effect, the optical matching program TRACE3D and

TRANSOPTR are selected. Then the results from TRACE3D and TRANSPOPTR are compared with our calculation, including gauss distribution and uniform distribution. However, since TRACE3D does not have spiral inflector element to match the beam optics of the injection line to the entrance of the central region, we add the spiral inflector element to TRACE3D and this mended software is TRACE3D_INF.

To get the transfer matrix of spiral inflector, the infinitesimal transfer matrix F(s) is introduced in the calculation and it is defined as , where M is the transfer matrix from s to s+ds, I is the identity matrix and s denotes longitudinal position along the transport system [5]. Then we add the transfer matrix M to TRACE3D, and thus it can be used to match the injection linetaking into accounts of the spiral inflector.

Based on the theory introduced above, we incorporate the spiral infector with tilted electrodes and constant magnetic field into TRACE3D, the matrix F[5] for untilted inflector is:

$$F = \begin{pmatrix} 0 & 1 & \frac{C}{2R} & 0 & 0 & 0 \\ -\frac{2}{A^2}+\frac{-C^2}{4R^2} & 0 & -\frac{3S}{2RA} & \frac{C}{2R} & 0 & -\frac{2}{A} \\ -\frac{C}{2\rho} & 0 & 0 & 1 & 0 & 0 \\ -\frac{3S}{2RA} & \frac{-C}{2R} & -\frac{1+3S^2}{4R^2} & 0 & 0 & -\frac{S}{R} \\ \frac{2}{A} & 0 & \frac{S}{R} & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Where A is the electric bending radius (or height) of the inflector. The variables C and S are defined as

$$C = \cos\left(s/A\right), \quad S = \sin\left(s/A\right)$$

R is the magnetic radius of the injected beam.

the matrix F [5] for tilted inflector is:

$$F = \begin{pmatrix} 0 & 1 & -2kC/A & 0 & 0 & 0 \\ F_{21} & 0 & F_{23} & -k'C/A & 0 & F_{26} \\ 2kC/A & 0 & 0 & 1 & 0 & 0 \\ F_{41} & k'C/A & F_{43} & 0 & 0 & 2/A \\ -2kC/A & 0 & -1/A & 0 & 0 & 1 \\ F_{61} & -k'C/A & F_{63} & -1/A & 0 & 0 \end{pmatrix}$$

Where k' the tilt parameter. The variables k is defined as: $k = \left(A/R + k'\right)/2$

The remaining matrix elements in the equations above are:

$$F_{21} = \frac{-S^2}{AR}\left(2k + \frac{k'}{1+k'^2S^2}\right) \qquad F_{43} = -k'SF_{23}$$

$$F_{23} = F_{41} = \frac{-S}{AR\left(1+k'^2S^2\right)} \qquad F_{61} = -(2k+k')\frac{C}{A^2}$$

$$F_{26} = (2k+k')\frac{S}{A} \qquad F_{63} = 2kk'\frac{SC}{A^2}$$

To check the correctness of the matrix, we compare the result of the above mentioned method with that calculated by CASINO. We choose the data of TR30 as our input data, which are A=2.5 , Rm = 1.9 , = -0.83. It can be seen that the two results are in accordance with each other.

Based on TRACE3D_INF and TRANSOPTR, we verify the calculation of the space charge effect, and compare the result with that computed by TRACE3D_INF and TRANSOPTR. The inflector entrance emittances are $\varepsilon_u = 32\pi mm \cdot mrad$ and $\varepsilon_h = 32\pi mm \cdot mrad$, and the beam intensity changes from 1mA to 10mA. Figure 4 show the result of the space charge effect of CASINO_SCE, TRACE3D_INF and TRANSOPTR. In these figures, the abscissa is the beam current I with unit mA, and the ordinate is the offset between the cases with and without space charge effect at the exit, the unit is cm. From the figures we can see that the result of the CASINO_SCE, TRACE3D_INF and TRANSOPTR is accordant with each other, and the space charge effect on the h direction is bigger than the u direction. However, there are still some differences in the three results, because the calculation of CASINO_SCE uses numerical tracing, while TRACE3D_INF and TRANSOPTR use matrix resolution. But there are not any quantity differences among them. The difference also exists in numerical value between TRACE3D_INF and TRANSOPTR, because TRACE3D_INF uses pure analytic matrix, while TRANSOPTR uses Runge-Kutta routine integrate matrix.

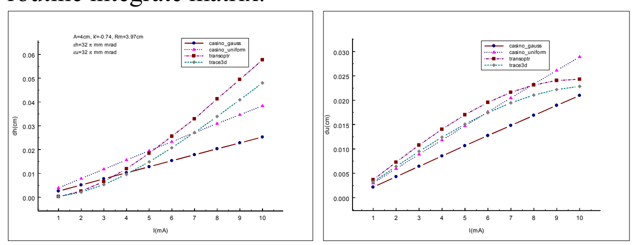

Figure 4: comparison of space charge effect for CASINO_SCE, TRACE3D and TRANSOPTR in h direction (left) and u direction (right)

CONCLUSION

Through the calculation, we can get the following conclusion:

The higher the beam intensity is, the stronger the space charge effect is. The space charge effect is stronger in the hr direction than in the ur direction

In general, the space charge effect of uniform distribution is stronger than the effect of the gauss distribution.

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